The Reverse Carleson Measure On The Bergman Spaces And Closed Range

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Abstract

We study the multiplication operators and the integration operators with closed range on the Bergman spaces. And we get the new characterization of the Reverse Carleson measure on the Bergman space by using the sampling property.

Key Words and Phrases: reverse Carleson measure, sampling set, integration operator, Bergman space, Hardy space, closed range, bounded below.

Let D be the open unit disk in complex plane C. For $z, w \in D$, $0 < r < 1$, let $\varphi_z(w) = \frac{z-w}{1-\bar{z}w}$ and let $\rho(z, w) = \left|\frac{z-w}{1-\bar{z}w}\right|$ and $D(w, r) = \{z \in D, \rho(w, z) < r\}$. Let $H(D)$ be the class of all analytic functions on D.

The space $\mathfrak{B}_\alpha$ of D is defined to be the space of analytic functions f on D such that

$$\|f\|_{\beta_\alpha} = |f(0)| + \|f\|_{\mathfrak{B}_\alpha} < +\infty,$$

where $\|f\|_{\mathfrak{B}_\alpha} = \sup_{z \in D} (1-|z|^2)^\alpha |f'(z)|$. Note that $\mathfrak{B}_1 = \mathfrak{B}$ is the Bloch space.

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The space $\mathfrak{B}_{\alpha,0}$ of D is defined to be the space of analytic functions f on D such that

$$(1-|z|^2)^{\alpha}|f'(z)| \to 0 \quad (|z| \to 1^-).$$

Note that $\mathfrak{B}_{1,0}=\mathfrak{B}_0$ is the little Bloch space.

The space $\mathfrak{B}^{\alpha}$ of D is defined to be the space of analytic functions f on D such that $\sup_{z\in D}(1-|z|^2)^{\alpha}|f(z)| < +\infty$.

Let $dA(z)$ be the area measure on D normalized so that the area of D is 1.

For $\alpha > -1$, the weighted Dirichret space D_p^{α} is defined to be the space of analytic functions f on D such that

$$\int_D (1-|z|^2)^{\alpha}|f'(z)|^p(\alpha+1)dA(z) < +\infty.$$

In the case of $\alpha=1$ and $p=2$, then $D_2^1=H^2$ is the Hardy space. In the case of $\alpha=2$ and $p=2$, then $D_2^2=L_a^2$ is the Bergman space.

For $\alpha > -1$, the weighted Bergman space $D_\alpha^p = L_a^p((1-|z|^2)^{\alpha}dA(z))$ is defined to be the space of analytic functions f on D such that

$$\int_D (1-|z|^2)^{\alpha}|f(z)|^p(\alpha+1)dA(z) < +\infty.$$

In the case of $\alpha=0$ and $p=2$, then $D_0^2=L_a^2$ is the Bergman space.

Let X be Banach spaces and let T be a linear operator from X into X. Then T is called to be bounded below on X if $\|Tf\| \geq C\|f\|$ for all $f \in X$ and positive constants $C>0$.

For g analytic on D, the operators I_g, J_g, M_g are defined by the following:

$$I_g(f)(z)=\int_0^z g(\zeta)f'(\zeta)d\zeta,\ J_g(f)(z)=\int_0^z f(\zeta)g'(\zeta)d\zeta,\ M_g(f)(z)=g(z)f(z).$$

If $g(z)=z$, then J_g is the integration operator. If $g(z)=\log\frac{1}{1-z}$, then J_g

is the Cešro operator.

For φ holomorphic self-map of D, the composition operator C_φ is defined by $C_\varphi(f)=f\circ\varphi$. Let $G_\epsilon=\varphi\left(\left\{z\in D, \frac{(1-|z|^2)|\varphi'(z)|}{1-|\varphi(z)|^2}\geq\varepsilon\right\}\right)$. By Schwarz-Pick lemma, the operator C_φ is bounded on the Bloch space $\mathfrak{B}$. It follows from Littlewood's subordination theorem that the operator C_φ is bounded on all the Bergman spaces. In [4], P. Ghatage and D. Zheng and Nina Zorboska determined the composition operators on the Bloch space with closed range using a sampling set G_ϵ for the Bloch space.

In [8], Ch. Pommerenke proved the following result with respect to the operator J_g:

Theorem 0.1. *For g analytic on D, the operator J_g is bounded on the Hardy space H^2 if and only if $g\in BMOA$.*

In [2], A. Aleman and A. G. Siskakis proved the following result with respect to the operator J_g:

Theorem 0.2. *For g analytic on D, for $p\geq 1$, the operator J_g is bounded on the Hardy space H^p if and only if $g\in BMOA$. And the operator J_g is compact on the Hardy space H^p if and only if $g\in VMOA$.*

In [3], A. Aleman and A. G. Siskakis proved the following result with respect to the operator J_g (See [3] with respect to the definition of the weighted Bergman spaces $L^p_{a,\omega}(D)$):

Theorem 0.3. *Let $p\geq 1$. Then for g analytic on D, the operator J_g is bounded on the weighted Bergman space $L^p_{a,\omega}(D)$ if and only if $g\in\mathfrak{B}$.* And *the operator J_g is compact on $L^p_{a,\omega}(D)$ if and only if* $\lim_{|z|\to 1^-}(1-|z|^2)|g'(z)|=0$.

In [11], we also proved the following result:

Theorem 0.4. *Let $\beta \geq 1$. Then the operator J_g: $\mathfrak{B} \to \mathfrak{B}_\beta$ is bounded if and only if*

$$\sup_{z \in D} (1 - |z|^2)^\beta \left(\log \frac{1}{1-|z|^2} \right) |g'(z)| < +\infty,$$

and the operator J_g: $\mathfrak{B} \to \mathfrak{B}_\beta$ is compact if and only if

$$\lim_{|z| \to 1^-} (1 - |z|^2)^\beta \left(\log \frac{1}{1-|z|^2} \right) |g'(z)| = 0.$$

And let $\alpha > 1$. Then the operator J_g: $\mathfrak{B}_\alpha \to \mathfrak{B}_\beta$ is bounded if and only if

$$\sup_{z \in D} (1 - |z|^2)^{\beta - \alpha + 1} |g'(z)| < +\infty.$$

And the operator J_g: $\mathfrak{B}_\alpha \to \mathfrak{B}_\beta$ is compact if and only if

$$\lim_{|z| \to 1^-} (1 - |z|^2)^{\beta - \alpha + 1} |g'(z)| = 0.$$

And let $0 < \alpha < 1$, and $\alpha \leq \beta$. Then the operator J_g: $\mathfrak{B}_\alpha \to \mathfrak{B}_\beta$ is bounded if and only if $g \in \mathfrak{B}_\beta$.
And the operator J_g: $\mathfrak{B}_\alpha \to \mathfrak{B}_\beta$ is compact if and only if

$$\lim_{|z| \to 1^-} (1 - |z|^2)^\beta |g'(z)| = 0, \quad i.e. \ g \in \mathfrak{B}_{\beta,0}.$$

In [12], we also proved the following result:

Theorem 0.5. *Let $\beta \geq \alpha > 0$. Then the operator I_g: $\mathfrak{B}_\alpha \to \mathfrak{B}_\beta$ is bounded if and only if*

$$\sup_{z \in D} (1 - |z|^2)^{\beta - \alpha} |g(z)| < +\infty.$$

And the operator I_g: $\mathfrak{B}_\alpha \to \mathfrak{B}_\beta$ is compact if and only if

$$\lim_{|z| \to 1^-} (1 - |z|^2)^{\beta - \alpha} |g(z)| = 0.$$

In [6], D. Luecking proved the following result with respect to the reverse Carleson measure:

Theorem 0.6. (D. Luecking) *Let* τ *be a bounded non-negative measurable function in* D. *Then there is a constant* $k > 0$ *such that*

$$\int_D |f'(z)|^2 \tau(z) \log \frac{1}{|z|^2}\, dA(z) \geq k \int_D |f'(z)|^2 \log \frac{1}{|z|^2}\, dA(z)$$

for all $f \in H^2$ *if and only if there exists a constant* $c > 0$ *such that the set* $G_c = \{z \in D\colon \tau(z) > c\}$ *satisfies the condition:*

(*) *There exists a constant* $\delta > 0$ *such that*

$$dA(G_c \cap D(\zeta, r)) > \delta dA(D \cap D(\zeta, r))$$

for all $\zeta \in \partial D$ *and* $r > 0$, *where* $D(\zeta, r)$ *is a disc with a center* ζ *and a radius* r.

In [7], D. Luecking proved the following result:

Theorem 0.7. (D. Luecking) *Let* $\alpha > -1$, *and let* μ *be a finite positive Borel measure on* D. *In order that there exists a constant* $C > 0$ *such that*

$$\left(\int_D |f'(z)|^2 d\mu(z)\right)^{\frac{1}{2}} \leq C\left(\int_D |f(z)|^2 (1+\alpha)(1-|z|^2)^\alpha dA(z)\right)^{\frac{1}{2}}$$

for all analytic functions f *if and only if there exists a constant* $C' > 0$ *such that*

$$\mu\left(\left\{z \in D,\ \rho(z, a) < \frac{1}{2}\right\}\right) \leq C'(1-|z|^2)^{4+\alpha}.$$

In [4], P. Ghatage and D. Zheng and Nina Zorboska determined the composition operators on the Bloch space that have a closed range using sampling set for $\mathfrak{B}$. So we also study when the operators I_g, J_g and M_g

and the composition operators are bounded below on the Bergman spaces and the (weighted) Bloch space using sampling set for weighted Bloch spaces. In particular, the fact that I_g have the closed range on the weighted Dirichlet space D_p^α is equivalent to "the reverse Carleson measure", i.e. the definition of I_g with the closed range on the weighted Dirichlet space D_p^α is the following:

$$\int_D |f'(z)|^p |g(z)|^p (\alpha+1)(1-|z|^2)^\alpha dA(z) \geq k \int_D |f'(z)|^p (\alpha+1)(1-|z|^2)^\alpha dA(z)$$

And it is exactly equal to the definition of the reverse Carleson measure. And we succeeded in characterizing the reverse Carleson measure by using new way completely that is different from theorem 0.6 (D. Luecking's result) in this paper (Theorem 2.4). And by characterizing the operator J_g with closed range, we also succeeded in characterizing the result that corresponds to theorem 0.7 (D. Luecking's result) in this paper (Theorem 2.8). Moreover we also succeeded in characterizing the multiplication operator with the closed range on the weighted Bergman spaces as well. In final section, we study the composition operators with the closed range on the weighted Bergman spaces as well.

Definition 1.1. *Let $\alpha > 0$. A set Γ of the open unit disk D is called a sampling set for $\mathfrak{B}^\alpha$ if there exists a positive constant $C > 0$ such that*

$$\sup_{z\in D}(1-|z|^2)^\alpha |f(z)| \leq C \sup_{z\in\Gamma}(1-|z|^2)^\alpha |f(z)|,$$

for all $f \in \mathfrak{B}^\alpha$.

Definition 1.2. *Let $\alpha > 0$. A set Γ of the open unit disk D is called a sampling set for $\mathfrak{B}_\alpha$ if there exists a positive constant $C > 0$ such* *that*

$$\sup_{z\in D}(1-|z|^2)^{\alpha}|f'(z)|\leq C\sup_{z\in \Gamma}(1-|z|^2)^{\alpha}|f'(z)|,$$

for all $f\in\mathfrak{B}_{\alpha}$.

By using a sampling set for $\mathfrak{B}_{\alpha}$, we can prove the following result with respect to the operator I_g:

Theorem 1.3. *Let $\beta\geq\alpha>0$ and $g\in H(D)$. The operator I_g: $\mathfrak{B}_{\alpha}\to\mathfrak{B}_{\beta}$ is bounded. Then the operator I_g: $\mathfrak{B}_{\alpha}\to\mathfrak{B}_{\beta}$ is bounded below if and only if there exists a positive constant $(1>)\varepsilon>0$ such that $\{z\in D,\ (1-|z|^2)^{\beta-\alpha}|g(z)|\geq\varepsilon\}$ is a sampling set for $\mathfrak{B}_{\alpha}$.*

Corollary 1.4. *Let $\alpha>0$ and $g\in H(D)$. The operator I_g is bounded on $\mathfrak{B}_{\alpha}$. Then the operator I_g is bounded below on $\mathfrak{B}_{\alpha}$ if and only if there exists a positive constant $\varepsilon>0$ such that $\{z\in D,\ |g(z)|\geq\varepsilon\}$ is a sampling set for $\mathfrak{B}_{\alpha}$.*

The following lemma is well-known result:

Lemma C. ([5]) *Let $\alpha>1$. For $f\in\mathfrak{B}_{\alpha}$, the norm*

$$|f(0)|+\sup_{z\in D}(1-|z|^2)^{\alpha}|f'(z)|$$

is equivalent to the norm

$$\sup_{z\in D}(1-|z|^2)^{\alpha-1}|f(z)|.$$

i.e. for some constant $C_1>0$ (independent of $f\in\mathfrak{B}_{\alpha}$),

$$\frac{1}{C_1}\sup_{z\in D}(1-|z|^2)^{\alpha-1}|f(z)| \le |f(0)| + \sup_{z\in D}(1-|z|^2)^{\alpha}|f'(z)|$$
$$\le C_1 \sup_{z\in D}(1-|z|^2)^{\alpha-1}|f(z)|.$$

By using a sampling set for $\mathfrak{B}^{\alpha}$ and Lemma C, we can prove the following result with respect to the operator J_g:

Theorem 1.5. *Let $\beta \ge \alpha > 1$ and $g \in H(D)$. The operator J_g: $\mathfrak{B}_\alpha \to \mathfrak{B}_\beta$ is bounded. Then the operator J_g: $\mathfrak{B}_\alpha \to \mathfrak{B}_\beta$ is bounded below if and only if there exists a positive constant $\varepsilon > 0$ such that $\{z \in D, (1-|z|^2)^{\beta-\alpha+1}|g'(z)| \ge \varepsilon\}$ is a sampling set for $\mathfrak{B}^{\alpha-1}$.*

Corollary 1.6. *Let $\alpha > 1$ and $g \in H(D)$. The operator J_g is bounded on $\mathfrak{B}_\alpha$. Then the operator J_g is bounded below on $\mathfrak{B}_\alpha$ if and only if there exists a positive constant $\varepsilon > 0$ such that $\{z \in D, (1-|z|^2)|g'(z)| \ge \varepsilon\}$ is a sampling set for $\mathfrak{B}^{\alpha-1}$.*

By using a sampling set for $\mathfrak{B}^{\alpha}$, we can prove the following result with respect to the multiplication operator M_g:

Theorem 1.7. *Let $\beta \ge \alpha > 1$ and $g \in H(D)$. Let the operator M_g: $\mathfrak{B}_\alpha \to \mathfrak{B}_\beta$ be bounded. Then the operator M_g: $\mathfrak{B}_\alpha \to \mathfrak{B}_\beta$ is bounded below if and only if there exists a positive constant $\varepsilon > 0$ such that $\{z \in D, (1-|z|^2)^{\beta-\alpha}|g(z)| \ge \varepsilon\}$ is a sampling set for $\mathfrak{B}^{\alpha-1}$.*

Corollary 1.8. *Let $\alpha > 1$ and $g \in H(D)$. Let the operator M_g be bounded on $\mathfrak{B}_\alpha$. Then the operator M_g is bounded below on $\mathfrak{B}_\alpha$ if and only if there exists a positive constant $\varepsilon > 0$ such that $\{z \in D, |g(z)| \ge \varepsilon\}$ is a sampling set for $\mathfrak{B}^{\alpha-1}$.*

We get the new characterization of the Reverse Carleson measure on the Bergman space by using the sampling property.

Definition 2.1. *The space BMOA is defined to be the space of analytic functions f on D such that* $\sup_{a\in D}\int_D (1-|\varphi_a(z)|^2)|f'(z)|^2 dA(z) < +\infty$.
In the case of $0<\alpha<1$, The space Q_α is defined to be the space of analytic functions f on D such that $\sup_{a\in D}\int_D (1-|\varphi_a(z)|^2)^\alpha |f'(z)|^2 dA(z) < +\infty$.

Lemma 2.2. *Let f be an analytic function on D. If $\alpha>1$, then $f \in \mathfrak{B}$ if and only if* $\sup_{a\in D}\int_D (1-|\varphi_a(z)|^2)^\alpha |f'(z)|^2 dA(z) < +\infty$.

Theorem 2.3. *Let $g\in H^\infty$. If the operator I_g: $H^2\to H^2$ is bounded below, then I_g: $BMOA\to BMOA$ is bounded below. If the operator I_g: $L_a^2\to L_a^2$ is bounded below, then I_g: $\mathfrak{B}\to\mathfrak{B}$ is bounded below. For $0<p<1$, if the operator I_g: $D_2^\alpha\to D_2^\alpha$ is bounded below, then I_g: $Q_\alpha\to Q_\alpha$ is bounded below.*

Theorem D. ([6]) *Let $\alpha>-1$. There is a constant $C>0$ such that*

$$\int_D |f'(z)|^2(1+\alpha)(1-|z|^2)^\alpha dA(z) \le C\int_G |f'(z)|^2(1+\alpha)(1-|z|^2)^\alpha dA(z)$$

for all $f\in D_2^\alpha$ if and only if a subset G of D satisfy the condition that there exist $\delta>0$ and $r>0$ such that $\delta|D(a,r)|\le|D(a,r)\cap G|$, where $|D(a,r)|$ is the (normalized) area of $D(a,r)$.

We determined the integration operators I_g on the Bergman spaces that have a closed range using sampling set for $\mathfrak{B}$. And the following theorem corresponds to Theorem 0.6:

Theorem 2.4. *Suppose that* $g \in H^{\infty}$. *Then there is a constant* $k > 0$ *such that*

$$\int_D |f'(z)|^2 |g(z)|^2 (1-|z|^2)^2 dA(z) \geq k \int_D |f'(z)|^2 (1-|z|^2)^2 dA(z)$$

for all $f \in L_a^2$ *if and only if there exists a positive constant* $\varepsilon > 0$ *such that* $\{z \in D,\ |g(z)| \geq \varepsilon\}$ *is a sampling set for* $\mathfrak{B}$.

Remark 2.5. Carefully examining the above theorem, we see the following are also the equivalent conditions respectively:

(2.5.1) $\sup_{z \in D}(1-|z|^2)|g(z)\varphi'_w(z)| \geq C$ for all $w \in D$.

(2.5.2) For any $\varepsilon < C$, $\rho(\Gamma, w) \leq R < 1$ for all $w \in D$, R depending only on ε, where $\Gamma = \{z \in D,\ |g(z)| \geq \varepsilon\}$.

Lemma 2.6. *Let* f *be an analytic function on* D. *Then* $f \in \mathfrak{B}_2$ *if and only if* $\sup_{a \in D} \int_D (1-|z|^2)^2 (1-|\varphi_a(z)|^2)^2 |f'(z)|^2 dA(z) < +\infty$.

Theorem 2.7. *Let* $g \in \mathfrak{B}$. *If* J_g: $L_a^2((1-|z|^2)^2 dA(z)) \to L_a^2((1-|z|^2)^2 dA(z))$ *is bounded below, then* J_g: $\mathfrak{B}_2 \to \mathfrak{B}_2$ *is bounded below.*

In [6], D. Leucking proved the following result:

Theorem D′. ([6]) *Let* $\alpha > -1$. *There is a constant* $C > 0$ *such that*

$$\int_D |f(z)|^2 (1+\alpha)(1-|z|^2)^{\alpha} dA(z) \leq C \int_G |f(z)|^2 (1+\alpha)(1-|z|^2)^{\alpha} dA(z)$$

for all $f \in L_a^2((1-|z|^2)^{\alpha} dA(z))$ *if and only if a subset* G *of* D *satisfy the condition that there exist* $\delta > 0$ *and* $r > 0$ *such that* $\delta|D(a, r)| \leq |D(a, r) \cap G|$, *where* $|D(a, r)|$ *is the (normalized) area of* $D(a, r)$.

We determined the integration operators J_g on the weighted Bergman spaces that have a closed range using sampling set for $\mathfrak{B}^1$. And the following theorem corresponds to Theorem 0.7:

Theorem 2.8. *Suppose that* $g \in \mathfrak{B}$. *Then there is a constant* $k > 0$ *such that*

$$\int_D |f(z)|^2 |g'(z)|^2 (1-|z|^2)^4 dA(z) \geq k \int_D |f'(z)|^2 (1-|z|^2)^4 dA(z)$$

for all $f \in L_a^2((1-|z|^2)^2 dA(z))$ *if and only if there exists a positive constant* $\varepsilon > 0$ *such that* $\{z \in D, (1-|z|^2)|g'(z)| \geq \varepsilon\}$ *is a sampling set for* $\mathfrak{B}^1$.

Remark 2.9. Carefully examining the above theorem, we see the following are also the equivalent conditions respectively:

(2.9.1) $\sup_{z \in D} (1-|z|^2)^2 |g'(z) \varphi'_w(z)| \geq C$ for all $w \in D$.

(2.9.2) For any $\varepsilon < C$, $\rho(\Gamma, w) \leq R < 1$ for all $w \in D$, R depending only on ε, where $\Gamma = \{z \in D, (1-|z|^2)|g'(z)| \geq \varepsilon\}$.

With respect to the multiplication operators, we can prove the following:

Theorem 2.10. *Let* $g \in H^\infty$. *If* M_g: $L_a^2((1-|z|^2)^2 dA(z)) \to L_a^2((1-|z|^2)^2 dA(z))$ *is bounded below, then* M_g: $\mathfrak{B}_2 \to \mathfrak{B}_2$ *is bounded below.*

We determined the multiplication operators M_g on the weighted Bergman spaces that have a closed range using sampling set for $\mathfrak{B}^1$.

Theorem 2.11. *Suppose that* $g \in H^\infty$. *Then there is a constant* k

> 0 *such that*

$$\int_D |f(z)|^2 |g(z)|^2 (1-|z|^2)^2 dA(z) \geq k \int_D |f(z)|^2 (1-|z|^2)^2 dA(z)$$

for all $f \in L_a^2((1-|z|^2)^2 dA(z))$ *if and only if there exists a positive constant* $\varepsilon > 0$ *such that* $\{z \in D, |g(z)| \geq \varepsilon\}$ *is a sampling set for* $\mathfrak{B}^1$.

Remark 2.12. Carefully examining the above theorem, we see the following are also the equivalent conditions respectively:

(2.12.1) $\sup_{z \in D} (1-|z|^2)^2 |g(z) \varphi'_w(z)| \geq C$ for all $w \in D$.

(2.12.2) For any $\varepsilon < C$, $\rho(\Gamma, w) \leq R < 1$ for all $w \in D$, R depending only on ε, where $\Gamma = \{z \in D, |g(z)| \geq \varepsilon\}$.

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