

Volume and Volatility in Tokyo Foreign Exchange Market and Their Contributing Factors

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Abstract

We found statistically significant effects of trading volume on volatility in USD/JPY tick data from Tokyo market. Their correlation shows intra-day variations. Their correlation has been an issue that theoretical works have not been successful. It is because it is difficult to straight forwardly investigate microstructure of asset market with tools of equilibrium analysis. These theoretical difficulties have not been much discussed in the microstructure literature. We elucidate them and propose an alternative approach to handle them. It is possible, using our approach, to explain volume and volatility correlation and its intra-day variation. Our approach applies stochastic processes. They are those from renewal theory and Poisson arrivals.

Foreign exchange dealers revise their expectations from time to time intra-day. We describe such revisions of expectation as a sequence of random switches between two states. This Markov process is one of the factors which contribute volume and volatility in FX market. Another contri-

buting factor is demand and supply of FX from macro fundamentals. We categorize effects of these contributing factors. (1) cycle effect: Shorter cycle of expectation revisions increases volume. (2) thin market effect: Expectation revisions give rise to variations in thickness of market. (3) heterogeneity effect: Dispersion of reservation prices makes difference in volatility (4) volume effect: Larger number of Poisson arrivals increases volatility. Using these effects we can explain intra-day variations of volume and volatility relationships.

Key Words : Foreign Exchange Rate, Heterogeneous Expectations, Microstructure, Volume and Volatility

1 Introduction

We found statistically significant effects of trading volume on volatility in USD/JPY tick data in Tokyo market. Their relationships show intra-day variations. "It takes volume to move prices." Karpoff (1987) documents such a link. In stock market, volume and the absolute value of price changes are correlated. However, theoretical works have not been successful to elucidate causes of the links. It is because it is difficult to straight forwardly investigate microstructure of asset market with tools of equilibrium analysis. In the following section we present volume and volatility of five minute intervals in FX market. They change with intra-day seasonality. In the third section, we discuss theoretical difficulties inherent in continuous auctions in asset markets. We propose alternative tools. In the last section we explain how our approach can be applied on empirical observations. We provide an outline of explanation about intra-day variations of volume and volatility correlation.

2 Empirical Observations

2.1 Volume and Volatility in Five Minute Intervals

We found statistically significant effects of volume on volatility in the spot USD/JPY tick data. We divided trading hours into five minute consecutive intervals. For each five minute interval, we calculated a standard deviation of price samples. For each interval, our data consist of daily samples of a pair of standard deviation and the number of transactions. Then we regressed the standard deviations on the sample sizes. Each interval has a regression result. The results show that volume and volatility relationships have intra-day variations. The regression coefficients are all positive except for lunch time period. The lunch time is quite different from other periods. It contains intervals with the maximum and the minimum regression coefficients. Our observations suggest that, since the volume and volatility correlation is not as simplistic as existing literature predicts, we need an alternative model which can reconcile intra-day variations.

We used tick data reported by four brokerage firms in Tokyo from June '95 to April '96. We divide daytime business hours from 8:30 a.m. to 5:30 p.m. into 108 of five minute intervals. For each interval, we calculated standard deviation of transaction prices if the interval of a given date contains at least two samples. In the following, we use a ward, price, rather than foreign exchange rate. What we calculated is standard deviations of prices, not log values. Then for each of 108 intervals, we regressed these standard deviations on number of transactions.

Average number of transactions per five-minute interval is graphed in

Figure 1. Its variations show intra-day patterns. In Tokyo, trading starts around 8:30 a.m. and it surges to a heavy trading session at 9:00 a.m.. It is the heaviest at 10:00 a.m.. The regression coefficients are shown in Figure 2. Although the trading volumes are thus different, the regression coefficients look stable in the entire morning sessions. They also look stable in the active after-lunch session. On the other hand, coefficients for lunch time intervals show more variations from the highest to the lowest which is negative. The regression coefficients are statistically significant and positive, except for lunch time. Figure 3 shows R^2 values. These values vary intra-day. The explanatory power of volume looks comparatively stable in the morning sessions. As for the lunch time, as in the case of the regression coefficients, there does not exist stable R^2 value.

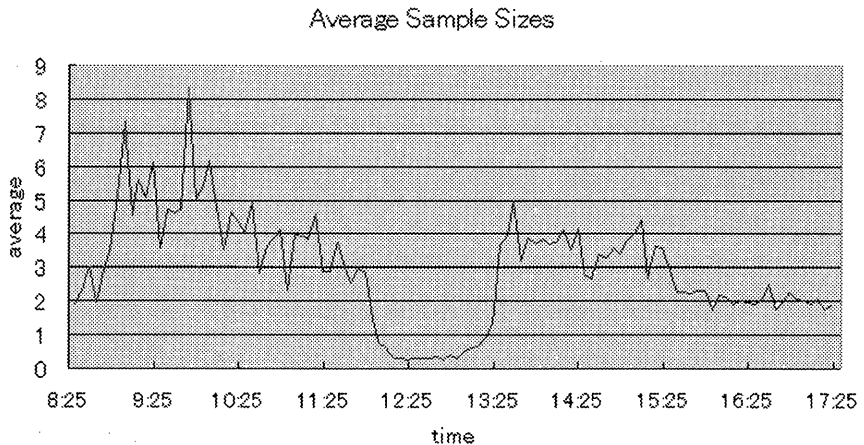


Figure 1 : Average number of transactions per 5 minutes. The averages are taken over 170 days. The ticks were reported by four brokerage firms in Tokyo. An information vendor named "Quick" provides tick data real-time to monitors in dealing rooms. Our data were obtained by downloading from such a subscribed monitor. Our data set covers from June 15, '95 to April 22, '96. Computers occasionally became down. So our data do not contain some of the trading days.

Intraday Variations of Regression Coefficients

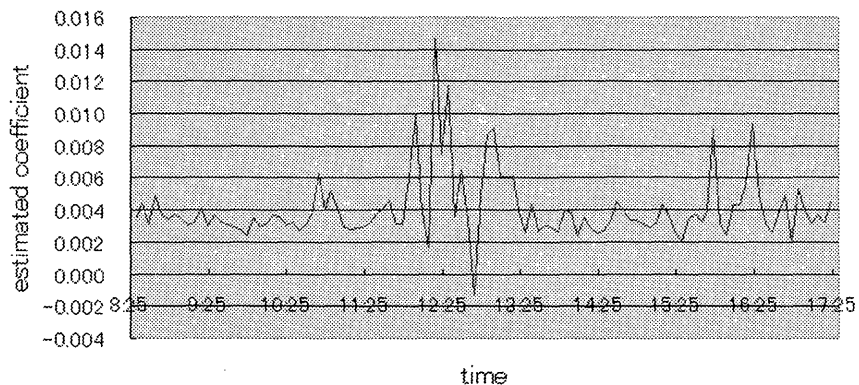


Figure 2 : Intra-day variations in regression coefficients. Standard deviations of transaction prices in 5 minute interval, if it contains at least two samples, are regressed on number of these samples.

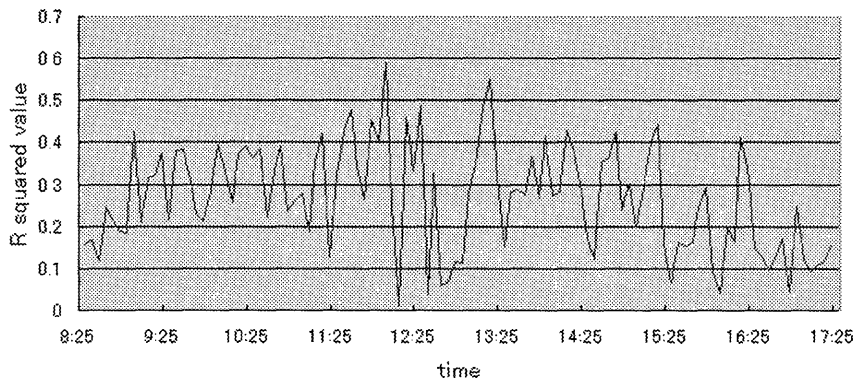
Intra-day Variation of R^2 

Figure 3 : Intra-day variation in R squared values

2.2 Five Periods of Intra-day Variations

We divide usual business hours into five periods as is shown in Table 1: M1, M2, L, A1, A2. Each of them has five minute interval's regression results. The average results are shown in Table 1. Period M2 and A1 have similar values for three items in Table 1. Also M1 has similar values of regression coefficients and R^2 . This implies that, as the trading volume increases, so does volatility. It also implies that the volatility increases, on average, at the same rate in these periods. This rate stays the same even though M1's volume is as big as 1.5 times of M2 and A1. The regression coefficient becomes larger in period A2. Meanwhile, R^2 value of A2 falls to about a half of M1, M2 and A1. During period A2, transaction volume becomes smaller toward evening. An average sample size per five minutes is almost half of M1.

Table 1 : Five Intra-day Trading Sessions

name of period	time	average of regression coefficients	average of R^2 values	average of sample size per 5 min.
M1	9:00 a.m. - 10:30 a.m.	0.0033	0.31	5.21
M2	10:30 a.m. - 12:00 noon	0.0035	0.32	3.51
L	12:00 noon - 1:30 p.m.	0.0062	0.29	0.55
A1	1:30 p.m. - 3:30 p.m.	0.0033	0.30	3.66
A2	3:30 p.m. - 5:30 p.m.	0.0042	0.16	2.10

It is desirable that FX microstructure models can explain observed intra-day variations of volume and volatility correlation, not just predicting sign of correlation coefficient. It is possible to do it by using our approach. We consider stochastic processes inherent in brokered FX auctions. Among stochastic mechanisms, we identify factors to influence volume and

volatility. Causes and effects are categorized as follows. There are four effects. The FX trading volume comes from two sources. One is due to difference in dealers' evaluation. The other comes from economy's fundamentals. This latter trading volume tends to increase the volatility (volume effect). Its effect on volatility depends on heterogeneity of expectations among FX dealers (heterogeneity effect). The volume effect can be cancelled out or amplified by the heterogeneity effect which can vary day by day. The heterogeneity effect lowers explanatory power of trading volume on volatility. The number of dealers who actively engage in trading is not constant through the day. Lunch time is the typical example. Thickness of the market changes intra-day. Thickness influences (thickness effect). Our model presupposes that FX dealers revise their intra-day expectations from time to time. As they revise their expectations, they trade to adjust their positions accordingly. A cycle of such expectation revisions could change its length. If the cycle becomes shorter, then transactions increase. Faster revisions increase volume but they do not change the expected number of dealers who actively trade. Hence volatility stays the same as before. In this case volume changes without volatility change (cycle effect).

3 Theoretical Framework

3.1 Volume and Volatility in Existing Literature

The existing microstructure literature predicts either positive or non-positive correlation between volume and volatility. The works have been mainly concerned with stock market. Often cited Admati and Pfleiderer (1988) deduces positive correlation between volatility and volume. As for FX market, Lyons (1997) predicts rather negative correlation. Lyons (1997)

provides a model called “simultaneous trade model of hot potato trading.” This model describes multiplier like effects of random fluctuations in order flow. An initial change go through dealer’s inventory controls and becomes multiple. Lyons (2001) says “trades are more informative when trading intensity is low”. Lyon’s “hot potato model” is clear-cut but it explains only one of causes of volume variability.

Lyons (1996) calls two existing hypotheses “hot potato view” and “event-uncertainty view.” According to event-uncertainty view, “trades are more informative when trading intensity is high.” While new information proliferates, informative transactions surge and an increased volatility accompanies. Inherent problem with such information based models is that being informative is not clearly defined. You may provide a qualitative structure for exogenous shocks to set prices. Still, traders’ quantitative interpretations and reactions do matter to set actual prices. Information based models have not formulated links between news and interpretation.

3.2 Difficulties to Model Price Formation in FX Market

There are difficulties to apply equilibrium analysis on the price formation process in FX market. The first of all, the auction is continuous. There is no specific length of time to define demand and supply. It is not clear which is equilibrium among a series of transaction prices. Nor is there a next period which “true value” manifests itself. The definition of equilibrium has to be explicitly defined by models, if their analyses require equilibrium. Microstructure models of information approach have not been satisfactory with this regard.

Continuous auction like in FX market lacks specific points in time that

demand and supply are submitted and hence that expectations are formed on. Our approach takes advantage of a peak or a trough of the expected time path. The expected time path may have peaks and troughs. Among them, the nearest in the future determines dealer's present action. As he takes advantage of up or down one by one, his profits increase.

The second difficulty is "a contest of guessing a winner of a beauty contest" situation. Various financial asset prices share this situation as in Keynes (1936). In this contest on contest, you vote for one of the contestants. You win if you have voted for the most popular one which collects the largest share of votes. If you are to predict the contest winner, you must consider how people's preferences disperse. Your own preference becomes irrelevant. You have to think of the average before you vote. More specifically you have to think of the mode of dispersed preferences. However, you may not vote at this stage. Guessing of others may go up to the next stage. You know that other people also are guessing the dispersion of opinions and that their votes also depend on "the perceived mode" just like you do. Hence, you will move on to trying to figure out a dispersion of "the perceived mode". Such process of guessing each other may continue to higher stages.

When you make a financial decision, you face the similar situation. In stead of standard of beauty, the key variable depends on transaction purpose. The reservation price could be a counterpart to the contest winner. Your relative bullishness is important. You look around. You try to guess dispersion of the reservation prices. Thus it is similar. However, in the financial market, choosing reservation price involves more complication. Difficulty of logical consistency arises. The trader's reservation price must

be an optimizing solution of some kind of model. This model, one way or another, must have price determination mechanism. But price determination itself relies on the perception on the reservation prices. For the financial market, an exogenous mechanism to set final value is not a truism. On the other hand, in case of the contest to guess a winner, you can simply start with a principle of the beauties in the eyes of beholders. The dispersion of preferences are exogenously given.

It merits to summarize this beauty contest problem in our context. Reservation prices are heterogeneous among dealers. Everyone takes this into consideration. He conjectures a distribution function which describes the dispersion of reservation prices. Using his own model of price determination, he chooses his reservation price. He knows every dealer is acting very much like him. However, he does not go up to the higher stage of considering what others have in mind. We assume he stops because it is not worth. This process is not as simple as considering the mode of "the conceived mode." The distribution function of the reservation price at a given time is random variable and is not directly observable. Dealers use their conjectured distribution function. Together with other conjectured variables and their own price determination models, they choose their reservation price. Hence there are multiple uncertain factors. The parameter estimates are very rough. Our dealer thinks he would not gain further precision. He does not go on to estimate "the mode of the conceived mode".

The beauty contest problem in our model, more specifically, takes a following form. A key variable is an expected peak or trough in the nearest future. Dealer's action hinges on it. Let x_j be this extremum of the j th

dealer's. This value is different among dealers. Random variable x_j 's have a density function $h(x)$. Together with this density, excess demand for the foreign exchange from the fundamentals also determines transaction price $x(t)$. For a given density $h(x)$, an arrival process of excess demand determines expected time path of transaction prices. The change in arrival pattern results in the peak or trough. How far price moves until then depends on the distribution of reservation prices. Dealers look around to figure out this distribution function. They can only conjecture it. As for excess demand from the fundamentals, dealers observe only its fraction. Based on such information, they choose reservation prices individually. Their actual dispersion may or may not coincide with the perceived density functions. Calibration is not easy. Hence, the heterogeneity of reservation prices does not vanish even after transactions take place.

Difficulty to know the distribution of reservation prices does not dissipate even if the market is more "more transparent." The dealers submit limit orders to the unique broker. The broker keeps this order list secret. Exception is a pair of the market's bid and ask, which are the best buying and selling prices. The broker keeps announcing these bid and ask. Thus the market is not "transparent." So the dealers have to estimate distribution of the limit order prices. Even if these prices are disclosed, it does not help to estimate the distribution of reservation prices. The list of limit orders does not necessarily reveal true perceptions. It is because there are possibilities of bluffing, which is an attempt of manipulation.

3.3 Characterizing Heterogeneous Opinions

In continuous auction, there is not specific point in time to compare all the traders' expectations. To identify difference, we compare the first peak

and trough of expected time path. Such an extremum determines dealer's present action. If a given dealer predicts upward price move, he would try to hold long position with the hope of selling his inventory later at profit. If we introduce risk neutrality, then the expected peak of transaction price will equal to his reservation price to sell. He will place limit order with that price. We assign a distribution function to these extrema. With risk neutrality assumption, these extrema become reservation prices. Thus we introduce a distribution function of reservation prices.

3.4 Two Sources of Order Flow

Change in trend in expected price occurs when parameters of order flow change. Knowing chronological arrivals of the order flow leads to the expected extremum. Large retail transactions with multinational corporations are examples with substantial influence. We consider order flow generating process in the following way. We introduce "*arrivals of buyers and sellers*" as in Garman (1976) and Amihud *et al* (1980). Arrivals of buyers and sellers play a role of demand and supply in a context of continuous auction. Prices increase if buyers arrives at the market faster than sellers. What is different from these preceding models is that we distinguish two sources to generate arrivals; one inside of the market and the other outside. The first source is dealers' revising expectations. The second source is re-

Table 2 : Switches between Two States of Expectation

state	expectation on price	chosen action
state0	not confident enough	maintain square position (zero inventory)
state1	have enough confident expectation on intra-day time path	construct open position upon entry; quote limit order price and wait to seek capital gain

tail transactions with customers. Their net value is excess demand from the fundamentals. These two sources generate stochastic order flow with different characteristics. Distinguishing them makes it possible to explain the volume and volatility intra-day variations and further to explain change in their correlation's sign.

We construct a process of revising expectation as follows. Dealers switch two states of expectation. In state 0, they are not confident enough to assume open position. They try to maintain square position, i.e. zero inventory. In state 1, they are confident enough in their expectations. They seek intra-day capital gains. They choose to have open positions based on the expected nearest peaks and troughs. Once they create open positions, they submit limit orders whose prices coincide with these peaks or troughs. They plan to close their positions at these prices with profits.

Switches of expectation states generate order flow. We have arrivals of buyers and sellers who hit the market's bid and ask. In our model, revising expectation always takes a form of switching expectation state. As the state changes, a dealer's optimal position changes. He tries to adjust. Immediately he hits one of the market rates. It depends on his reservation price and the market rates whether he is a buyer or seller. As he enters state 1, he picks a value for the extrema randomly. He compares this value and the market's bid and ask. Then he hits bid or ask. Only exception is the case where his value falls between the market rates. Had this occurred, he does not hit the market rate. Instead he quotes both bid and ask. Exiting state 1 also generate arrivals of buyers and sellers who hit the market rates. If he is in state 1, he is very likely to have open position. As he exits state 1, he has to trade to close his position. He hits one of the market

rates. Exceptional case is the same as before. His limit order prices coincide the market rates and he has square position. This situation takes place again with small probability. Thus each time when switching of expectation occurs, a buyer or seller arrives and hits the market rate almost always.

The second source to generate the order flow is economy's fundamentals. Demand and supply of foreign currency by macro economy first appear as dealers' retail transactions. Then arrivals of retail transactions immediately change into order flow in wholesale market. This immediate transformation occurs as follows. If a dealer has a retail transaction, he counterbalances it in the wholesale market right away. He acts in this way whether he is in state 0 or state 1. The reasons are as follows. If he is in state 0, he does not want to have open position. So he counterbalances the retail transaction. If he is in state 1, he must have optimal open position. The retail transaction perturbs it. He counterbalances the retail transaction immediately as a consequence of the following assumptions. First, the dealer is risk neutral. Second, a quantity of transaction is one unit for both retail and wholesale transactions. Third, the maximum open position allowed to assume is also one transaction unit. This regulation is imposed exogenously by his employer. If he chooses to have open position, the optimal position must be at its maximum since he is risk neutral. He must have constructed it already. If he has a retail transaction then, his position deviates from this desired level. He may not be able to have more inventory because the constraint is binding. Or alternatively his inventory level may fall below the optimal. In either case, he tries to recover the desired position immediately. He counterbalances the retail transaction by hitting one of the market rates. Thus whether he is in state 1 or not, an arrival of retail transaction changes into an immediate arrival in the market. The exceptional cases have small

probabilities.

The dealers observe only their own retail transactions. Aggregated figures are not publicly available. Also the distribution of reservation prices is not observable. Although some dealers may communicate themselves, it still does not cover the entire market. Order flow is observable. However dealers cannot distinguish its source of generation. Due to limitation of information, dealers come to have different reservation prices. Heterogeneity does not vanish.

3.5 Approximation by Median of Quoted Prices

Different opinions give rise to transactions. With this reason, dealers trade and have open positions. This takes place without order flow from the fundamentals. They create open positions. Thus created positions satisfies accounting identity. They are net zero. We take advantage of this identity. We approximate movement of transaction prices by a median of limit order prices. Together with assumptions of identical limit on the open position and one unit of the transaction quantity, the number of dealers with long positions is the same as those with short positions.

Transaction always takes place at the one of the market rates. This means, together with the accounting identity, that we can track the transaction price by the median. If we consider the simplest case such that only expectation switches generate order flow, the last transaction price coincides, exactly or close to, 50 percent quantile of limit order prices. If the number of dealers who are quoting prices is odd, the price exactly coincides 50 percent quantile.

If odd number of dealers submitted orders, a dealer whose reservation

price is the median must have zero inventory. He must be quoting both buying and selling prices. His prices are the market bid and ask. We assume here that bid/ask spread is negligible if they are quoted by one dealer. If an arrival occurs, the transaction price must be his price, which is the median. If the number of dealers is even, the transaction price will be equal to one of the market rates. Although they are not the median itself, they are close to it. Therefore we can approximate the transaction prices with the median of limit order prices. The limit order prices are equal to reservation prices under our model assumptions. We substitute the median of reservation prices for the transaction prices and analyze the median's movements.

The open position identity still holds when excess demand is not zero. Excess demand is aggregated retail transactions. They do not balance most of time intra-day. Dealers as a whole absorb excess demand. It implies that the net position of dealers is equal to (negative of) excess demand. Again we can use this accounting identity to characterize random move of transaction prices. As the dealers absorb excess demand, the transaction price deviates from the median. Excess demand determines how many dealers have to reverse positions. For a given necessary number of dealers, how far prices have to change depends on the distribution of reservation prices. We analyze the median, or its modified value in order to characterize transaction price volatility.

3.6 Contributing Factors of Volume and Volatility

Our regression results show that volume has statistically significant positive effect on volatility. However volume's explanatory power remains small. Also we found that coefficients are intra-day non-stationary. We

consider that empirical data correspond to various parameter values of contributing factors. In the following we sort out their effects. They are summarized in Table 3.

Table 3 : Effects on Volume and Volatility

factor	volume	volatility
shorter cycle of switches of expectation states	+ (cycle effect)	no change if ratio of those who in state 1 unchanged
smaller ratio of those who in state 1	no change if cycle length unchanged	+ (thickness effect)
increase in retail transactions	+	+ (volume effect)
more heterogeneous expectations	no change	+ (heterogeneity effect)

The process of revising expectation is the random switch between two states. Sojourn time in each state follows exponential distribution. Let θ_0 and θ_1 be their parameters. The expected sojourn time in state j is given by $\frac{1}{\theta_j}$. A ratio of dealers who are in state 1 is given by $\frac{\theta_0}{\theta_0 + \theta_1}$. This is an asymptotic result given in Ross (1997). Larger value of θ_j makes the switching cycle shorter. As it becomes shorter, trading volume increases. If two parameters increase proportionately, the expected ratio of dealers in a state remains unchanged. Then volume increases without added volatility. We call such effect of cycle length “**cycle effect.**”

On the other hand, if the ratio of two states changes, whether the cycle length changes or not, it has an effect on volatility. If the ratio of state 1 decreases, the number of limit order decreases. Then volatility increases. There are two reasons. First, the median of the smaller samples has larger

variance. Second, a given amount of excess demand, larger change in price is necessary. Thus the change in the ratio of state 1 influences volatility through the thickness of the market. We call it "**thickness effect**" as in Table 3. As far as the switching expectation state is concerned, its influence on volatility occurs only through the expected number of dealers in state 1.

Retail transactions is another contributing factor. If the expected retail transactions increases, volatility increases, *ceteris paribus*. This causality is deduced as one of the characteristics of Poisson arrival process. Prices fluctuate as buyers and sellers arrive unevenly. In other words, the net buyer's arrivals change the price. Two sources generate order flow. However, as far as uneven arrivals concern, only retail transactions contribute volatility change. The net buyer's arrivals due to retail transactions is defined excess demand. It is a random variable which is defined as the difference of two random variables. We assume the arrivals of retail buyers and sellers are respective Poisson process. Let λ_b and λ_s be their parameter. Poisson arrivals have neat characteristics. Firstly the variance and the expected value of the number of arrivals are equal. Hence, λ_j for $j = b, s$ is expected value and also variance. Secondly a sum of Poisson arrivals is also Poisson. Variance of this sum is given by $\lambda_b + \lambda_s$. Since retail transactions becomes order flow in the market immediately, order flow they generate are also Poisson. Excess demand is a random variable defined as difference of two Poisson arrivals. This random variable converges to Normal distribution as shown in Johnson *et al* (1993). Its variance is given by $\lambda_b + \lambda_s$. Therefore, as the retail transactions' expected volume increases, the variance of excess demand increases. Hence, volatility also increases. We call this "**volume effect**" of the retail transactions.

Dealers as a whole absorb excess demand. For a given excess demand, dealers reverse their positions and absorb it. During this process, the limit order prices are hit and prices change. How far prices have to change depends on the distribution of limit order prices. As they scatter in a wider range, the price change will be larger. As dealers have more heterogeneous reservation prices, transaction prices will move more for a given excess demand. We call this “heterogeneity effect.”

Figure 4 describes categorized effects. First suppose only the expectation state switching generates order flow. Let point *A* signify a given pair of a standard deviation of a median of limit order prices and an expected value of trading volume per unit time. The median is a substitute for the transaction price. So point *A* indicates a pair of the median’s volatility and the volume due to switching. Next we introduce excess demand. For a

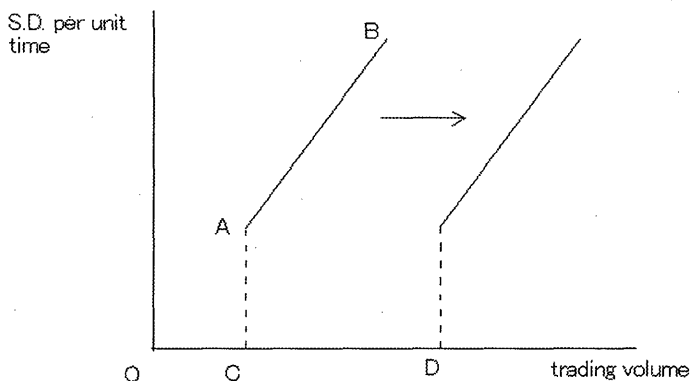


Figure 4 : Consequence of Cycle Effect. A slope of segment *AB* is determined by heterogeneity effect and thickness effect. Cycle effect shifts segment *AB* horizontally. If renewal cycle becomes shorter, cycle effect shifts the segment *AB* outward. Segment *OC* is volume generated by switching expectation states. segment *CD* is generated by retail transactions.

given number of limit orders, volatility increases as order flow due to retail transactions is added. Heterogeneity and thickness effects change the slope of line AB . Sample pairs of volatility and volume per unit time will scatter along line AB . Volume effect creates samples of larger volume and volatility. Cycle effect shifts line AB samples horizontally.

4 Empirical Application and Conclusion

Our model can explain empirical observations reported in Table 1. Late afternoon period $A2$ has a steeper regression line and lower R^2 value compare with $M1$, $M2$ and $A1$. Late afternoon, toward evening, pursuing capital gains come to require overnight open position hence requires more of confidence. So, in $A2$, dealers exit state 1 more than entering it. Sojourn time parameter θ_0 decreases while θ_1 increases. An expected sojourn time in state 1 becomes shorter. As a result, the number of limit orders decreases. This leads to thickness effect. At the same time, we have cycle effect. Figure 5 shows consequences of the thickness and cycle effects. During period $A2$, shift of volume and volatility line is taking place. Samples from five minute intervals accordingly shift too. If we do regression on these data, we would have regression result like line CD . We would have a larger regression coefficient and a smaller R^2 value, compare with $M1$, $M2$ and $A1$. This explains what we report about $A2$ in Table 1. Thus it is possible, using our approach, to explain volume and volatility relationships in foreign exchange market.

We used spot USD/JPY tick data. We divided trading hours into five minute intervals. Regression results show that trading volume in a five minute interval increases its volatility. Degree of such tendency depends on

time of the day. It varies with intra-day patterns. It is possible, using our approach, to explain the volume volatility correlation and its intra-day patterns. These are issues that existing microstructure literature has failed to reconcile. It is because continuous auctions in asset markets have theoretical difficulties. On those markets, equilibrium analysis cannot be straight forwardly applied. Instead, we introduce stochastic processes as analytical tools. Using our approach, we elucidate contributing factors of volume and volatility. Further we can explain intra-day of their correlation. Our approach looks promising. It can be applied on other financial markets. One of the issues for the future research is to formulate stochastic structure of auctions in a more rigorous manner.

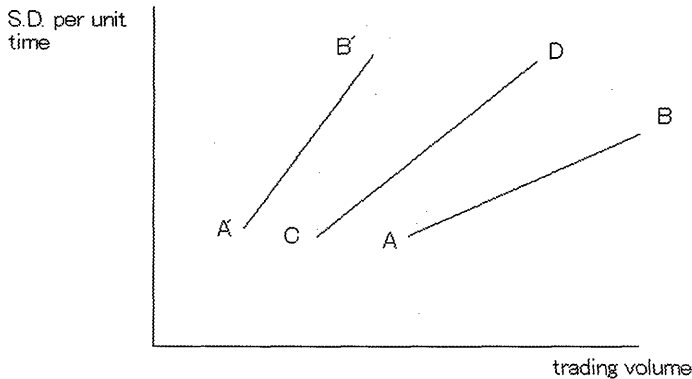


Figure 5 : Shift of Volume Volatility Line during Late Afternoon. In late afternoon period of A2, the cycle and thickness effects occur at the same time. Volume volatility line gradually moves to $A'B'$. If we regress samples scattered in the shifting area, you will have a result like line CD . Its slope is higher than the one for the previous periods.

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A Regression Results

About each 5 minute intervals, standard deviations of transaction prices are regressed on the number of transactions. Null hypothesis $\beta=0$ is tested. In most cases, coefficients are positive and their t-values are significant with 5% significance level. Exceptions are about half of lunch time period L, and two cases in late afternoon period A2. We use (*) to indicate cases such that null hypothesis cannot be rejected with 5% level.

Table 4 : Five Minute Intervals in M1

starting time	Regression Coefficient	t-Value	number of days
8:30	0.003594	4.143	94
8:35	0.00441	4.677	109
8:40	0.003129	4.044	124
8:45	0.004877	5.249	86
8:50	0.003658	5.576	118
8:55	0.003437	5.356	126
9:00	0.003784	5.999	162
9:05	0.003446	11.010	165
9:10	0.003042	6.338	153
9:15	0.003277	8.543	159
9:20	0.004087	8.508	154
9:25	0.003081	9.796	162
9:30	0.003715	6.408	150
9:35	0.003326	9.730	155
9:40	0.003142	9.649	151
9:45	0.00294	8.352	159
9:50	0.002786	7.040	167
9:55	0.002392	6.627	166
10:00	0.003526	7.891	158
10:05	0.002989	10.250	162
10:10	0.003193	8.666	153
10:15	0.003667	6.859	137
10:20	0.003646	9.696	159
10:25	0.003008	9.718	150

Table 5 : Five Minute Intervals in M2 and L

starting time	Regression Coefficient	t-Value	number of days
10:30	0.003248	9.017	144
10:35	0.00276	9.749	155
10:40	0.003137	5.986	126
10:45	0.003848	8.164	136
10:50	0.006304	9.392	139
10:55	0.004035	6.699	147
11:00	0.005266	6.203	112
11:05	0.004161	7.416	146
11:10	0.002972	5.657	140
11:15	0.002806	8.833	145
11:20	0.002848	10.220	145
11:25	0.002945	3.937	110
11:30	0.003239	7.551	121
11:35	0.003751	9.966	134
11:40	0.004124	10.300	118
11:45	0.004617	7.219	102
11:50	0.003149	6.238	110
11:55	0.003088	9.324	107
12:00	0.005939	6.232	60
12:05	0.009957	6.373	30
12:10	0.003856	2.570*	22
12:15	0.001630	0.287*	9
12:20	0.014650	2.609	10
12:25	0.007473	2.002*	10
12:30	0.011680	2.174*	7
12:35	0.003589	0.694*	14
12:40	0.006545	2.201	12
12:45	0.003364	0.748*	11
12:50	-0.001363	-0.660*	8
12:55	0.004954	1.363*	16
13:00	0.008685	1.121*	12
13:05	0.009100	2.913	24
13:10	0.006146	3.085	20
13:15	0.006138	4.568	24
13:20	0.006125	5.874	30
13:25	0.003918	4.710	51

Table 6 : Five Minute Intervals in A1

starting time	Regression Coefficient	t-Value	number of days
13:30	0.002650	5.013	143
13:35	0.004450	7.460	145
13:40	0.002740	7.898	156
13:45	0.003071	6.985	131
13:50	0.002960	9.157	147
13:55	0.002616	7.064	139
14:00	0.003995	10.120	147
14:05	0.003856	7.206	139
14:10	0.002417	7.242	137
14:15	0.003522	10.290	143
14:20	0.002906	9.196	141
14:25	0.002540	7.465	141
14:30	0.002692	5.007	122
14:35	0.003259	4.176	125
14:40	0.004519	8.138	124
14:45	0.003998	8.603	132
14:50	0.003390	9.915	134
14:55	0.003405	6.678	144
15:00	0.003099	7.979	148
15:05	0.002967	6.197	156
15:10	0.003110	7.965	156
15:15	0.004325	8.924	120
15:20	0.003630	10.580	142
15:25	0.002581	5.008	137

Table 7 : Five Minute Intervals in A2

starting time	Regression Coefficient	t-Value	number of days
15:30	0.002049	2.888	124
15:35	0.003472	4.330	100
15:40	0.003807	4.247	101
15:45	0.003289	4.408	103
15:50	0.003876	5.680	102
15:55	0.009032	6.294	98
16:00	0.003112	2.934	85
16:05	0.002471	2.067*	103
16:10	0.004330	4.875	100
16:15	0.004313	4.237	92
16:20	0.005644	7.996	93
16:25	0.009386	7.177	100
16:30	0.005005	3.974	94
16:35	0.003164	3.685	92
16:40	0.002638	3.398	109
16:45	0.003823	3.426	85
16:50	0.004895	4.322	93
16:55	0.001997	2.139*	103
17:00	0.005327	5.692	101
17:05	0.004016	3.662	99
17:10	0.003235	3.051	95
17:15	0.003682	3.246	90
17:20	0.003206	3.079	73
17:25	0.004593	3.950	86