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Supplement to the paper "An asymptotic equivalence of the cross-data and predictive estimators"

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This article supplements Ogasawara (2019).

S1. Bernoulli distribution

S1.1 Bernoulli distribution under canonical parametrization

$$\begin{aligned}
&\Pr(x\,|\,\theta) = \left\{\frac{1}{1+\exp(-\theta)}\right\}^{x}\left\{\frac{\exp(-\theta)}{1+\exp(-\theta)}\right\}^{1-x} \quad (x=0,1),\\
&= \frac{\exp\{-(1-x)\theta\}}{1+\exp(-\theta)}, \quad \left(\hat{\theta}_{\mathrm{ML}(-i)} = \ln\frac{\bar{x}_{(-i)}}{1-\bar{x}_{(-i)}}\right),\\
&D_{(-)} = \sum_{i=1}^{n}\left[(1-x_i)\omega\,\hat{\theta}_{\mathrm{ML}(-i)} + \ln\{1+\exp(-\omega\,\hat{\theta}_{\mathrm{ML}(-i)})\}\right],\\
&\frac{\partial D_{(-)}}{\partial\omega} = \sum_{i=1}^{n}\left\{(1-x_i)\,\hat{\theta}_{\mathrm{ML}(-i)} - \frac{\exp(-\omega\,\hat{\theta}_{\mathrm{ML}(-i)})}{1+\exp(-\omega\,\hat{\theta}_{\mathrm{ML}(-i)})}\hat{\theta}_{\mathrm{ML}(-i)}\right\},\\
&= \sum_{i=1}^{n}\left\{1-x_i - \frac{\{\bar{x}_{(-i)}/(1-\bar{x}_{(-i)})\}^{-\omega}}{1+\{\bar{x}_{(-i)}/(1-\bar{x}_{(-i)})\}^{-\omega}}\right\}\ln\frac{\bar{x}_{(-i)}}{1-\bar{x}_{(-i)}}\\
&= \sum_{i=1}^{n}\left\{\frac{1}{1+\{\bar{x}_{(-i)}/(1-\bar{x}_{(-i)})\}^{-\omega}} - x_i\right\}\ln\frac{\bar{x}_{(-i)}}{1-\bar{x}_{(-i)}},
\end{aligned} \tag{S1.1}$$

where

$$\bar{x}_{(-i)} = \bar{x} - (n-1)^{-1}(x_i - \bar{x}) \ (i=1,\ldots,n)\,. \tag{S1.2}$$

To solve $\partial D_{(-)}/\partial\omega = 0$, we use