# Effect of Stochastic Participation in Demographic Donor-Recipient Game ${ }^{1}$ 

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#### Abstract

We consider the effect of stochastic participation in game on the emergence of cooperation in Demographic Donor-Recipient Game. In usual model setting, players are supposed to play games actually with probability one if they are faced to play games. But sometimes players may want to play games, and sometimes may not. We incorporate this tendency to play games into usual Demographic model by assuming that players play game actually with their inheriting probabilities if they are faced to play games. In Demographic Donor-Recipient game, players are initially randomly distributed in square lattice of cells. In each period, players move locally to random cell in neighbors or globally to random unoccupied cell in the whole lattice, and face to play multiple games against local neighbors or against randomly selected global players. In each game, one player is selected as Donor and the other as Recipient randomly and they play the game actually with the product of two Donor's and Recipient's probabilities of participation. Donor has two moves Cooperate or Defect; Cooperate means Donor pays cost for Recipient to receive benefit. Defect means Donor does nothing. We restrict value of these probabilities to be one of $\mathrm{H}(\mathrm{igh}), \mathrm{L}(\mathrm{ow})$ probability, or one. If wealth (accumulated payoff) of player becomes negative or his age becomes greater than his lifetime, he dies. If his wealth becomes greater than some amount and there is unoccupied cell in neighbors, he has an offspring. We show, by Agent-Based Simulation, that the emergence rate of cooperation is fairy small if initial population contains players who have Low probability of participation if he is a Donor and probability of one if he is a Recipient, but introducing variableness of these probabilities promotes the emergence of cooperation.


Keywords: stochastic participation, demographic game, Donor-Recipient game, emergence of cooperation, generalized reciprocity, Agent-Based Simulation

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## 1 Introduction

Emergence of cooperation in repeated dilemma game is a very fascinating and important topic. This paper investigates the effect of stochastic participation on the emergence of cooperation in demographic Donor-Recipient (DR) game. DR game is a two-person game where one player is randomly selected as Donor and the other as Recipient. Donor has two moves, Cooperate and Defect. Cooperate means Donor pays cost $c$ in order for Recipient to receive benefit $b(b>c>0)$. Defect means Donor does nothing. Note that Recipient has no move.

Epstein [1] introduces demographic model. He shows the emergence of cooperation where AllC and AllD are initially randomly distributed in a square lattice of cells. In each period, players move locally (that is, to random cell within the neighboring 4 cells, that is, north, west, south, and east cells; von Neumann neighbors, if unoccupied) and play Prisoner's Dilemma (PD) game against local (neighboring) player(s). If wealth (accumulated payoff) of a player becomes negative or his age becomes greater than his lifetime, he dies. If his wealth becomes greater than some amount and there is an unoccupied cell in von Neumann neighbors, he has an offspring and gives the offspring some amount from his wealth. Thus the local interaction in the spatial structure is an important element in the emergence of cooperation. Namekata and Namekata [2] extend Epstein's original model discussed above by introducing global move, global play, and Reluctant players into demographic PD game. Reluctant players delay replying to changes and use extended forms of tit for tat (TFT). They consider the effect of Reluctant players on the emergence of cooperation and show cases where the reluctance promotes the emergence of cooperation. Here TFT Cooperates at first game and at later games uses the same move as the opponent did in the previous game. Thus the reluctance to respond the opponent's change is also an important element in the emergence of cooperation. Namekata and Namekata [3] examine the effect of move-play pattern on the emergence of cooperation and the distribution of strategies. They restrict patters of move and play of a player to simple structure; local or global, where local or global means that with high probability the player moves (plays) locally or globally, respectively. They show that cooperative strategies evolutionarily tend to move and play locally, defective do not, and AllC and AllD are abundant unless all strategies initially play locally.

Nowak and Sigmund [4] consider the emergence of cooperation in different setting where two players are randomly matched, one is selected as Donor and the other as Recipient at random, and play DR game at each period. Frequency of a strategy at the next period is proportional to the payoff of the strategy earned at the current period, which is different from that in our demographic model. The chance that the same two players meet again over periods is very small. Every player has his own image score that takes on some range, is initially zero, and increases or decreases by one if he cooperates or defects, respectively. Donor decides his move (Cooperate or Defect) depending on the opponent's image score. Riolo et al. [5] deal with similar repeated DR game setting where, instead of im-
age score, every player has his own tag and tolerance and Donor cooperates only if the difference between his tag and the opponent's is smaller than his tolerance.

Szabó and Hauert [6] consider the effect of voluntary participation on the emergence of cooperation in PD games on square lattice or random regular graphs. Besides usual AllC and AllD in PD game they introduce the third player called Loner in their model. When every player plays a game against Loner, he and the Loner always obtain the fixed payoff that is better than the payoff between two AllD's and worse than the payoff between two AllC's. Thus in their voluntary participation model two players actually play different game other than PD game if at least one of them is Loner.

In general, reciprocity explains the emergence of cooperation in several situations [7] where players act according to their experience or to the previous actions of the opponent: Direct reciprocity assumes that a player plays games with the same opponent repeatedly and he determines his move depending on moves of the same opponent. If a player plays games repeatedly and the opponents may not be the same one, indirect (downstream) reciprocity assumes that the player determines his move to the current opponent depending on the previous moves of this current opponent, or indirect upstream reciprocity, or generalized reciprocity, assumes that the player determines his move to the current opponent depending on the previous experience of his own. Since a player in our model and Namekata and Namekata [2,3] determines his move depending on his own previous experience, they deal with generalized reciprocity. Nowak and Sigmund [4] deal with indirect (downstream) reciprocity because Donor determines his move to his opponent Recipient depending on the image score of the Recipient that relates to the previous moves of the Recipient. There is no reciprocity, either direct or indirect in the model of Riolo et al. [5] because Donor's move does not depend on the opponent's previous moves as well as his own previous experience. The reluctance introduced in $[2,3]$ is an important element promoting the cooperation in generalized reciprocity (indirect upstream reciprocity).

In this paper we introduce stochastic participation in game: Each player has his own two inheriting participation probabilities, one as a Donor and the other as a Recipient. Thus, if two players are faced to play a game, the game between a Donor and a Recipient is actually played with the product of two probabilities, the participation probability of the Donor and that of the Recipient. Althogh Szabó and Hauert [6] introduce Loner who is unwilling to participate in the social enterprise and play a different game from PD game, we introduce stochastic participation to deal directly with this unwillingness to participate in the game. In our model every player plays the same DR game but he has possibly different inheriting participation probabilities from those of others. We investigate the effect of stochastic participation in game on the emergence of cooperation in Demographic Donor-Recipient Game.

In Section 2, we explain our model in detail. In Section 3, results of simulation are discussed. And Section 4 concludes the paper.

## 2 Model

We start with extending TFT as follows in order to introduce reluctant strategy: Let $m=2 ; t=0, \ldots, m+1 ; s=0, \ldots, m$. Strategy ( $m, t ; s$ ) is illustrated in Fig 1. It has $m+1$ inner states. The inner states are numbered $0,1, \ldots, m$; thus $m$ is the largest state number. State $i$ is labeled $\mathrm{D}_{i}$ if $i<t$ or $\mathrm{C}_{i}$ if not. If current state is labeled C or D , then the strategy prescribes using C or D , respectively. In other words, the strategy prescribes using D if the current state $i<t$ and using C if not; thus the value $t$ is the threshold which determines the move of a player. Initial state is state $s$; its label is $\mathrm{D}_{s}$ if $s<t$ or $\mathrm{C}_{s}$ if not. If current state is $i$, then the next state is $\min \{i+1, m\}$ or $\max \{i-1,0\}$ given that the opponent uses C or D , respectively, in this game. If $m>1$, then the strategy may delay replying to its opponent's change. Thus strategy ( $m, t ; s$ ) is an extended form of TFT. To sum up, our strategies are expressed as ( $m, t ; s$ ); $m$ is the largest state number, $t$ is the threshold, and $s$ is the initial state number. The initial state is denoted as $(m, t ; *)$ if it is determined randomly. We also omit the initial state like ( $m, t$ ) if we have no need to specify it. Note that reluctant strategy ( $m, t ; s$ ) by itself decides its move to the current opponent depending on the previous experience of its own, meaning indirect upstream reciprocity, that is, generalized reciprocity. AllC is denoted by $(2,0)$ and AllD by $(2,3)$.


Fig. 1. Strategy component ( $m, t ; s$ ) in case of $t<s<m$. Circles denote inner states. Initial state is the state pointed by arrow labeled "initial state". Threshold divides states into two subclasses; one prescribes using D and the other using C . The transition between states occurs along the arrow labeled C or D if the opponent uses C or D , respectively.

We deal with DR game as a stage game. DR game is a two-person game where one player is selected as Donor and the other as Recipient randomly. Donor has two moves, Cooperate (C) and Defect (D). C means Donor pays cost $c$ in order for Recipient to receive benefit $b(b>c>0)$. Defect means Donor does nothing. Note that Recipient has no move. We assume that each player is faced to play 6 games against (possibly different) players at each period. Since it is common in demographic dilemma game that the sum of payoffs of a player, in two successive games once as Donor and once as Recipient, to be positive if the opponent uses C and negative if D and the worst sum of a player is equal to the best sum in absolute value, we transform the original payoffs to new ones by subtracting constant $x$. Constant $x$ is given by $(b-c) / 4$. We set $b=5$ and $c=1$ in this paper. (cf. $b / c=10$ in
[4] and $b / c=5 / 3,2,2.5,10 / 3,5.10$, and 20 in [5]). Table 1 shows the transformed payoff matrix of DR game.

In this paper, we introduce stochastic participation in game: Each player has his own two inheriting participation probabilities, rateOfPlayGameAsDonor (rPlayAsD)

Table 1. Payoff matrix of DR game.

|  |  | Recipient |
| :--- | :--- | :--- |
| Donor | C | $-c-x=-2, b-x=4$ |
|  | D | $-x=-1,-x=-1$ | as a Donor and rateOfPlayGameAsRecipient (rPlayAsR). If two players are faced to play a game, the game between a Donor and a Recipient is actually played with the product of two probabilities, $r$ PlayAsD of the Donor and rPlayAsR of the Recipient. We restrict value of these probabilities to be one of $\mathrm{H}(\mathrm{igh})(=0.95)$, $\mathrm{L}(\mathrm{ow})(=0.6)$ probability, or I $(=1.0)$.

A player has the following properties that are inherited from parents to offspring; rPlayAsD, rPlayAsR, expected Cooperation rate (ecCr), rateOfGlobalMove ( $r G M$ ), rateOfGlobalEncounter ( $r G E$ ), and strategy; whose initial distributions are summarized in Table 2.

Table 2. Initial distribution of inheriting properties

| property | initial distribution |
| :---: | :---: |
| $\begin{aligned} & (\text { rPlayAsD } \\ & \text { rPlayAsR }) \end{aligned}$ | We deal with 14 distributions; LL, LI, IL, HH, HI, IH, II, LI+HI, IL+IH, $\mathrm{LL}+\mathrm{HH}+\mathrm{LI}+\mathrm{HI}, \mathrm{LL}+\mathrm{HH}+\mathrm{IL}+\mathrm{IH}, \mathrm{LL}+\mathrm{HH}, \mathrm{LI}+\mathrm{HI}+\mathrm{IL}+\mathrm{IH}$, and $\mathrm{LL}+\mathrm{HH}+\mathrm{LI}+\mathrm{HI}+\mathrm{IL}+\mathrm{IH}$. The first letter and the second of two letters indicate the value of $r$ PlayAsD and that of $r$ PlayAsR. L, H, and I are 0.6 (Low), 0.95 (High), and 1.0 (I), respectively. Further, letters connected with " + " means that one of two letters is selected randomly among them and the selected two letters determines $r$ PlayAsD and $r$ PlayAsR, respectively. For example, LI+HI means " $r$ PlayAsD=0.6 and $r$ PlayAs $R=1.0$ " or " $r$ PlayAsD $=0.95$ and $r$ PlayAs $R=1.0$ " is selected randomly. |
| $e c \mathrm{Cr}$ | Uniformly distributed in interval ( $0.2,0.5$ ). |
| $(r G M, r G E)$ | We deal with distribution, $\{(1 / 4) 11,(1 / 4) l \mathrm{lg},(1 / 4) \mathrm{gl},(1 / 4) \mathrm{gg}\}$. For example, gl means $r G M$ is distributed in interval g and $r G E$ in interval 1 , where $\mathrm{l}:=(0.05,0.2)$ and $\mathrm{g}:=(0.8,0.95)$, indicating to move globally and encounter locally. $\{(1 / 4) l l,(1 / 4) \lg ,(1 / 4) \mathrm{gl},(1 / 4) \mathrm{gg}\}$ means $r G M$ and $r G E$ are selected randomly among $11, \mathrm{lg}, \mathrm{gl}$, and gg . |
| strategy | We deal with distribution, Rlct-2:=\{(1/4)(2,0), (1/4)(2,1;*), (1/4)(2,2;*), $(1 / 4)(2,3)\}$. The notation means that with probability $1 / 4$ strategy $(2,0)$ (AllC) is selected, with probability $1 / 4$ strategy $(2,1 ; *)$ is selected, and so on, where * indicates that initial state is selected randomly. Note that initially $50 \%$ of players use C on the average since both AllC and AllD are included with the same probability and so are both ( $m, t ; *)$ and ( $m, m-t+1 ; *$ ). Rlct-2 means Reluctant strategies with $m=2$. |

In period $0, N(=100)$ players are randomly located in 30-by-30 lattice of cells. The left and right border of the lattice are connected. If a player moves outside,
for example, from the right border, then he comes inside from the left border. The upper and lower border are connected similarly. Players use strategies of ( $m, t ; s$ ) form. Initial wealth of every player is 6 . Their initial (integer valued) age is randomly distributed between 0 and deathAge ( $=50$ ).

In each period, each player ( $\left.1^{\text {st }}\right)$ moves, and ( $\left.2^{\text {nd }}\right)$ plays DR games given by Table 1 against other players. Positive payoff needs opponent's C. (The detailed description of $\left(1^{\text {st }}\right)$ move and $\left(2^{\text {nd }}\right)$ play is given in Table 3.) The payoff of the game is added to his wealth. If the resultant wealth is greater than fissionWealth $(=10)$ and there is an unoccupied cell in von Neumann neighbors, then the player has an offspring and give the offspring 6 units from his wealth. His age is increased by one. If the resultant wealth becomes negative or his age is greater than deathAge $(=50)$, then he dies. Then next period starts.

Table 3. Detailed description
(1) With probability $r G M$, a player moves to random unoccupied cell in the whole lattice. If there is no such cell, he stays at the current cell. Or with probability $1-r G M$, a player moves to random cell in von Neumann neighbors if it is unoccupied. If there is no such cell, he stays at the current cell.
(2) With probability $r G E$, the opponent whom a player encounters to play DR game is selected at random from all players (except himself) in the whole lattice. Or with probability $1-r G E$, the opponent is selected at random from von Neumann neighbors (no interaction if none in the neighbors). The DR game is actually played with the product of two probabilities, $r$ PlayAsD of the Donor and $r$ PlayAsR of the Recipient. This process is repeated 6 times. (Opponents are possibly different.)
(1) describes move and (2) describes play in detail.

In our simulation we use synchronous updating, that is, in each period, all players move, then all players play, then all players have an offspring if possible. We remark that the initial state of the offspring's strategy is set to the current state of the parent's strategy. There is a small mutationRate $(=0.05)$ with which they are not inherited. Initial distribution of inheriting properties given in Table 2 is also used when mutation occurs. We assume that with errorRate ( $=0.05$ ) a player makes mistake when he makes his move. Thus AllC may Defect sometime.

Note that the initial distribution of strategy is Rlct-2 (including AllC, (2,1), $(2,2)$, and AllD). Also that the initial distribution of $(r G M, r G E)$ has simple structures; with high probability a player moves and encounters locally or globally, thus there are 4 move-encounter patters such as $11, \mathrm{lg}, \mathrm{gl}$, and gg .

Especially note that the initial distribution of (rPlayAsD, rPlayAsR) has also simple structure; $r$ PlayAs $X$ takes one of $\mathrm{L}=0.6, \mathrm{H}=0.95$, and $\mathrm{I}=1.0$. The initial distribution of (rPlayAsD, rPlayAsR) is one of 14 distributions, LL, LI, IL, HH, HI, IH, II, LI+HI, IL+IH, LL+HH+LI+HI, LL+HH+IL+IH, LL+HH, LI+HI+IL+IH, and LL+HH+LI+HI+IL+IH. We exclude LH and HL because of simplicity. But we include, for example, LI because LI implies a type of players who Follow-

Donor when they decide to participate in a game or not. We call this type of player LI or Low FollowDonor type. Also IH implies a type of players who FollowRecipient. We call them IH or High FollowRecipient type. We also include LL $(\mathrm{HH})$ because they imply a type of players who do not discriminate their role between Donor and Recipient when he decides to participate in a game or not. We call them LL (HH) or Low (High) NoDiscrimination type. The last 7 distributions, $\mathrm{LI}+\mathrm{HI}, \mathrm{IL}+\mathrm{IH}, \mathrm{LL}+\mathrm{HH}+\mathrm{LI}+\mathrm{HI}, \mathrm{LL}+\mathrm{HH}+\mathrm{IL}+\mathrm{IH}, \mathrm{LL}+\mathrm{HH}, \mathrm{LI}+\mathrm{HI}+\mathrm{IL}+\mathrm{IH}$, and $\mathrm{LL}+\mathrm{HH}+\mathrm{LI}+\mathrm{HI}+\mathrm{IL}+\mathrm{IH}$ are symmetric in the sense that the number of players with Low rPlayAs $X$ value is equal to those with High $r$ PlayAs $X$ value on the average, thus they are practically plausible initial distribution. The other 7 distributions are only reference ones. The last 3 distributions, $\mathrm{LL}+\mathrm{HH}, \mathrm{LI}+\mathrm{HI}+\mathrm{IL}+\mathrm{IH}$, and $\mathrm{LL}+\mathrm{HH}+\mathrm{LI}+\mathrm{HI}+\mathrm{IL}+\mathrm{IH}$, are of special interest because they are further symmetric between two roles, Donor and Recipient. We investigate the effect of stochastic participation on the emergence of cooperation first in fixed participation type case, that is, inheriting participation probabilities, $r$ PlayAs $D$ and $r$ PlayAs $R$ of a player are fixed throughout his lifetime. Next we investigate the effect of stochastic participation in variable participation type case; players vary their rPlayAsX according to their experience as Recipient.

## 2.1 variable participation type case

In real life, good experience makes people want to play game more often and bad experience makes them want to play game less often. We introduce this feature in to our model as follows: Players keep the results of the last 10 DR games as Recipient. After every DR game as Recipient, the player revises their $r$ PlayAs $X$ according to the comparison between (his experienced Cooperation rate of the last 10 DR games as Recipient) $=$ (the number of experienced C moves of the last 10 DR games as Recipient)/10 and his expected Cooperation rate (ecCr) as follows:

1. Set $r$ PlayAs $X$ to be H if $r$ PlayAs $X$ is L and (his experienced Cooperation rate of the last 10 DR games as Recipient) is larger than or equal to his $e c \mathrm{Cr}$.
2. Set $r$ PlayAs $X$ to be L if $r$ PlayAs $X$ is H and (his experienced Cooperation rate of the last 10 DR games as Recipient) is less than his ecCr.

Here expected Cooperation rate (ecCr) of a player is his inheriting property and expresses his subjective rate at which he expects the society is cooperative. Note that we assume ecCr is uniformly distributed in interval $(0.2,0.5)$ and thus its average is 0.35 in Table 2. After every game as Recipient the player changes his participation type, from LI to HI, from IL to IH, or from LL to HH if his experienced Cr is larger than or equal to his subjective $e c C r$, and from HI to LI , from IH to IL, or from HH to LL if his experience Cr is less than his subjective $e c \mathrm{Cr}$. As we will see in the next section this variableness promotes cooperation.

## 3 Simulation and Result

Our purpose to simulate our model is to examine the effect of stochastic participation on the emergence of cooperation. We use Repast Simphony 2.2 ( http://repast.sourceforge.net/ ) to simulate our model.

We execute 300 runs of simulations in each different setting. We judge that the cooperation emerges in a run if there are more than 100 players and the average C rate is greater than 0.2 at period 500 , where the average C rate at a period is the average of the player's average $C$ rate at the period over all players and the player's average C rate at the period is defined as the number of C moves used by the player divided by the number of games played as Donor at the period. (We interpret $0 / 0$ as 0 .) This average $C$ rate is the rate at which we see cooperative move C as an outside observer. We call a run in which the cooperation emerges a successful run. Since negative wealth of a player means his death in our model and he has a lifetime, it is necessary for many players to use C so that the population is not extinct. We are interested in the emergence rate of cooperation (Ce) that is the rate at which the cooperation emerges.

## 3.1 fixed participation type case

What is the effect of introducing stochastic participation on the emergence of cooperation? We summarize the emergence rate of cooperation, Ce , in Table 4 for fixed participation type case. The first column indicates initial distribution of (rPlayAsD, rPlayAsR) and the second Ce in fixed participation type case. Its graph is depicted in Fig. 2. Ce for LL is 0.293 and is very small (the minimum) compared with other values. Ce for II is not the maximum over 14 distributions. The maximum is attained at IH.

We draw the horizontal straight line " 0.530 " that has the same value as LI (Low FollowDonor type) in Fig. 2. The initial distributions for which Ce is less than 0.530 are $\mathrm{LL}, \mathrm{IL}, \mathrm{LI}+\mathrm{HI}$, $\mathrm{LL}+\mathrm{HH}+\mathrm{LI}+\mathrm{HI}, \mathrm{LI}+\mathrm{HI}+\mathrm{IL}+\mathrm{IH}$, and

Table 4. Ce in fixed participation type case

| Distribution of $r$ PlayAs $X$ | Ce |
| :--- | :---: |
| LL | 0.293 |
| LI | 0.530 |
| IL | 0.497 |
| HH | 0.640 |
| HI | 0.670 |
| IH | 0.693 |
| II | 0.633 |
| LI+HI | 0.500 |
| IL+IH | 0.597 |
| LL+HH+LI+HI | 0.460 |
| LL+HH+IL+IH | 0.583 |
| LL+HH | 0.570 |
| LI+HI+IL+IH | 0.433 |
| LL+HH+LI+HI+IL+IH | 0.447 | $\mathrm{LL}+\mathrm{HH}+\mathrm{LI}+\mathrm{HI}+\mathrm{IL}+\mathrm{IH}$. They include LI except IL and LL. We conclude the

following observation:

1. (fixed participation type case) The emergence rate of cooperation, Ce, for LL is very small. Ce is fairy small if the initial distribution contains LI or Low FollowDonor type. Ce for IL is also fairy small.


Fig. 2. Ce in fixed participation type case

## 3.2 variable participation type case

What is the effect of further introducing variableness of participation type on the emergence of cooperation? We summarize the emergence rate of cooperation, Ce , in Table 5 for variable participation type case (as well as fixed participation type case). The first

Table 5. Ce and Ce_v in variable participation type case

| Distribution of $r$ PlayAs $X$ | Ce | Ce_v | Ce_v/Ce |
| :--- | :---: | :---: | :---: |
| LI+HI | 0.500 | 0.650 | 1.30 |
| IL+IH | 0.597 | 0.653 | 1.09 |
| LL+HH+LI+HI | 0.460 | 0.720 | 1.57 |
| LL+HH+IL+IH | 0.583 | 0.650 | 1.11 |
| LL+HH | 0.570 | 0.727 | 1.27 |
| LI+HI+IL+IH | 0.433 | 0.587 | 1.35 |
| LL+HH+LI+HI+IL+IH | 0.447 | 0.610 | 1.37 | column indicates initial distribution of participation types and the third column (Ce_v) Ce in variable participation type case (the second column (Ce) Ce in fixed participation type case as well, as a reference). Its graph is depicted in Fig. 3. We assume that any type initially not in the



Fig. 3. Ce in fixed and variable participation type case
distribution cannot be born later in variable participation type case. Thus the last 7 initial distributions of Table 4 are our interest because both LI and HI, both IL and IH , or both LL and HH should be in the initial distribution. Every value in Ce_v column is larger than the corresponding value (in the same row) in Ce column and also than 0.530 . Thus the variableness of rPlayAsX promotes the emergence of cooperation. Note that in Table 5 the ratio Ce _v/Ce is larger than or equal to 1.30 only for the distributions including LI. Thus the variableness of $r$ PlayAs $X$ is more effective in the promotion of the emergence of cooperation only for the distribution including LI.

We next examine the average distribution of participation types at period 500 over successful runs. Fig. 4 and 5 shows them in fixed and variable participation type case for the initial distribution, $\mathrm{LL}+\mathrm{HH}+\mathrm{LI}+\mathrm{HI}+\mathrm{IL}+\mathrm{IH}$. In fixed participation type case only LI type has large share. In variable participation type case HI type has the largest share, LI type has the second share, and the others have very small share. That is, Fig. 4 and 5 have the feature that only one or two bars are large. Thus introducing variableness of participation type makes the rate of LI type smaller and the rate of HI type larger, thus the ratio $\mathrm{HI} / \mathrm{LI}$ larger. The distributions of participation types at period 500 for other initial distributions of participation types have the same feature. Keeping Fig. 4 and 5 in mind, we are interested in, for example, the ratio, rate of HI type / rate of LI type if at least one of two rates is not small. For example, if the initial distribution is LL+HH+LI+HI+IL+IH, then the distribution of participation type at period 500 over successful runs is the form of $\left(r_{\mathrm{LL}}, r_{\mathrm{HH}}, r_{\mathrm{LI}}, r_{\mathrm{H}}, r_{\mathrm{IL}}, r_{\mathrm{IH}}\right)\left(r_{\mathrm{LL}}+r_{\mathrm{HH}}+r_{\mathrm{LI}}+r_{\mathrm{HI}}+r_{\mathrm{IL}}+r_{\mathrm{IH}}=1\right)$. We are interested in the ratio $r_{\mathrm{HH}} / r_{\mathrm{LL}}$ if at least one of $r_{\mathrm{HH}}$ and $r_{\mathrm{LL}}$ is larger than 0.16 , the ratio $r_{\mathrm{HI}} / r_{\mathrm{LI}}$ if at least one of $r_{\mathrm{HI}}$ and $r_{\mathrm{LI}}$ is larger than 0.16 and so on. These ratios in fixed and variable participation type case are shown in Table 6 where, for example, $r_{\mathrm{HH}} / r_{\mathrm{LL}}$ is expressed as HH/LL. "-" means that the corresponding ratio is not available. "- -


Fig. 4. Distribution of participation types at period 500 (fixed participation type)


Fig. 5. Distribution of participation types at period 500 (variable participation type)
" means that the ratio is also not available because the corresponding two rates are both small and less than 0.16. In Table 6 for the initial distributions including LI, that is, $\mathrm{LI}+\mathrm{HI}, \mathrm{LL}+\mathrm{HH}+\mathrm{LI}+\mathrm{HI}, \mathrm{LI}+\mathrm{HI}+\mathrm{IL}+\mathrm{IH}$, and $\mathrm{LL}+\mathrm{HH}+\mathrm{LI}+\mathrm{HI}+\mathrm{IL}+\mathrm{IH}, \mathrm{HI} / \mathrm{LI}$ is very small (less than 0.015), which implies LI or Low FollowDonor type dominates the population. In Fig. 3 the corresponding Ce's of these initial distributions including LI are fairy small (less than 0.530 ). We conclude that the dominance of LI or Low FollowDonor type is the reason why Ce is fairy small in fixed participation type case. Next HI/LI_v (HI/LI in variable participation type case) is not small (around 1.7) for the initial distributions including LI in Table 6, which implies HI or High FollowDonor type has larger share compared with LI or Low FollowDonor type in variable participation type case. In Table 5 the corresponding

Table 6. The ration HH/LL in fixed case and HH/LL_v in variable case, etc.

|  | HH/LL | HH/LL_v | HI/LI | HI/LI_v | IH/IL | IH/IL_v |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| LI+HI | - | - | 0.0141 | 1.80 |  | - |
| IL+IH | - | - | - | - | 66.0 | 5.77 |
| LL+HH+LI+HI | - | -- | 0.0139 | 1.68 | - | - |
| LL+HH+IL+IH | 1.59 | 1.91 | - | - | -- | -- |
| LL+HH | 1.98 | 2.25 | - | - | - | - |
| LI+HI+IL+IH | - | - | 0.00884 | 1.78 | -- | -- |
| LL+HH+LI+HI+IL+IH | -- | -- | 0.0182 | 1.66 | -- | -- |

"-" means the entity is not available. "- -" means, for example, both HH and LL is less than 0.16.

Ce_v/Ce's of the initial distributions including LI are larger than or equal to 1.30 , which implied that the variableness of participation type is more effective in the promotion the emergence of cooperation. We conclude that the larger share of HI or High FollowDonor compared with LI or Low FollowDonor is the reason why Ce in variable participation type case is effectively larger than that in fixed participation type case. In Table 6, HH/LL is already larger than 1.5 and HH/LL_v is larger than HH/LL for LL+HH+IL+IH and LL+HH. Thus the larger share of HH or High NoDiscrimination type compared with LL or Low NoDiscrimination type is the reason why Ce in fixed participation type case is already large. We cannot judge whether IH/IL and IH/IL_v for IL+IH distribution in Table 6 have direct relation with Ce or not. We conclude the following observation in relation to variable participation type case:
2. (variable participation type case compared with fixed case) Introducing variableness of participation type promotes the emergence rate of cooperation, Ce , compared with fixed participation type case. Especially this promotion is more effective if the initial distribution includes LI or Low FollowDonor type. This promotion is because (a) HI or High FollowDonor type has larger share compared with LI or Low FollowDonor type at period 500 in variable participation type case, or (b) the ratio of the rate HH and the rate LL in variable participation type case is larger than the corresponding ratio in fixed participation type case.

### 3.3 One sample path from period 0 to period 500

Let us pick up one sample path from period 0 to period 500 for the initial distribution $\mathrm{LL}+\mathrm{HH}+\mathrm{LI}+\mathrm{HI}+\mathrm{IL}+\mathrm{IH}$ in variable participation type case and discuss why HI or High FollowDonor type has larger share compared with LI or Low Follow-

Donor type. First Fig 6 shows the scatter diagram of two rates, the rate of AllC and that of $(2,1)$ in the average distribution of strategies at period 500 over successful runs for $\mathrm{LL}+\mathrm{HH}+\mathrm{LI}+\mathrm{HI}+\mathrm{IL}+\mathrm{IH}$ in variable participation type case. The rate of $(2,2)$ is very small (smaller than 0.08 and not shown in detail). We pick up one successful run that is surrounded with circle in Fig. 6. The distribution of strategies in this run from period 0 to period 500 is depicted in Fig. 7. The rate of $(2,1)$ peaks at period 13 , decreases until around period 50 , and then remains at the same level. The rates of AllC and $(2,2)$ almost vanish until around 10.

Next we examine distribution of move-encounter patterns of the strategy $(2,1)$ in the run from period 0 to period 500. It is depicted in Fig. 8. The local move-


Fig. 6. Scatter diagram of the rate of AllC and that of $(2,1)$ in the average distribution of strategies at period 500 over successful runs for $\mathrm{LL}+\mathrm{HH}+\mathrm{LI}+\mathrm{HI}+\mathrm{IL}+\mathrm{IH}$ in variable participation type case


Fig. 7. Distribution of strategies from period 0 to period 500 for $\mathrm{LL}+\mathrm{HH}+\mathrm{LI}+\mathrm{HI}+\mathrm{IL}+\mathrm{IH}$ in variable participation type case
local encounter pattern peaks at period 13, then decreases and has the similar shape as the rate of $(2,1)$ in Fig. 7. Any other move-play patters almost vanish from the beginning. Thus almost all players using strategy $(2,1)$ move locally and encounter locally.

Next we examine the distribution of participation types of $(2,1)$ in the run from period 0 to period 500. It is depicted in Fig. 9. The rate of participation type HI peaks at period 12, decreases, and remains at the same level (around 0.5). The rate of participation type LI finally remains at the same level (around 0.06). The other rates almost vanish until around period 10 . Thus $(2,1) \mathrm{HI}$ type has the largest share, $(2,1)$ LI type has the second share, and other participation types of $(2,1)$ vanish.


Fig. 8. Distribution of move-encounter patterns from period 0 to period 500 for $\mathrm{LL}+\mathrm{HH}+\mathrm{LI}+\mathrm{HI}+\mathrm{IL}+\mathrm{IH}$ in variable participation type case


Fig. 9. Distribution of participation types from period 0 to period 500 for $\mathrm{LL}+\mathrm{HH}+\mathrm{LI}+\mathrm{HI}+\mathrm{IL}+\mathrm{IH}$ in variable participation type case

Let us ask a question why $(2,1) \mathrm{HI}$ type has the largest share in this variable participation type case. In variable participation type case, players revise their participation type according to their recent ( 10 games) experience as Recipient. In order for $(2,1) \mathrm{HI}$ type to increase and keep its share, $(2,1)$ needs to be more often cooperated by other $(2,1)$. We guess some elements of this story as follows:

1. (Local move local - encounter of cooperative strategy) Many $(2,1)$ move locally and encounter locally.
2. (Cluster of cooperative strategies) Many $(2,1)$ cluster together.
3. (Cooperation in cluster) Many $(2,1)$ in the cluster cooperate with each other.
4. (Maintenance of HI by cooperation) $(2,1) \mathrm{LI}$ type varies to $(2,1) \mathrm{HI}$ type or $(2,1) \mathrm{HI}$ type remains the same by the variableness of participation type.

Fig. 8 implies 1 (local move local encounter of cooperative strategy). Similarity of two graphs, $(2,1) 11$ and $(2,1) \mathrm{HI}$, in Fig. 8 and 9, implies the coincidence of 1 (local move and local encounter of cooperative strategy) and 4 (maintenance of HI by cooperation). The detail from period 0 to period 17 of Fig. 8 and 9 is depicted in Fig. 10 and confirms the similarity of two graphs. The coincidence of 1 and 4 are related through 2 (cluster of cooperative strategies) and 3 (cooperation in cluster).

We see the spatial distribution of strategies and participation types and how cooperative strategies suddenly cluster together and the cluster grows in Fig. 11. In Fig. 11 the color of AllC, $(2,1),(2,2)$ and AllD are cyan, blue, magenta, and red, respectively. A player is represented as rectangle or oval. The direction of the longer side of a rectangle or the longer axis of oval determines the participation type of the player. Vertical direction implies HI and horizontal direction LI. (Not vertical but) More vertical implies HH or IH. (Not horizontal but) More horizontal implies LL or IL. In Fig 11 the distributions of strategies and participation types at


Fig. 10. The rates $(2,1) 11$ and $(2,1) \mathrm{HI}$ in detail


Fig. 11. Spatial distribution of strategies and participation types
period 3, 4, 12 and 16 are shown in this order (left to right then top to down). A cluster is surrounded by circle in Fig. 11. One cluster of $(2,1)$ suddenly appears at period 4. This cluster grows through period 12 to 16 . Other two clusters of $(2,1)$ also suddenly appear after period 12 until period 16 . They also grow. $(2,1)$ strategies in these clusters already have almost HI type. Thus we guess that many players of $(2,1) \mathrm{HI}$ type in these cluster remain the same type by the cooperation in the cluster and the variableness of participation type.

## 4 Conclusion

We introduce stochastic participation in demographic DR game and investigate the effect of stochastic participation on the emergence of cooperation. We show, by Agent-Based Simulation, that the emergence rate of cooperation is fairy small if initial population contains players who have low participation probability if he is a Donor and probability of one if he is a Recipient, but introducing variableness of participation probability in terms of his recent experience as a Recipient promotes the emergence of cooperation. We also discuss why the variableness of participation probability promotes the cooperation.

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