No. 5

Business Cycles with Asset Price Bubbles and the Role of Monetary Policy

by

Hiroshi Shibuya

June 1993

Department of Economics

Otaru University of Commerce

Business Cycles with Asset Price Bubbles and the Role of Monetary Policy

by

Hiroshi Shibuya*

June 1993

Department of Economics

Otaru University of Commerce

Address for Correspondence:

Department of Economics
Otaru University of Commerce
Midori 3-5-21
Otaru, Hokkaido 047
Japan
Tel. (0134)23-1101

* I would like to thank seminar participants at Otaru University of Commerce for their helpful comments.

"[A] theory of equilibrium growth badly needed--and still needs--a theory of deviations from the equilibrium growth path. ... [The] fundamental intellectual need is for a common understanding of medium-run departures from equilibrium growth. That is the stuff of everyday macroeconomics," [Robert Solow, Nobel Lecture, December 8, 1987]

Abstract

This paper presents a dynamic general equilibrium model that incorporates profit-maximizing banks and *inside money* into a *decentralized* Ramsey economy. It offers an alternative to the monetary equilibrium model of Sidrauski with money in the utility function and that of Stockman with a cash-in-advance constraint, both of which are based on a *centralized* Ramsey model with *outside money*. The model shows how an expansionary monetary policy triggers the business cycle with an asset price bubble. The magnitude of the business cycle and the asset price bubble depend not only on the *size* but also on the anticipated *duration* of the monetary policy. Moreover, if the end of the expansionary monetary policy is followed by a banking crisis, the economy could experience a prolonged period of economic slowdown. Thus, the model is capable of analyzing the phenomenon of *disequilibrium dynamics*.

I. Introduction

Since the mid-1980s, the United States, Japan, and the United Kingdom have gone through a business cycle that is characterized by an asset price bubble. They have experienced a cycle of economic boom with low interest rates and soaring asset prices, followed by economic slowdown with monetary tightening and plummeting asset prices. These developments have raised many important theoretical as well as policy questions: Can asset price bubbles occur in a macroeconomy with rational economic agents and market clearing? If so, what is its mechanism? What is the role of monetary policy in the business cycle with asset price bubbles? What are the implications of these developments for inflation and income distribution? No general equilibrium models exist that can address these questions.[1]

The purpose of this paper is to construct a dynamic general equilibrium model with a banking system and to shed new light on the transmission channel of monetary policy and the mechanism of the business cycle with asset price bubbles. The model focusses on dynamic interactions between interest rates, the asset price, and real economic activities, as well as income distribution between labor and capital. In the model, the business cycle is triggered by a change in the cost of credit intermediation. Although the cost of credit intermediation can be affected by many factors such as financial deregulation and innovations, a banking crisis and monetary policy are two of the most important factors.

Monetary policy affects macroeconomic dynamics by changing the cost of credit intermediation: For example, a reduced central bank lending rate engineered by an expansionary monetary policy reduces the cost of banking

---

[1] The monetary model of Brunner and Meltzer (1976) may be used to analyze such a phenomenon. Their model, however, is not based on individual optimization and is limited to temporary equilibrium analysis.

operation, which translates into a reduced spread between loan and deposit rates of commercial banks. The reduced interest rate spread means a reduced cost of credit intermediation for the economic system. An expansionary monetary policy therefore acts like a positive productivity shock to the economic system. Conversely, a restrictive monetary policy acts like a negative productivity shock. The magnitude of the resulting business cycle depends not only on the *size* but also on the anticipated *duration* of the monetary policy.

A small change in the cost of credit intermediation (the interest rate spread) can produce a large deviation of the economy from the equilibrium growth path. The deviation of the cost of credit intermediation from the natural level that is consistent with the equilibrium growth path temporarily dislocates the economy onto a divergent growth path, initiating a process of disequilibrium dynamics. When the cost of credit intermediation returns to the natural level, the economy will move back to the convergent equilibrium growth path. Thus, the economy can diverge significantly from the equilibrium growth path while the cost of credit intermediation deviates from the natural level.

It is real interest rates, rather than a nominal quantity of money, that play a main role in the transmission of monetary policy. In this respect, our theory is similar in spirit to Wicksell's (1936) "cumulative process," in which a divergence between the "natural" and "market" rates of interest triggers the Wicksellian disequilibrium dynamics. The economy may be away from full equilibrium path for a long period of time. Wicksell's model, however, is incomplete because it keeps the real side fixed and focuses only on price dynamics. Our model illustrates how price dynamics interacts with quantity dynamics in the course of the business cycle.

The business cycle of our model is characterized by asset price bubbles, namely, sharp rises and falls in the asset price relative to the product price (Tobin's q). We therefore present a dynamic general equilibrium model of asset price bubbles in the course of macroeconomic dynamics. We show, in particular, how a prolonged period of expansionary monetary policy can produce an asset price bubble. Again, the magnitude of the bubble depends not only on the *size* but also on the anticipated *duration* of expansionary monetary policy. The longer the expected duration of the expansionary monetary policy, the greater the size of an eventual bubble burst.

We define an asset price bubble as a temporary departure of the asset price from its *macroeconomic fundamental*: namely, the asset value that is consistent with the *equilibrium* growth path of the economy. This macroeconomic bubble is different from a microeconomic bubble of Tirole (1982, 1985), which is defined as a deviation of the asset price from its *microeconomic fundamental*: namely, the present discounted value of dividends. Tirole (1982) shows that an asset price bubble in the microeconomic sense cannot not exist in a perfect foresight equilibrium with infinitely-lived agents. We show that an asset price bubble in the macroeconomic sense can exist even in a perfect foresight equilibrium with infinitely-lived agents if the economy is decentralized.

The model suggests a new concept of inflation and its relation to disequilibrium dynamics. First, it shows that a higher asset price implies a higher intertemporal cost-of-living for labor. In other words, the asset price (Tobin's q) is a fundamental index for inflation in a dynamic context. Next, it shows that a deviation of the economy from the equilibrium growth path that is caused by a monetary policy shock is reflected in the movement of the asset price. Thus, the model links the dynamic concept of inflation (defined as an increase in the intertemporal cost-of-living for labor) to a dynamic disequilibrium state of the real economy.

The same transmission mechanism also suggests the dynamic effects of a banking crisis. Both a monetary policy shock and a banking crisis affect macroeconomic dynamics through their effects on the cost of credit intermediation. For example, Bernanke (1983) has argued that a banking crisis played a major role in the propagation of the Great Depression by raising the cost of credit intermediation. In this respect, our model emulates the dynamic effects of a banking crisis *à la* Bernanke. In particular, it reconciles the deep and protracted period of economic slowdown (a suboptimal state of the economy) with the postulates of rational economic agents and market clearing. A banking crisis increases the cost of credit intermediation and thereby affects intertemporal market opportunities and allocations; it can drive the economy off course for a prolonged period of time.

The rest of this paper is organized as follows: Section II constructs a dynamic general equilibrium model of a macroeconomy with a banking system. It presents the maximization problems of banks, households, and firms, and then describes the market equilibrium and the system of equations that defines macroeconomic dynamics. Section III analyzes the effects of expansionary and restrictive monetary policies as well as a banking crisis on macroeconomic dynamics, and discusses the role of monetary policy. Section IV concludes this paper.

II. A Decentralized Ramsey Economy With Inside Money

This section develops a dynamic general equilibrium model with a banking system, which is built on the decentralized Ramsey model of Abel and Blanchard (1983) and Blanchard and Fischer (1989). They use their model mainly to analyze the dynamic effects of fiscal policy. We show that their model can be easily adapted to incorporate a banking system (a central bank and commercial banks) and thereby to analyze the dynamic effects of monetary policies and a banking crisis in a dynamic general equilibrium framework. Moreover, our model with the banking system exhibits the phenomenon of the business cycle with asset price bubbles.

We introduce (1) profit-maximizing banks and (2) the Clower (money-in-advance) constraints on households and firms.[2] Money is supplied by identical banks as bank loans and held by households and firms as bank deposits, which are then used as *a means of payments* for consumption and investment. In other words, we introduce commercial banks and *inside money* (credit money) into a *decentralized* Ramsey model, and define the role of money as a means of payments. Thus, we present an alternative to the monetary equilibrium model of Sidrauski (1967) with money in the utility function and that of Stockman (1981) with a cash-in-advance constraint, both of which are based on a *centralized* Ramsey model with *outside money*.

A. The Basic Framework

The economy is a pure credit economy and consists of a central bank, commercial banks, firms, and households. The balance sheets of these

[2] In a monetary economy, "money buys goods, goods buy money, but goods do not buy goods." See Clower (1967).

economic agents are summarized in Table 1 where plus signs indicate assets while minus signs indicate liabilities. Profit-maximizing commercial banks determine money supply (bank loans) while meeting the reserve requirement. The central bank supplies reserves to commercial banks and implements its monetary policy by changing the central bank lending rate on reserves or the required reserve ratio. Households and firms determine money demand (bank deposits), which they use as the means of payments for their consumption and investment transactions.

Intertemporal consumption plans are made by optimizing identical households, which also supply one unit of labor inelastically. Total consumption (in present value) is an increasing function of total wealth (human and nonhuman), and its rate of change over time is an increasing function of the real interest rate. Intertemporal investment plans are made by value-maximizing identical firms, which carry out production with labor and capital as inputs. Investment is an increasing function of the ratio of the asset price to the product price (Tobin's q).

The equality of aggregate demand and supply is maintained at every point in time by the endogenous adjustment of the term structure of real interest rates $\{r_t\}_0^\infty$. The term structure of real interest rates coordinates the intertemporal consumption and investment plans of households and firms. These real interest rates prevail in the bond and loan markets ($r = r^b = r^l$). On the other hand, the spread between the loan and deposit rates ($z = r^l - r^d$) is determined in the money market at the intersection of the money supply and demand curves. The deposit rate (r^d) is then determined given the real interest rate and the spread.

We assume for notational simplicity that the number of banks, firms and households are the same so that per bank, per household and per firm values coincide. Therefore, all variables in the text are in per capita terms. Finally,

Table 1

The Balance Sheets of Economic Agents

	Reserves	C.B. Loans	Bank Loans	Deposits	Bonds	Equity	Net Worth
Central Bank	−R	+L					
Banks	+R	−L	+l	−m			
Firms			−l	$+m^f$	−b	−e	+qk
Households				$+m^h$	+b	+e	−V
Interest Rate	0	r^c	r	r^d	r		

all variables in the model presented below are *real* variables unless otherwise indicated.

B. Money and the Banking System

Money supply (bank loans) is determined by profit-maximizing commercial banks at each point of time. Each identical bank decides the amount of loans to households and firms so as to maximize its profits subject to the balance sheet condition and the reserve requirement as follows:

$$\begin{aligned} \max \quad & \Pi = r_t l_t - r^d_t m_t - r^c_t R_t - C(l_t; \theta_t) \\ \text{s.t.} \quad & l_t = m_t \quad \text{and} \quad R_t = \tau_t l_t \end{aligned} \tag{1}$$

where l represents bank loans; m bank deposits; R reserves held at the central bank; τ the required reserve ratio; r the commercial bank lending rate; r^d the deposit rate; r^c the central bank lending rate; $C(l; \theta)$ the commercial bank's operational cost function; and θ a cost parameter for the bank. The cost function $C(l; \theta)$ is assumed to be twice continuously differentiable and strictly convex in l and θ.

From the first-order condition, the money supply function is obtained as follows:

$$\begin{gathered} z_t \equiv r_t - r^d_t = C_l(l_t; \theta_t) + \tau_t r^c_t \equiv D(l_t; \tau_t, r^c_t, \theta_t) \\ \text{or} \\ m^s_t = l_t = l(z_t; \tau_t, r^c_t, \theta_t) \end{gathered} \tag{2}$$

where $l(z; .)$ is the inverse function of $D(l; .)$ and satisfies $l_z < 0$, $l_\tau < 0$, $l_{r^c} < 0$, and $l_\theta < 0$; $m^s = l(z; \tau, r^c, \theta)$ represents the money supply function; and z $(= r-r^d)$ is the spread between loan and deposit rates, which represents the *cost of credit intermediation* for the economy.

Monetary policy shifts the money supply curve. The central bank can control the money supply curve by changing the required reserve ratio (τ) or the central bank lending rate (r^c).[3] An expansionary monetary policy shifts the money supply curve to the right in the (l, z) plane, and thus raises the interest rate spread given the money demand curve derived below (Chart 1). A restrictive monetary policy, on the other hand, shifts the money supply curve to the left in the (l, z) plane, and thus reduces the interest rate spread given the money demand curve.

A *banking crisis* shifts the money supply curve to the left because it raises the cost of banking operation (θ). Bernanke (1983) has argued that a banking crisis during the Great Depression raised the costs of channeling funds from households (savers) to firms (ultimate borrowers), thereby adversely affecting macroeconomic performance. In our model, such macroeconomic effects of a banking crisis are captured by an increase in the value of cost parameter θ, which shifts the money supply curve to the left and raises the cost of credit intermediation, which is determined at the intersection of the money supply and demand curves in money market equilibrium.

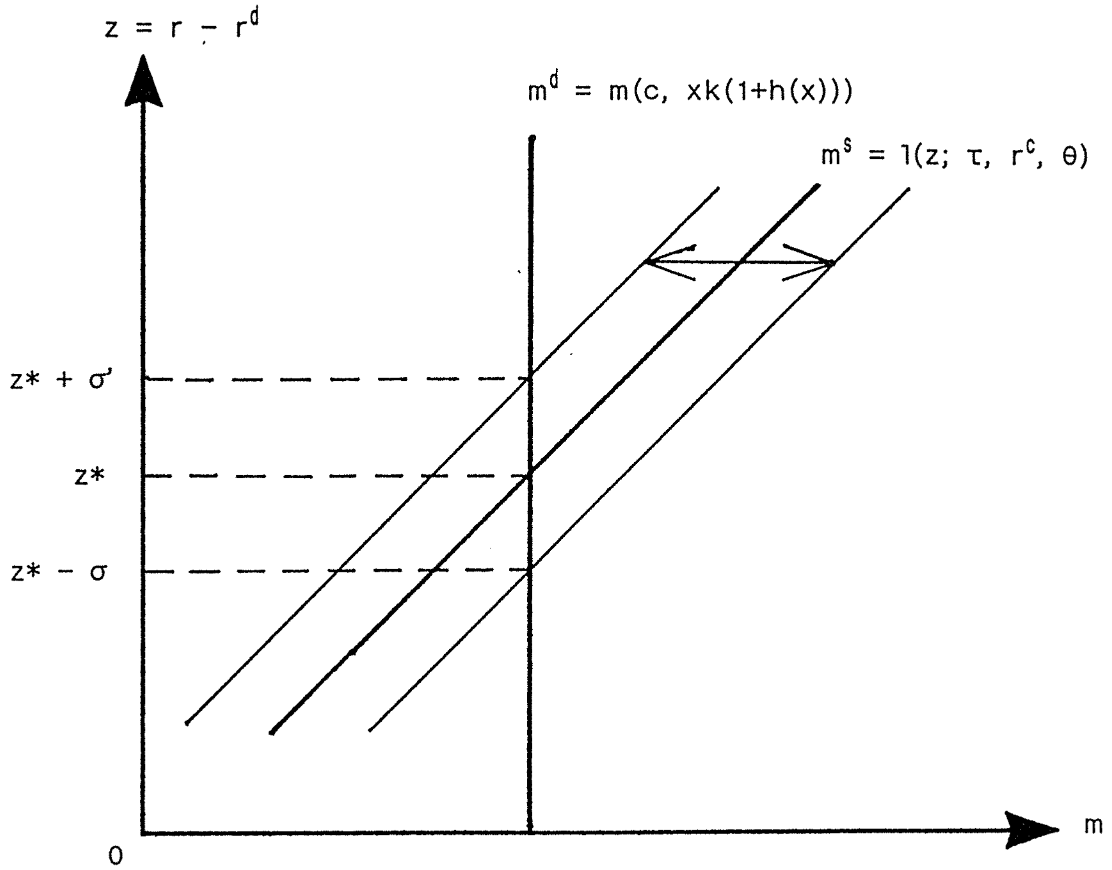
Money demand (bank deposits) is determined by the Clower constraints on households and firms at each point of time. In a monetary economy, "money buys goods, goods buy money, but goods do not buy goods." Money is therefore demanded for transactions purposes by households (m^h) for purchasing consumption goods and by firms (m^f) for purchasing investment goods. Thus, total demand for money is given as the sum of these two

---

[3] The central bank lending rate of our model includes not only the official discount rate, but also the federal funds rate in the United States, and the call rate in Japan. Bernanke and Blinder (1992) present empirical evidence that the federal funds rate is a good indicator of monetary policy actions.

Chart 1

Money Demand and Supply Curves



demands:

$$m_t^d = m_t^h + m_t^f$$
$$= c_t + x_t k_t (1 + h(x)) \qquad (3)$$

where c_t represents consumption and $x_t k_t(1+h(x_t))$ represents investment with adjustment costs in units of output. Thus, we can express the demand for money function as $m^d = m(c, xk(1+h(x)))$. In the next sections, we examine how such consumption and investment are determined by households and firms.

C. Value Maximization by Firms

Each identical firm decides on the time path of investment, facing the time path of wage bills and interest rates. As labor is in fixed supply by assumption, there is one worker per firm. Investment (i) incurs an adjustment cost of ih(i/k) where k is capital stock. The adjustment cost function h(.) is assumed to be nonnegative and convex with a minimum value of zero when net investment is zero (i.e., $h(\delta)=0$ where δ = rate of depreciation). Each firm has to hold money balance (bank deposits) equivalent to the amount of investment expenditure, which incurs transaction costs at the rate of the interest rate spread (z_t).

Thus, the firm's maximization problem can be written as follows:

$$\max \quad \int_0^\infty [f(k_t) - x_t k_t (1 + h(x_t)) - w_t - z_t m^f{}_t] \rho_t \, dt$$

$$\text{s.t.} \quad \dot{k}_t = (x_t - \delta) k_t \quad \text{and} \quad m^f{}_t = x_t k_t (1 + h(x_t)) \qquad (4)$$

$$\text{where} \quad f'(.) > 0, \quad f''(.) < 0, \quad f'(0) = \infty, \quad k_0 = \overline{k_0} > 0,$$

$$\text{and} \quad \rho_t = \exp\{-\int_0^t r_v \, dv\}.$$

The Hamiltonian for this problem is:

$$\rho_t [f(k_t) - (1 + z_t) k_t x_t (1 + h(x_t)) - w_t + q_t(i_t - \delta k_t)] \tag{5}$$

where q_t is the shadow price of an additional unit of capital, which also represents the equilibrium price of capital measured in units of output.

Let us define $x_t = i_t/k_t$ and $H(x_t) = x_t(1+h(x_t))$. Then, the optimality conditions are obtained as follows:

$$H'(x_t) = (1 + z_t)^{-1} q_t \tag{6}$$

$$\dot{q}_t = (r_t + \delta) q_t - [f'(k_t) + (1 + z_t) x_t^2 h'(x_t)] \tag{7}$$

$$\lim_{t \to \infty} \rho_t q_t k_t = 0 \tag{8}$$

The first condition states that the marginal cost of investment must be equal to the equilibrium price of capital (measured in units of output) adjusted for the cost of holding money or the transaction cost ($q_t/(1+z_t)$).

The second condition can be solved subject to the transversality condition as follows:

$$q_t = \int_t^{\infty} [f'(k_s) + (1 + z_s) x_s^2 h'(x_s)] \exp\{ - \int_t^s (r_v + \delta) dv \} ds \tag{9}$$

This shows that the equilibrium asset price (q_t) is the present discounted value of marginal products of investment where the discount rate depends on the term structure of market-determined interest rates.

D. Financing Scheme

Next, let us look at the *financing scheme* of the firm. We assume that firms finance replacement investment by retained earnings and net investment by bank loans ($1-m^f=m^h$) and bond issues (b). Then, the firm's cash-flow (η) is given by the following:

$$\eta_t = f(k_t) - k_t x_t (1 + h(x_t)) - w_t - r_t (m_t^h + b_t) - z_t m_t^f \tag{10}$$

The dividends (π) are given by the following:

$$\pi_t = f(k_t) - k_t \delta (1 + h(\delta)) - w_t - r_t (m_t^h + b_t) - z_t m_t^f \tag{11}$$

Therefore, new bonds are issued as follows:

$$\begin{aligned} \dot{b}_t + \dot{m}_t^h &= k_t [\, x_t (1 + h(x_t)) - \delta (1 + h(\delta)) \,] \\ &= k_t [\, x_t (1 + h(x_t)) - \delta] \end{aligned} \tag{12}$$

This shows that household savings are equal to net investment.

From the balance sheet condition, the following relationships hold between the values of capital (V), bonds (b), equity (e), and capital stock (k):

$$\begin{aligned} V_t &= q_t k_t = b_t + (1_t - m_t^f) + e_t = b_t + m_t^h + e_t \\ \text{where} \quad e_t &= \rho_t^{-1} \int_t^{\infty} \pi_s \, \rho_s \, ds \end{aligned} \tag{13}$$

This simply shows that the value of the firm is equal to the sum of its liabilities and owners' equity and that it is independent of the method of financing. In other words, all financing schemes are equivalent in the sense that they lead to the same optimal path of $\{c_t\}$ and $\{i_t\}$. This is one of the main implications of the *Modigliani–Miller theorem*. The above relationship also indicates that Tobin's "marginal q" and "average q" are equal in our model.[4]

[4] This equality depends on the linear homogeneity of the production and adjustment cost functions in labor, capital and investment. This condition is satisfied as all variables in the model are in per capita terms. See Hayashi (1982).

E. Utility Maximization by Households

Each household supplies one unit of labor inelastically and receives wage w_t. His income is the sum of wages, interests on deposits and bonds, and dividends. The income must be allocated either to current consumption or to savings in the form of deposits and bonds. The household's problem is to choose a sequence of consumption $\{c_t\}^{\infty}_0$ subject to the wealth constraint under the term structure of real interest rates $\{r_t\}^{\infty}_0$. Therefore, the household's maximization problem can be written as follows:

$$\max \quad \int_0^{\infty} U(c_t)\, e^{-\beta t}\, dt$$

$$\text{s.t.} \quad c_t + (\dot{b}_t + \dot{m}^h_t) = \pi_t + w_t + r_t(b_t + m^h_t) - z_t m^h_t \tag{14}$$

$$\text{and} \quad m^h_t = c_t$$

The Hamiltonian for this problem is:

$$e^{-\beta t} [\; U(c_t) + \lambda_t \{ \pi_t + w_t + r_t(b_t + m^h_t) - (1 + z_t) c_t \}] \tag{15}$$

Thus, the optimality conditions are given by:

$$U'(c_t) = \lambda_t\, (1 + z_t) \tag{16}$$

$$\dot{\lambda}_t = (\beta - r_t)\, \lambda_t \tag{17}$$

$$\lim_{t \to \infty} (b_t + m^h_t)\, \lambda_t\, e^{-\beta t} = 0 \tag{18}$$

From these conditions, we can obtain the present value of marginal utilities in units of utils as follows:

$$\lambda_t = \int_t^\infty \exp\{-\int_t^s(\beta - r_v)\, dv\}\, ds \tag{19}$$

This together with equation (17) determine the path of the rate of change of marginal utility and thereby the path of the rate of change of consumption.

The level of this consumption path is determined by the following household wealth constraint:

$$\begin{aligned}\int_0^\infty (1 + z_t)\, c_t\, \rho_t\, dt &= \{ b_0 + m_0^h + \int_0^\infty \pi_t \rho_t\, dt \} + \int_0^\infty w_t \rho_t\, dt \\ &= q_0\, k_0 + \int_0^\infty w_t\, \rho_t\, dt \\ &= V + V*\end{aligned} \tag{20}$$

This shows that total consumption plus associated transaction costs equal to the sum of nonhuman wealth (V) and human wealth (V*). It also shows how q enters the consumption decision of households through the wealth effect.

F. Intertemporal Cost-of-Living and Income Distribution

The household wealth constraint also suggests a new measure of inflation in a dynamic general equilibrium context. Let us rewrite the wealth constraint as follows:

$$q_0 = \int_0^\infty [\ (1 + z_t)\, c_t - w_t\ ]\, \rho_t\, dt \ /\ k_0 \tag{21}$$

This shows that q_0 represents not only the equilibrium price of capital but also an intertemporal cost-of-living relative to wages. If q raises, income distribution will shift from labor to capital. In general equilibrium, this means an increase in the *intertemporal cost-of-living for labor*. Thus, q measures inflation (defined as an increase in the intertemporal cost-of-living for labor) in a dynamic general equilibrium model.[5]

G. Market Equilibrium and Macroeconomic Dynamics

Households and firms make optimal decisions, taking the term structure of interest rates as given. Therefore, aggregate supply and demand are brought into equality by endogenous adjustment of the term structure of interest rates $\{r_t\}^{\infty}_0$:

$$\begin{aligned} f(k_t) &= c_t + k_t x_t (1 + h(x_t)) + z_t m_t \\ &= (1 + z_t) c_t + (1 + z_t) k_t x_t (1 + h(x_t)) \\ &\qquad \text{or} \\ (1 + z_t)^{-1} f(k_t) &= c_t + k_t x_t (1 + h(x_t)) \end{aligned} \tag{22}$$

This goods market equilibrium condition suggests that the cost of credit intermediation (z_t) represents a transaction cost in the monetary economy.

It can be shown that there exists a term structure of interest rates $\{r_t\}^{\infty}_0$ that makes the centralized and decentralized Ramsey economies to be equivalent.[6] Under such a term structure of interest rates, the behavior of consumption, investment and output in our decentralized economy becomes equivalent to that of the following centralized Ramsey economy:

$$\begin{aligned} &\max \quad \int_0^{\infty} u(c_t)\, e^{-\beta t}\, dt \\ &\text{s.t.} \quad (1 + z_t)^{-1} f(k_t) = c_t + k_t x_t (1 + h(x_t)) \\ &\text{and} \quad \dot{k}_t = k_t (x_t - \delta) \end{aligned} \tag{23}$$

---

[5] Shibuya (1992) derives an analogous index of inflation from a discrete-time partial equilibrium model.

[6] The proof is analogous to that in Abel and Blanchard (1983).

The Hamiltonian for this problem is:

$$e^{-\beta t} [U((1 + z_t)^{-1} f(k_t) - k_t x_t (1 + h(x_t))) + \mu_t k_t (x_t - \delta)] \tag{24}$$

Therefore, the optimality conditions for the centralized economy are obtained as follows:

$$U'(c_t) \; H'(x_t) = \mu_t \quad (= \lambda_t \, q_t) \tag{25}$$

$$\dot{\mu}_t = (\beta + \delta) \mu_t - U'(c_t) [(1 + z_t)^{-1} f'(k_t) + x_t^2 h'(x_t)] \tag{26}$$

$$\lim_{t \to \infty} e^{-\beta t} \mu_t k_t = 0 \tag{27}$$

To sum up: the *macroeconomic dynamics* of our decentralized market economy is completely determined by these equations. In the centralized economy, real interest rates are *internalized* so that they do not appear in the above equations.

III. Monetary Policies and Disequilibrium Dynamics

In the previous section, we have shown how the central bank can shift the money supply (bank credit) curve with the use of its monetary policy instruments such as the central bank lending rate and the required reserve ratio. We have also shown how the macroeconomic dynamics of our economy can be described by a system of basic differential equations. This section examines the dynamic effects of monetary policies on the real economy.

A. Monetary policies

We assume that there exists a natural level of the cost of credit intermediation z* (the interest rate spread between loan and deposit rates) that is determined by the banking technology and that is consistent with the *equilibrium* growth path of the economy. Then, *a monetary policy* in our model is defined by triplet (σ, t_0, t_1), which consists of an *unanticipated* rise in the cost of credit intermediation by σ at t_0 and an *anticipated* fall in the costs by σ at t_1. In other words, *expansionary* monetary policy $(-\sigma, t_0, t_1)$ temporarily reduces z_t such that:

$$z_t = \begin{cases} z^* - \sigma & \text{for } t_0 \le t \le t_1 \\ z^* & \text{otherwise} \end{cases} \tag{28}$$

In contrast, restrictive monetary policy (σ, t_0, t_1) temporarily increases z_t such that:

$$z_t = \begin{cases} z^* + \sigma & \text{for } t_0 \le t \le t_1 \\ z^* & \text{otherwise} \end{cases} \tag{29}$$

It is important to note that a monetary policy is *unanticipated* at the time of initial implementation but once implemented its duration is *anticipated,* and that it causes a temporary deviation of the cost of credit intermediation from the natural level.

B. Macroeconomic Dynamics and Steady State

To study the dynamic effects of monetary policies in the decentralized economy, we can start with examining their effects on the centralized economy. Once the behavior of consumption, investment, and output is characterized in the centralized economy, it is relatively easy to deduce the movements of the equilibrium asset price and the market clearing term structure of interest rates in the decentralized market economy. From the previous analysis of our model, the system of fundamental equations that describes macroeconomic dynamics is given as follows (time subscript t's are omitted):

$$c + k x (1 + h(x)) = (1 + z)^{-1} f(k) \tag{30}$$

$$U'(c) H'(x) = \mu = \lambda q \tag{31}$$

$$\dot{\mu} = (\beta + \delta)\mu - U'(c) [(1 + z)^{-1} f'(k) + x^2 h'(x)] \tag{32}$$

$$\dot{k} = k (x - \delta) \tag{33}$$

These equations suggest that, from the viewpoint of macroeconomic dynamics, monetary policy acts like a transitory productivity shock to the economic system.

The system of equations (30)–(33) defines the macroeconomic dynamics of the centralized economy in the (k, x, c, μ) space. We can eliminate c in equation (31) using equation (30), and then differentiate it with respect to time. Next, we can eliminate $d\mu/dt$ in the equation using equation (32) and obtain the following differential equation in terms of dk/dt and dx/dt:

$$B_1 \dot{x} = B_2 + B_3 \dot{k} \tag{34}$$

where

$$B_1 \equiv H''(x) + \alpha(c)\{H'(x)\}^2 k > 0$$

$$B_2 \equiv (\beta + \delta) H'(x) - x^2 h'(x) - (1 + z)^{-1} f'(k) \tag{35}$$

$$B_3 \equiv \alpha(c) H'(x) [(1 + z)^{-1} f'(k) - H(x)]$$

in which $\alpha(c) = -U''(c)/U'(c)$. This equation, together with equation (33), characterizes the macroeconomic dynamics of the centralized economy in the (k, x) plane. Since the macroeconomic dynamics of the centralized and decentralized economies are equivalent, it is straightforward to characterize that of the decentralized economy in the (k, q/(1+z)) plane using $H'(x) = q/(1+z)$. Since $d[q/(1+z)]/dx = H''(x) > 0$, the phase diagrams in the (k, x) and (k, q/(1+z)) planes are homeomorphic and share the same characteristics. As the economy has a unique saddle point equilibrium in the (k, x) plane, it carries over to the saddle point equilibrium in the (k, q/(1+z)) plane.[7]

The steady state of this macroeconomic system is obtained by setting dk/dt = 0 and dx/dt = 0 as follows:

$$(1 + z)^{-1} f'(k*) = (\beta + \delta) H'(\delta) - \delta^2 h'(\delta) \tag{36}$$

This implies that a rise in the cost of credit intermediation (z) reduces the steady-state level of capital (k*) and conversely a fall in the cost of credit intermediation increases the steady-state level of capital.

---

[7] To draw the phase diagram: First, draw the loci dk/dt = 0, $B_2 = 0$, and $B_3 = 0$ in the (k, x) plane, assuming that $\alpha(c)$ = constant and noting that $B_2 = 0$ and $B_3 = 0$ have the same value of k for $x = \beta + \delta$ and that $B_2 = 0$ and dk/dt = 0 intersect at k = k*. We see that these loci divide the (k, x) plane into seven regions. Second, check the signs of B_2, B_3, dk/dt, dx/dt for each region. Third, using these signs, we can draw the phase diagram in the (k, x) plane. The phase diagram has a saddle point in the (k, x) plane.

C. Asset Price Bubbles and the Business Cycle

Now let us examine the dynamic effects of expansionary monetary policy $(-\sigma, t_0, t_1)$ on the decentralized market economy. Suppose that the economy is initially on the balanced growth path. Then, the expansionary monetary policy generates a business cycle with an asset price bubble. To see this, let us examine backward what happens at time t_1. While z_t jumps up at time t_1, costate variable μ cannot have an *anticipated* discontinuity.[8] Therefore, c and x must satisfy the following two relations:

$$H'(x)\,U''(c)\frac{dc}{dz} + H''(x)\,U'(c)\frac{dx}{dz} = \frac{d\mu}{dz} = 0 \tag{37}$$

$$\frac{dc}{dz} + \bar{k}\,(1 + h(x))\frac{dx}{dz} + \bar{k}\,x\,h'(x)\frac{dx}{dz} = -f(\bar{k})\,(1 + z)^{-2} \tag{38}$$

Thus, we obtain the following derivatives of consumption and investment with respect to the cost of credit intermediation:

$$\frac{dc}{dz} = -\frac{f(\bar{k})}{(1 + z)^2[1 + \bar{k}\,H'(x)\,(\alpha(c)/\beta(x))]} < 0 \tag{39}$$

and

$$\frac{dx}{dz} = -\frac{\alpha(c)\,f(\bar{k})}{\beta(x)\,(1 + z)^2[1 + \bar{k}\,H'(x)\,(\alpha(c)/\beta(x))]} < 0 \tag{40}$$

where

$$\alpha(c) \equiv -\frac{U''(c)}{U'(c)} > 0 \qquad \text{and} \qquad \beta(x) \equiv \frac{H''(x)}{H'(x)} > 0 \tag{41}$$

In other words, an increase in z at time t_1 reduces both consumption and investment.

---

8 See Kemp and Long (1977).

This result in turn implies that an increase in the cost of credit intermediation at time t_1 reduces the equilibrium asset price $(q/(1+z))$ and increases the shadow price of consumption $(\lambda\ (1+z))$:

$$\frac{d[q/(1+z)]}{dz} < 0 \qquad \text{and} \qquad \frac{d[\lambda\,(1+z)]}{dz} > 0 \tag{42}$$

These results follow directly from the optimality conditions of consumption and investment decisions in the decentralized economy that we have studied in the previous section.

When the expansionary monetary policy ends at time t_1, the cost of credit intermediation returns to the natural level thereafter. Thus, the economy must be on the equilibrium growth path after time t_1, and eventually converges to the balanced growth path.[9] Therefore, the above analysis implies that the economy must move from a divergent disequilibrium path onto the convergent equilibrium growth path at time t_1.[10] This in turn implies that the economy must move from the initial balanced growth path to a divergent disequilibrium path at time t_0 when the central bank initially implements the *unanticipated* expansionary monetary policy.[11]

From these analyses, we can draw phase diagrams that describes the dynamic effects of monetary policies.[12] Chart 2 shows how an expansionary monetary policy triggers a business cycles with an asset price bubble. It

---

[9] This follows from the transversality condition.

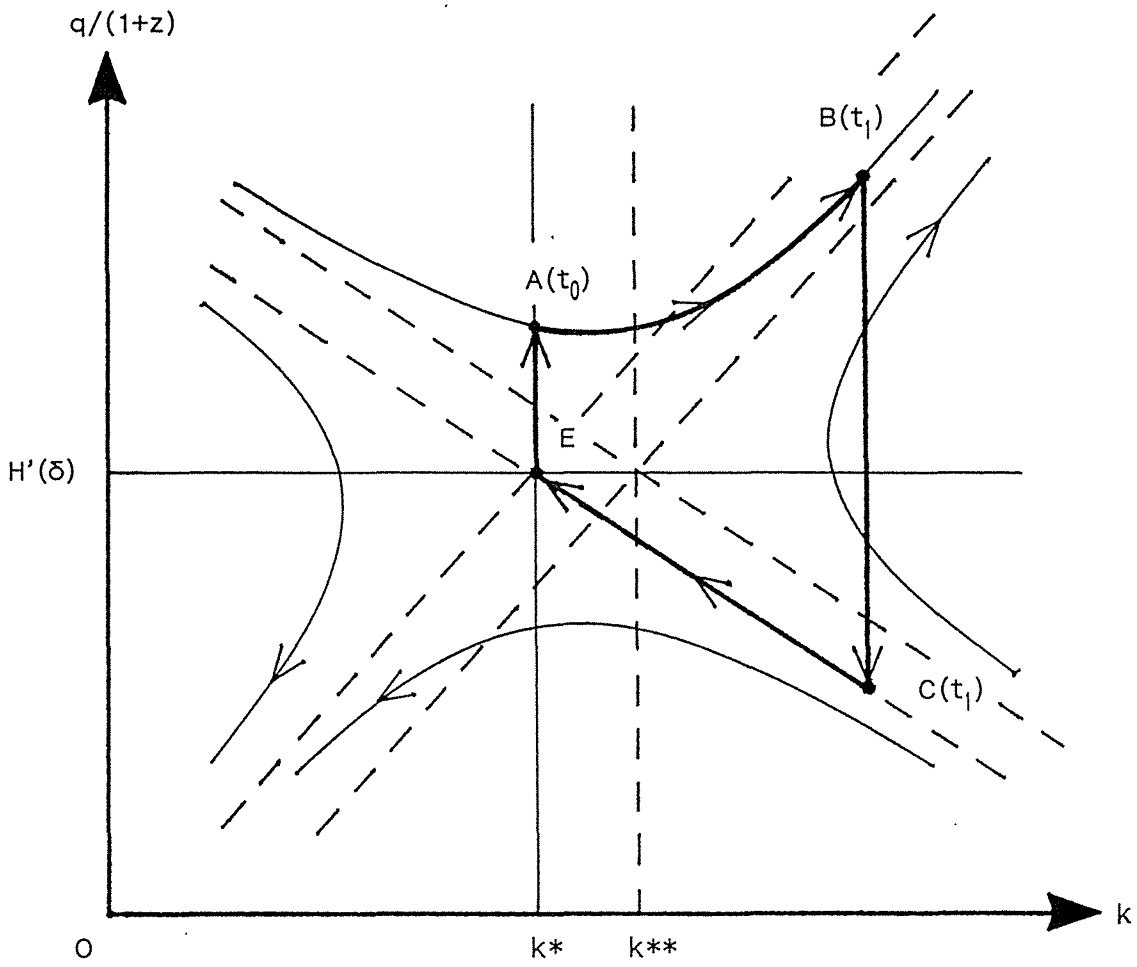
[10] The interest rate must be infinitely negative at time t_1 for an instant in order to generate the discontinuous falls in consumption and investment. This feature is due to the discontinuity in z at time t_1 and could be eliminated either by moving to discrete time or by making z continuous with respect to time.

[11] See Kemp and Long (1977) and Abel and Blanchard (1983) for technical details.

[12] The following phase diagrams are drawn assuming that $\alpha(c) = -U''(c)/U'(c)$ is constant. In steady state, $q^*/(1+z^*)$ is equal to $H'(\delta)$, which is *one* under the assumption of $h(\delta) = 0$ and $h'(\delta) = 0$, and k^* satisfies $f'(k^*)/(1+z^*) = \beta + \delta$.

Chart 2

A Business Cycle with an Asset Price Bubble
Triggered by an Expansionary Monetary Policy



first triggers a jump in the asset price and investment at the time of initial monetary easing (t_0). And thereafter the asset price and investment increase along a divergent disequilibrium growth path, accumulating capital until the end of the expansionary monetary policy (t_1). This temporary accumulation of capital helps smooth out the effects of the increased cost of credit intermediation on consumption and investment at time t_1. However, the smoothing is not complete because of adjustment costs, and there is a discontinuous fall in consumption and investment at the time of monetary tightening (t_1).

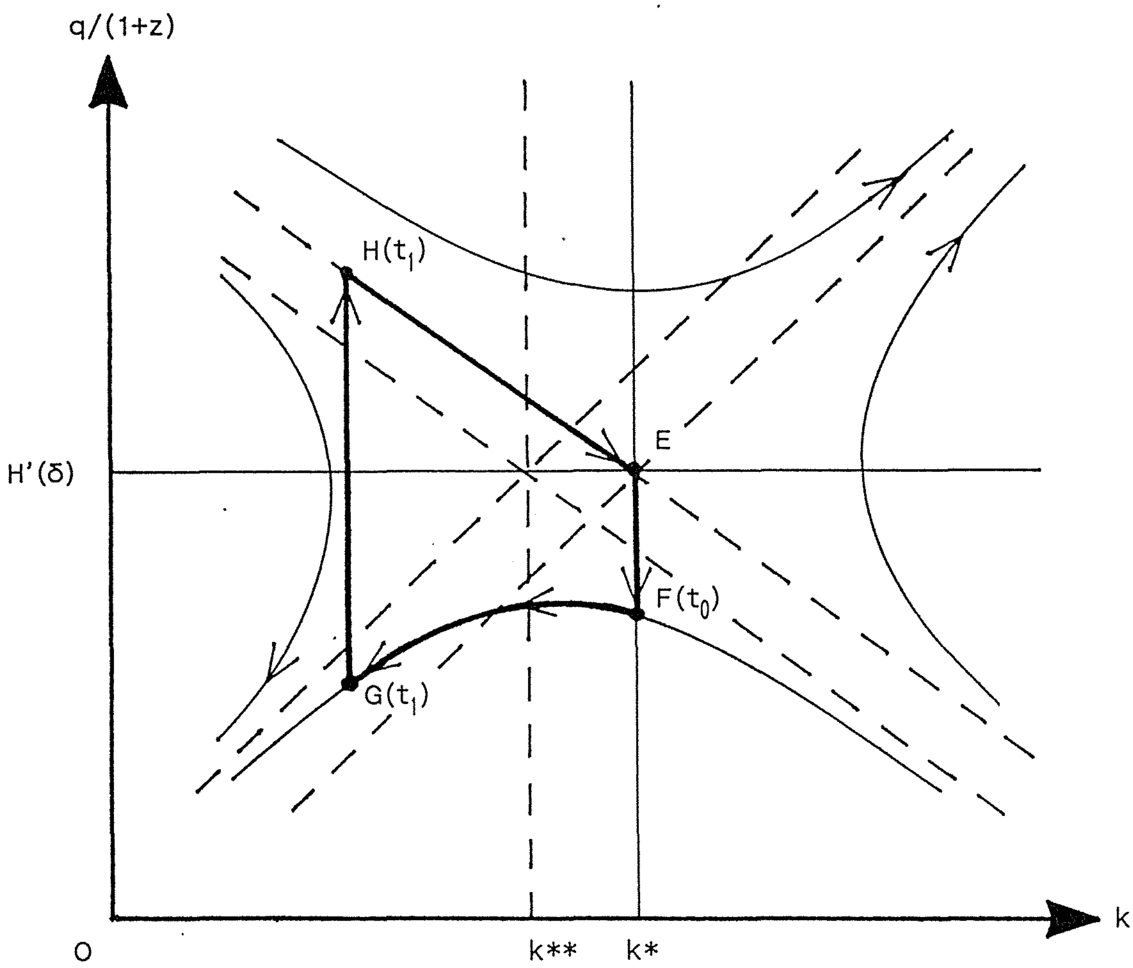
Using the same arguments, we can show that restrictive monetary policy or banking crisis (σ, t_0, t_1) generates a business cycle with asset price deflation (Chart 3). An rise in the cost of credit intermediation caused by either an unanticipated restrictive monetary policy or an unanticipated banking crisis first triggers a discontinuous drop in the asset price and investment, which dislocates the economy from the balanced growth path to a divergent disequilibrium path. The economy then moves along this disequilibrium path during the *anticipated* duration of the restrictive monetary policy or the banking crisis. When the restrictive monetary policy or the banking crisis finally ends, the economy jumps back to the convergent equilibrium path this time and eventually returns to the balanced growth path of the economy.

There are many important instances in economic history that the end of expansionary monetary policy caused financial distress and led to a banking crisis, creating a "boom-and-bust" business cycle. Let us consider, therefore, the case in which the end of expansionary monetary policy ($-\sigma$, t_0, t_1) causes *unanticipated* banking crisis ($\sigma*$, t_1, t_2) at time t_1.[13] First, the dynamic

[13] The basic characteristics of the disequilibrium path will remain the same even if the banking crisis at time t_1 is anticipated at time t_0. The specific path to be taken, however, will be different between the two cases.

Chart 3

A Business Cycle with Asset Price Deflation
Triggered by a Restrictive Monetary Policy
or a Banking Crisis



process is essentially the same as that of Chart 2 up to time t_1. But, the banking crisis at time t_1 increases the cost of credit intermediation by σ^* above the natural level, which dislocates the economy from point C to point D as shown in Chart 4. Then, the rest of the dynamic process is analogous to that of Chart 3. The economy moves along a divergent disequilibrium path from point D to point F until time t_2 at which the banking crisis ends. At time t_2, the economy jumps up to point G on the equilibrium growth path, and thereafter converges to the balanced growth path represented by point E. This completes the monetary-and-real business cycle with an asset price bubble and a banking crisis.[14]

D. A Link between Dynamic Disequilibrium and Inflation

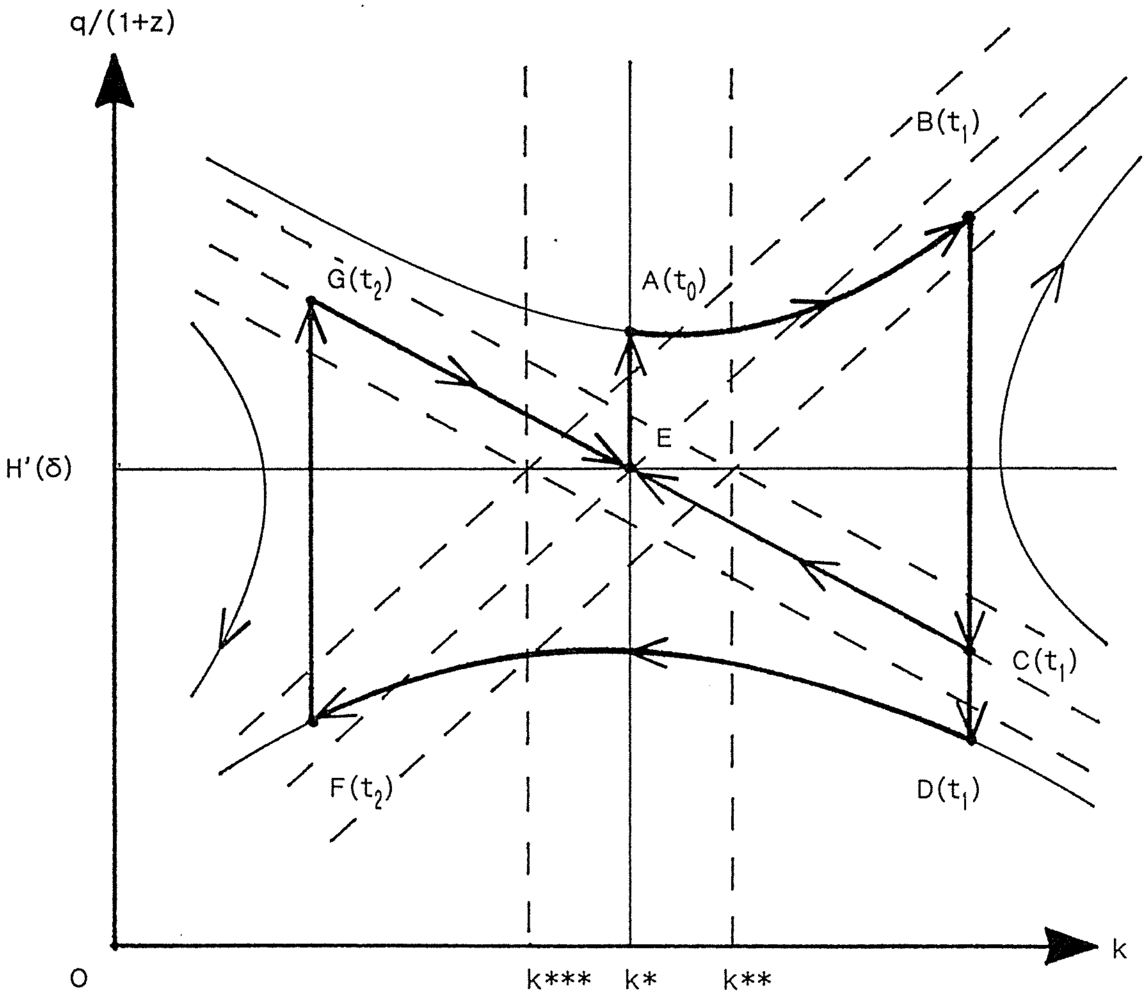
We have already pointed out that the asset price measures the intertemporal cost-of-living for labor in general equilibrium. We have also shown that the asset price signals a deviation of the cost of credit intermediation from the natural level or a deviation of the economy from the equilibrium growth path that is induced by a monetary policy shock. In this way, the asset price links a dynamic disequilibrium state of the real economy with inflation defined as an increase in the intertemporal cost-of-living for labor. In other words, the business cycle with asset price bubbles that is artificially caused by an expansionary monetary policy is associated with inflation in the context of dynamic general equilibrium or an *unwarranted* shift in income distribution from labor to capital.

---

[14] These business cycles may be interpreted as a modern representation of the Wicksellian cumulative process. It is interesting to note that Myrdal (1939) has emphasized the importance of "Tobin's q" in the Wicksellian monetary theory and explicitly related a deviation of "Tobin's q" from one with the start of a Wicksellian cumulative process.

Chart 4

A Monetary-and-Real Business Cycle
with an Asset Price Bubble and a Banking Crisis



E. The Role of Monetary Policy

What does the above analysis say about the role of monetary policy? It suggests that if the ultimate goal of monetary policy is to guide the economy onto a "non-inflationary balanced growth" path, it should aim at keeping the cost of credit intermediation at the natural level through time. To achieve this goal, however, the central bank needs an indicator of the dynamic state of the macroeconomy that can signal a deviation of the cost of credit intermediation from the natural level. Our analysis suggests that the asset price works as such an indicator.

The appropriate interpretation of asset price movements, however, requires further elaboration. For example, we can think of three different situations that the central bank must distinguish in order to achieve non-inflationary balanced growth through the asset-price-based monetary policy. The first situation is simply when the economy is already on the balanced growth path. The role of monetary policy in this case is to prevent itself from becoming a major source of macroeconomic disturbances. In particular, the central bank should avoid implementing a wrong policy that brings the economy off the balanced growth path, starting a business cycle with asset price inflation and deflation (Chart 2).

The second situation is when the economy is off the balanced growth path due to a banking crisis, which raises the cost of credit intermediation. The role of monetary policy in this case is to offset the temporary rise in the cost of credit intermediation until the banking crisis is over so that it can prevent the economy from going through a business cycle with asset price deflation.[15] For example, in the case of Chart 4, monetary policy can offset

---

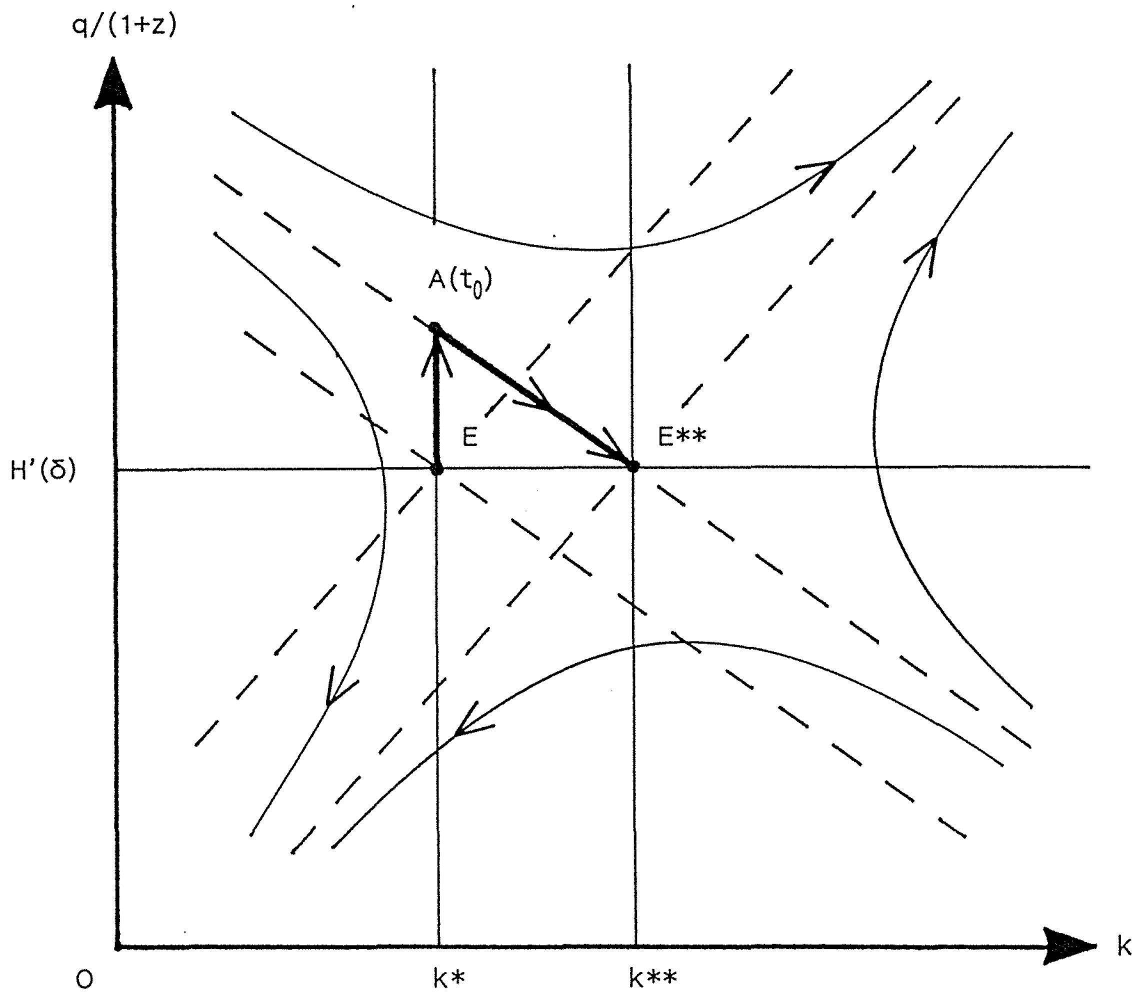
[15] At the same time, it is important for the monetary authorities to control the banking crisis for earlier resolution so that its duration and thus the recession can be minimized.

the rise in the cost of credit intermediation ($\sigma*$) at time t_1 so that the economy can move from point C to point E along the convergent equilibrium path, instead of going through the recessionary phase (C->D->F->G->E).

The third situation is when a productivity shock shifts the steady-state of the economy. For example, we can think of a case in which financial deregulation and innovations permanently reduce the cost of credit intermediation. In this case, the central bank should let the economy respond optimally to such a productivity shock without monetary interventions (Chart 5). A monetary policy that is aimed at offsetting the productivity shock in this case would dislocate the economy from the convergent equilibrium growth path and would trigger a new round of the business cycle with asset price deflation.

Chart 5

The Optimal Adjustment to a Permanent Productivity Shock
Such as Financial Deregulation and Innovations



IV. Concluding Remarks

We have developed a dynamic general equilibrium model that incorporates a banking sector and *inside money* into the *decentralized* Ramsey economy. Our model therefore provides an alternative to the monetary equilibrium model of Sidrauski (1967) with money in the utility function and that of Stockman (1981) with a cash-in-advance constraint, both of which are based on a *centralized* Ramsey economy with *outside money.* We have shown that monetary policies affect macroeconomic dynamics through the same mechanism as "real" shocks. Thus, our model can integrate both "monetary" and "real" business cycle theories within the common framework of the decentralized Ramsey economy. It suggests that we can analyze the dynamic effects of monetary and fiscal policies as well as productivity shocks within the same framework presented in Abel and Blanchard (1983) and Blanchard and Fischer (1989).

Moreover, our model emulates the business cycle with asset price bubbles. It shows how an expansionary monetary policy reduces the cost of credit intermediation and thus triggers the business cycle. The expansionary monetary policy temporarily dislocates the economy off the *equilibrium* growth path onto a divergent *disequilibrium* growth path. Meanwhile the asset price keep rising accumulating capital stock and then drop discontinuously as the expansionary monetary policy comes to an end. The economy then moves back to the equilibrium growth path and thereafter converges to the balanced growth path. However, if the end of the expansionary monetary policy is followed by a banking crisis, the economy could experience a prolonged period of economic slowdown like the Great Depression. Thus, the model is capable of analyzing the phenomenon of *disequilibrium dynamics.*

Let us conclude this paper summarizing its main features: First, it has shown how monetary policies can affect the real economy through the credit channel of monetary transmission, and also how the dynamic effects of monetary policies can be analyzed in a general equilibrium model. Second, it has shown how monetary policies and a banking crisis can generate business cycles with asset price inflation and deflation in a decentralized market economy with rational economic agents and market clearing. Third, it has linked a dynamic disequilibrium state of the real economy with inflation defined as an increase in the intertemporal cost of living for labor. Fourth, it has proposed that the central bank keep the cost of credit intermediation at the natural level through time so that the economy converges to the non-inflationary balanced growth path and adjusts optimally to productivity shocks. Fifth, in this context, it has argued that the asset price can be a useful indicator for monetary policy because it conveys information about the state of dynamic disequilibrium and inflation.

References

Abel, Andrew and Olivier Blanchard (1983): "An Intertemporal Model of Saving and Investment," *Econometrica* 51: 675-92.

Bernanke, Ben (1983): "Nonmonetary Effects of the Financial Crisis in the Propagation of the Great Depression," *American Economic Review* 73 (3): 257-76.

Bernanke, Ben and Alan Blinder (1992): "The Federal Funds Rate and the Channels of Monetary Transmission," American Economic Review 83 (4): 901-21.

Blanchard, Olivier and Stanley Fischer (1989): *Lectures on Macroeconomics*, Cambridge: The MIT Press.

Brunner, Karl and Allan Meltzer (1976): "An Aggregative Theory for a Closed Economy," in Jerome Stein (ed.), *Monetarism*, Amsterdam: North-Holland: 69-103.

Clower, Robert (1967): "A Reconsideration of the Microfoundation of Monetary Theory," *Western Economic Journal* 6: 1-8.

Hayashi, Fumio (1982): "Tobin's Marginal and Average q: A Neoclassical Interpretation," *Econometrica* 50: 213-24.

Kemp, Murray and Ngo Van Long (1977): "Optimal Control Problems with Integrands Discontinuous with Respect to Time," *The Economic Record* 53: 405-20.

Myrdal, Gunnar (1939): *Monetary Equilibrium*, New York: Augustus M. Kelley.

Shibuya, Hiroshi (1992): "Dynamic Equilibrium Price Index: Asset Price and Inflation," *Bank of Japan Monetary and Economic Studies* 10 (1): 95-109.

Sidrauski, Miguel (1967): "Rational Choice and Patterns of Growth in a Monetary Economy," *American Economic Review* 57 (Papers and Proceedings): 534-44.

Solow, Robert (1987): "Growth Theory and After," Nobel Lecture, December 8, 1987, in Robert Solow, *Growth Theory: An Exposition*, Oxford University Press.

Stockman, Alan (1981): "Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy," *Journal of Monetary Economics* 8: 387-93.

Tirole, Jean (1982): "On the Possibility of Speculation under Rational Expectations," *Econometrica* 50: 1163-81.

Tirole, Jean (1985): "Asset Bubbles and Overlapping Generations," *Econometrica* 53: 1499-1528.

Wicksell, Knut (1936): *Interest and Prices: A Study of the Causes Regulating the Value of Money*, New York: Augustus M. Kelley.

This Discussion Paper Series is published by the Institute of Economic Research and integrates two old ones published separately by the Department of Economics and the Department of Commerce.

Discussion Paper Series
Institute of Economic Research
Otaru University of Commerce

No.	Title	Author/s	Date
1.	ﾎｰｷﾝｽﾞ=ｻｲﾓﾝの条件に関する諸説の統合について	ﾀﾞｽｸﾞﾌﾟﾀ,ﾃﾞｨﾊﾞﾝｶｰ	Jul.1992
2.	Motivation and Causal Inferences in the Budgetary Control	Yoshihiro Naka	Aug.1992
3.	Проблемы управления рабочей силой на предприятиях Дальнего Востока(социологические аспекты)	Анатолий Михайлович Шкуркин	Nov.1992
4.	Dynamic Tax Incidence in a Finite Horizon Model	Jun-ichi Itaya	Jan.1993
5.	Business Cycles with Asset Price Bubbles and the Role of Monetary Policy	Hiroshi Shibuya	Jun.1993

Discussion Paper Series
Department of Economics
Otaru University of Commerce

No.	Title	Author/s	Date
1.	Monetary Policy in a Model of International Trade with a Sector Sticky Wage Rate	Takashi Fukushima & Hideki Funatsu	Feb.1985
2.	Export Credit Insurance	Hideki Funatsu	Feb.1985
3.	Asset Trading in an Overlapping-Generations Model:Efficiency of Competitive Equilibrium	Kenji Yamamoto	Oct.1985
4.	Asset Trading in an Overlapping-Generations Model with Production Shocks	Kenji Yamamoto	Oct.1985
5.	Immiserizing Investment in a Vertically Related International Trade	Masao Satake	Mar.1986
6.	Dynamic Tax Incidence in a Two-Class Economy	Jun-ichi Itaya	May 1986
7.	A Three Factor Model of International Trade with Minimum Wage Rates	Hideki Funatsu	May 1986
8.	A Note of the Maximum Number of Firms with Equal Market Share in a Quantity Setting Supergame	Masaru Uzawa	Nov.1986
9.	Tax Incidence in a Two-Sector Growing Economy with Perfect Foresight	Jun-ichi Itaya	May 1987
10.	Two Kinds of Information in Price Search	Kaoru Endo & Teruya Nagao	Aug.1987
11.	On the Hedging and Investment Behavior of the Competitive Firm under Price Uncertainty	Jun-ichi Itaya	Oct.1987

12.	Tax Incidence in a Two-Sector Growing Economy with Perfect Foresight:Long-Run Analysis	Jun-ichi Itaya	Mar.1988
13.	Comparative Statics for the Private Provision of Public Goods in a Conjectural Variations Model with Heterogeneous Agents	Dipankar Dasgupta & Jun-ichi Itaya	Mar.1991
14.	Capital Accumulation Game of Multifirms with External Adjustemnt Costs	Jun-ichi Itaya	Mar.1991
15.	Using the Correct Economic Interpretation to Prove the Hawkins-Simon-Nikaido Theorem:One More Note	Dipankar Dasgupta	Jul.1991
16.	Transversality Condition in Infinite Time Horizon Concave Problems	Tomoichi Shinotsuka	Oct.1991

Discussion Paper Series
Department of Commerce
Otaru University of Commerce

No.	Title	Author/s	Date
1.	分権化組織における部門間調整と情報ｲﾝｾﾝﾃｨﾌﾞ･ｼｽﾃﾑの設計	井上正 & 鵜野好文	Apr.1985
2.	日本的雇用慣行とその経済合理性	井上正 & 鵜野好文	May 1989

Institute of Economic Research, Otaru University of Commerce
3-5-21, Midori, Otaru, Hokkaido 047, Japan　Tel.0134-23-1101

小樽商科大学経済研究所
〒047　北海道小樽市緑3丁目5番21号　Tel.0134-23-1101㈹