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Consumption Taxation and Tax Prepayment
Approach in Dynamic General Equilibrium Models
with Consumer Durables

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Abstract

This paper reexamines the dynamic impacts of the proportional consumption tax in a perfect foresight model of general equilibrium with nondurable and durable consumption goods as well as productive capital. In contrast to the previous literature, it is shown that the consumption tax is not neutral with respect to the consumption/savings decisions, even if labor supply is fixed and if the tax revenues are fully returned to consumers in a lump-sum way. Moreover its increase reduces the overall welfare defined along the transitional path as well as in the ultimate steady state equilibrium. Although how the consumption tax affects the speed of capital accumulation along the transitional path depends on the form of utility functions, in a model with plausible preferences and reasonable parameter values, an increase in the consumption tax *adversely* affects that speed, i.e., savings.

1. Introduction

A number of prominent economists, e.g., Fisher(1937), Kaldor(1955), and Feldstein (1978), have proposed consumption or expenditure taxation as an alternative to the existing income tax system on the grounds that a consumption tax would not directly distort intertemporal consumption/savings decisions. Indeed, there is widespread belief that a reform toward consumption taxation would promote capital accumulation and thus improve social welfare. Recently this conventional wisdom has been positively confirmed by several authors using *dynamic general equilibrium* models with solid microfoundations for an individual's intertemporal behavior as long as labor supply is fixed [Schenone(1975), Summers(1981), Sin (1982, 1987), Abel and Blanchard (1983), Itaya (1991)]¹.

Most previous studies do not explicitly include consumer durable goods. While the assumption of a homogeneous consumption good is a simplifying abstraction, expenditures on durables and consumption of nondurables have quite different dynamic behavior. Many durable goods come in large, lumpy, and expensive units, so that durables must be purchased in discrete units and thus consumers will not make smooth adjustment over time. In macroeconomic time series data, it is well-known that durable expenditures will be highly serially correlated compared to nondurable consumption and display much more volatility over the business cycle than do nondurables. Since consumer durables comprise a significant fraction of total consumption in national income accounts [for example, the share of durable expenditures (including semi-durable goods but excluding housing services) in total consumption is approximately 23 percent in Canada, 19 percent in Japan, 26 percent in U.K., and 21 percent in U.S.A. in 1991], the behavior of durable expenditures plays an important role in determining the macroeconomic aggregates of consumption

and savings.

Furthermore, the most popular way of treating consumer durables is to introduce such goods into the utility function directly. The inclusion of the *stock* of durables in the consumer's utility function stands for various benefits or pleasure to the holder, such as prestige, power, bequests, insurance against certain kinds of risks, and certain psychic consumption benefits (art, jewelry, etc.). Despite these apparent differences between the two types of consumptions, insufficient attention has been paid to the role of durables in the literature of dynamic tax incidence. At least two papers have tried to remedy the relative neglect of durables; Brennan and Nellor(1982) and Grieson and Musgrave(1985) have investigated the effects of consumption taxation on savings behavior in the *two-period partial equilibrium* model which incorporates wealth into the utility function. They showed that, in the presence of psychic returns from the holding of wealth, the consumption tax is *not* neutral in the sense that its change affects intertemporal relative prices, and moreover encourages savings. Unfortunately, the restrictive structure of their models limits the general validity of their results.²

Since the purchase of a durable good can be regarded as savings under the *cash-flow approach* to the consumption tax, it would be exempt from the tax. However, the services yielded by the durable good over time should be included as "*consumption*", and hence added to the tax base. How can one measure the annual flow of services produced by a house or a refrigerator? Proponents of a consumption tax suggest a *tax prepayment* (or *yield exemption*) approach for durables to deal with this problem. When the original durable investment is undertaken, it is taxed as if it were consumption. No taxes are levied on the imputed yields generated by such a investment in succeeding periods. Hence, imputation problems are

avoided. Moreover, the tax prepayment approach indeed yields the same amount in present value terms as would have been collected as annual taxation. For these reasons, it is argued by those who support the consumption tax that this tax would be administratively simpler than the income tax. Nevertheless, none of the existing literature has rigorously demonstrated whether, *under the tax prepayment approach*, changes in the consumption tax may or may not affect the consumption/savings decisions within a *general equilibrium framework*.

Therefore, we shall construct a simple *dynamic general equilibrium* model in which consumer durables enter the utility function directly and are a substitute for productive capital as a store of value. By extending individual's horizon to infinity, it enables us to analyze the effects of consumption taxation on the transitional dynamics of capital accumulation and on the overall welfare accrued during the entire path of adjustment to the new steady state equilibrium. It is shown that, *under the tax prepayment approach*, changes in the proportional (flat-rate) consumption tax may distort the *individual's* consumption/savings decisions by affecting either the marginal rate of substitution between nondurable and durable consumption or the portfolio between durables and productive capital, *even if labor supply is fixed and if the tax revenues are fully rebated to consumers in a lump-sum way*. Thus the resulting distortions lead to a failure of the neutrality in the *macroeconomic aggregates*, being determined by market-clearing conditions, in a dynamic economy.

Section 2 constructs a basic model and analyzes the effects of the consumption tax on macroeconomic aggregates and welfare in the steady state. Section 3 analyzes its transitional effects on the speed of capital accumulation in a numerical model with a constant relative risk aversion utility function (CRRA) of the composite Cobb-Douglas bundle of

nondurables and durables as well as a variety of models with different preferences. Section 4 analyzes its welfare implications by comparing between the welfare in the initial steady state equilibrium and the welfare accrued along the transitional path as well as in the ultimate steady state equilibrium. Section 5 analyzes the differential incidence of consumption taxation by comparing it with income taxation in the steady state, when lump-sum taxation is not available. Section 6 summarizes the main conclusions and discusses possible extensions of the model.

2. The Model

Consider a representative consumer who is infinitely lived and has perfect foresight. He enjoys utility from his current flow of the nondurable consumption good, c , and from the services flowing from his stock of the durable consumption good, h . His utility functional is given by:

$$\int_0^{\infty} u(c, h) e^{-\rho t} dt, \quad (1)$$

where $u(c, h)$ is an increasing, strictly concave in c and h , and twice-continuously differentiable function, and ρ is the constant subject rate of time preference. The stock of durables enters his instantaneous utility function directly, because services are assumed to be proportional to stocks.

The consumer can hold his wealth in the form of capital, k , and consumer durable goods. Thus his budget constraint can be expressed by

$$(1 + \tau_c)(c + \dot{h}) + \dot{k} + \delta(k + h) = f(k) + x, \quad (2a)$$

$$h(0) = h_0 > 0 \text{ and } k(0) = k_0 > 0, \quad (2b)$$

where τ_c is the *proportional* consumption tax rate, x is a lump-sum transfer from the government equal to $\tau_c(c+\dot{h})$, h_0 and k_0 are respectively the initial stocks of durables and productive capital, and the dot denotes the time derivative.³

Output is produced using a stock of productive capital and inelastically supplied labor, according to the constant-returns-to-scale neoclassical production function $f(k)$. Output may be used for three alternatives: consumed as nondurable consumption goods, added to the stock of durable consumption goods, and added to the stock of productive capital. Since all these goods are assumed to be perfect substitutes in production, their before-tax relative prices are fixed at unity. For simplicity, we further assume that both nondurables and durables are normal goods, that both durables and productive capital depreciate exponentially at the same rate δ , and that the population growth rate is zero.⁴

The representative consumer chooses the time paths of his consumption and asset holdings to maximize (1) subject to (2), taking the time paths of wages and interest rates, tax rates, and transfer payments as given. Denote λ and μ as costate variables of the current Hamiltonian associated with (2a) and the slack variable identity, $z=\dot{h}$. Assuming an interior solution, straightforward application of Pontryagin's Maximum Principle yields

$$u_c(c, h) = \lambda(1+\tau_c), \quad (3a)$$

$$\frac{\dot{\lambda}}{\lambda} = \rho + \delta - f'(k), \quad (3b)$$

$$\frac{\dot{\mu}}{\mu} = \rho + \frac{\delta}{1+\tau_c} - \frac{u_h(c, h)}{u_c(c, h)}, \quad (3c)$$

$$-\lambda(1+\tau_c) + \mu = 0, \quad (3d)$$

$$c + \dot{h} + \dot{k} + \delta(k + h) - f(k) = 0, \quad (3e)$$

$$\lim_{t \rightarrow \infty} \lambda(t)k(t)\exp(-\rho t) = 0, \quad (3f)$$

$$\lim_{t \rightarrow \infty} \mu(t)h(t)\exp(-\rho t) = 0, \quad (3g)$$

where u_i denotes the first-order partial derivative of u with respect to argument i . Eqs.(3b) and (3c) are the evolution equations of the shadow prices of capital and durables, respectively. Eq.(3e) is the goods market clearing condition. It may be noted that if durables neither have any utility (i.e., $u_h=0$) nor depreciate at all (i.e., $\delta=0$), then the consumption tax rate drops out of this system, so changes in τ_c leave all real variables unchanged along the transitional path as well as in the steady state.⁵

In the steady-state, setting $\dot{\lambda}=0$, $\dot{\mu}=0$, $\dot{k}=0$, and $\dot{h}=0$ gives:

$$f'(k^*) = \rho + \delta, \quad (4a)$$

$$u_h(c^*, h^*)/u_c(c^*, h^*) = \rho + \frac{\delta}{1+\tau_c}, \quad (4b)$$

$$f(k^*) = c^* + \delta(k^* + h^*), \quad (4c)$$

where the starred variables pertain to the steady state values. The modified golden rule condition (4a) *solely* determines the steady state capital stock, because of the *fixed* discount rate of the representative agent. Hence, the steady state capital stock is independent of changes in the consumption tax. By contrast, the steady state levels of nondurable and durable consumption are affected by the consumption tax. This occurs because the consumption tax affects the marginal rate of substitution (henceforth MRS) between nondurable and durable consumption through the

implicit rental price of durables, $\delta/(1+\tau_c)$, in (4b).

Given k^* , differentiating (4b) and (4c) with respect to τ_c and rearranging yields

$$\frac{dc^*}{d\tau_c} = -J^{-1} \frac{\delta^2 u_c}{(1+\tau_c)^2} < 0 \quad \text{and} \quad \frac{dh^*}{d\tau_c} = J^{-1} \frac{\delta u_c}{(1+\tau_c)^2} > 0, \quad (5)$$

where $J \equiv \delta(u_{ch}u_c - u_{hc}u_{cc})/u_c - (u_{hh}u_c - u_{hc}u_{ch})/u_c > 0$ due to the normality assumption. Moreover, we can show that

$$\frac{du^*}{d\tau_c} = J^{-1} \frac{\delta u_c^2}{(1+\tau_c)^2} \left(\rho + \frac{\delta}{1+\tau_c} - \delta \right) \begin{matrix} \geq \\ < \end{matrix} 0 \quad \text{according as } \rho + \frac{\delta}{1+\tau_c} \begin{matrix} \geq \\ < \end{matrix} \delta, \quad (6)$$

where $u^* \equiv u(c^*, h^*)$. An increase in τ_c leads to a fall in the implicit rental price of durables (or the user cost of durables), thus stimulating the steady state demand for durables while depressing that for nondurables. Although both goods are subject to the same tax, the tax base does not include the depreciation of durables, and thus the rental price of durables becomes cheaper with τ_c relative to nondurables. Moreover, if $\rho + \frac{\delta}{1+\tau_c}$ is greater (smaller) than δ , steady state welfare rises (falls), for $\rho + \frac{\delta}{1+\tau_c}$ and δ can be interpreted as the utility gain in terms of the marginal utility of nondurable consumption and the opportunity cost in terms of foregone nondurable consumption by holding an additional unit of durables, respectively.

However, the invariance of the steady state capital stock is not robust. Once the modified golden rule condition (4a) is allowed to depend *directly* on nondurable and/or durable consumption, the neutrality in terms of steady state capital fails to hold. For example, if the discount rate depends on durable as well as nondurable consumption, steady state capital is affected by alternative rates of the consumption tax through variations in the endogenous discount rate;⁶ alternatively, if the stock

of productive capital enters the utility function in addition to durables, the modified golden rule condition depends on the stock of durables also.⁷ On the other hand, in the overlapping generations model that incorporates durables into the utility function, the lifetime consumption pattern is determined together with savings decision and portfolio choice when the young solves their optimization problem. Since this pattern in turn determines individual savings and thus the demand for capital, changes in the consumption tax induce them to alter the portfolio between capital and durables, thereby affecting steady state capital.⁸

Alternatively, if depreciation deduction is allowed for durables only, then the budget constraint must be amended to

$$(1+\tau_c)(c + \dot{h} - \theta\delta h) + \dot{k} + \delta(k + h) = f(k) + \hat{x}, \quad (7)$$

where θ ($0 \leq \theta \leq 1$) is the share of the economic depreciation to be deducted and $\hat{x} = \tau_c \{(c + \dot{h}) - \theta\delta h\}$. Then condition (3c) becomes

$$\frac{\dot{\mu}}{\mu} = \rho + \left(\frac{1}{1+\tau_c} - \theta \right) \delta - \frac{u_h(c, h)}{u_c(c, h)}, \quad (8)$$

while the other conditions remain unchanged. Observation of (8) immediately reveals that nonneutrality still holds except for $\theta = 1/(1+\tau_c)$. It should be noted that, even if true economic depreciation ($\theta = 1$) is allowed, neutrality does not emerge except the case where the initial tax rate equals zero. This nonneutrality result stands in sharp contrast to the well-known fact that allowing true economic depreciation for business capital in conjunction with allowing interest deductability implies that the corporate income tax is neutral with respect to the investment decisions of firms [see Samuelson (1964)].

When $\theta = 1/(1+\tau_c)$, the budget equation (7) is reduced to

$$(1+\tau_c)(c + \dot{h}) + \dot{k} + \delta k = f(k) + \hat{x}.$$

By substituting $\theta=1/(1+\tau_c)$ into (8), we see that the consumption tax disappears and thus is neutral along the transitional path as well as in the steady state. In this case, both nondurables and durables are taxed *symmetrically* in the sense that *all* expenditures on durables as well as nondurables are subject to the tax by deducting certain fraction of the depreciation of durables from the tax base. Hence, their relative price (i.e., the implicit rental price of durables) becomes ρ , and thus is not distorted by the consumption tax. If the tax authority adheres to implementing the policy of neutral taxation, it needs information on the size of the *true economic depreciation* of durables, δh . However, it is generally very difficult to know the *exact* value of such depreciation, because of the absence of the perfect resale markets for all used durable goods in actual economies. Therefore, such a policy would require us to give up one of the administrative advantages of the consumption tax in that it is *not* necessary to access the magnitude of depreciation.

3. Transitional Analysis

In this section we shall analyze how a change in the consumption tax affects the transitional dynamic path of capital accumulation in this model. Differentiating (3d) with respect to time and substituting (3b) and (3c) into the resulting equation, we obtain

$$\frac{u_h(c, h)}{u_c(c, h)} = f'(k) - \delta + \frac{\delta}{1+\tau_c}. \quad (9)$$

We solve (9) for h in terms of c and k , and differentiate the expression for h with respect to time to get

$$\dot{h} = h_c \dot{c} + h_k \dot{k}, \quad (10)$$

where h_i ($i=c,k$) is the partial derivative of $h(c,k)$ with respect to the corresponding argument. Differentiating (3a) with respect to time, substituting (3b) and (10) into the resulting expression, and substituting (10) into (3e), we can rewrite these equations as

$$\begin{bmatrix} \dot{c} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} \frac{cu_{cc} + cu_{ch}h_c}{u_c} & \frac{cu_{ch}h_k}{u_c} \\ h_c & 1+h_k \end{bmatrix}^{-1} \begin{bmatrix} c\{(\rho+\delta)-f'(k)\} \\ f(k)-c-\delta\{h(c,k,\tau_c)+k\} \end{bmatrix}. \quad (11)$$

Assuming the *CRRRA* utility function of the form:

$$u(c,h) = \frac{(c^\alpha h^{1-\alpha})^{1-\gamma}}{1-\gamma}, \quad \text{for } \gamma > 0, \gamma \neq 1,$$

$$u(c,h) = \alpha \ln c + (1-\alpha) \ln h, \quad \text{for } \gamma = 1,$$

we obtain the following linear approximation of the system (11) around the steady state:

$$\begin{bmatrix} \dot{c} \\ \dot{k} \end{bmatrix} = \Delta^{-1} \begin{bmatrix} (\gamma-1)\alpha f''h^*(1+\delta h_c) & -(1+h_k)f''c^* - (\gamma-1)\alpha f''h^*(\rho-\delta h_k) \\ \gamma(1+\delta h_c) & f''h^* - \gamma(\rho-\delta h_k) \end{bmatrix} \begin{bmatrix} c-c^* \\ k-k^* \end{bmatrix}, \quad (12)$$

where $\Delta \equiv f''h^* \left(\rho + \frac{\delta}{1+\tau_c} \right)^{-1} \{ \alpha\gamma + (1-\alpha) \} - \gamma < 0$, $h_c \equiv h^*/c^* > 0$, and $h_k \equiv -\alpha h^{*2} f'' / (1-\alpha) c^*$

> 0 . Following Fisher (1979), in order to investigate analytically the effect of the consumption tax on the speed of capital accumulation, we have to know the signs of the derivatives regarding the trace and the determinant of the matrix appearing on the R.H.S of (12) with respect to τ_c . However, these signs are opposite, thus making the effect on that speed ambiguous. The main reason for this ambiguity can be explained as follows; A permanent increase in τ_c reduces the implicit rental price for durables, thereby stimulating the demand for durables, but depressing

that for nondurables. The immediate reduction in nondurable consumption discriminates in favor of the accumulation of productive capital through (3e). In addition, since the total wealth stock (i.e., $k+h$) is instantaneously fixed, the rise in the demand for durables leads to a fall in the demand for productive capital, thereby lowering the total output $f(k)$. The fall in output depresses the rate of capital accumulation through (3e). Since these two effects are in opposite directions,⁹ the overall effect on the rate of capital accumulation may be positive or negative depending on the relative strength of these two effects.

To gain some idea of the magnitudes involved, we solve numerically the effects of the consumption tax on the negative roots. To calibrate further the model specified above, we assume a Cobb-Douglas production function with capital's share of total income 25 percent and that an annual depreciation rate of the capital stock is 4 percent of total income. These values can easily be obtained from casual examination of national income accounts. On the other hand, Bernanke (1985) found that the annual depreciation rate of durables is about 20 percent. Since we have applied the common depreciation rate to durables and capital, the appropriate value lies between these two values, so that two values of the depreciation rate are chosen; 10 and 15 percent. The rate of time preference is set equal to 4 percent. Most of other studies have used values between 1 percent and 4 percent for the rate of time preference. A relatively higher value of 4 percent is chosen so as to offset the bias caused by the infinite horizon assumption on the grounds that a higher rate of time preference is equivalent to a shorter time horizon. There is considerable econometrics evidence on the intertemporal elasticity of consumption, $1/\gamma$, but the range of these estimates is quite large and

there is no consensus on its value. Therefore, we allow values of the intertemporal elasticity of consumption to range between .1, .5, .8, 1.0, 2.0, 5.0, and 10.0. The share of nondurables in total consumption in national income accounts for the parameter α may be smaller than the true value, because some of nondurable expenditures are included in the category of *services* (e.g., medical and educational expenses). Since there is such uncertainty about the size of this parameter, three values of α are considered; .6, .7, and .8.

Table 1 lists absolute values of the unique negative roots over a wide range of values for α , γ , and τ_c . From table 1, two things should be noted. First, for most values of α and γ , an increase in the consumption tax rate would depress the speed of capital accumulation on the transitional path. In other words, this negative effect on the rate of capital formation is extremely robust within a wide range of values for the parameters of the individual's utility function. It would be fair to say that increasing the consumption tax discourages savings in the presence of durable goods. However, this highly likely result is opposite to that of Brennan and Nellor (1983) in which the imposition of the consumption tax would encourage savings. The main reason for this difference is that both the purchase of wealth and its consumption benefits are exempt from the tax in their model. Such a favorable tax treatment of wealth (i.e., durables) would lead to an increase in savings compared to income taxation. Secondly, when the intertemporal elasticity of consumption is lower (i.e., .1 and .5) and when α is relatively lower, it is likely that larger absolute values of the negative roots are associated with higher rates of the consumption tax. The smaller the intertemporal elasticity, the stronger a desire for a smoother the consumption path, and the lesser the rate of change of consumption for a

given value of the excess of the marginal product over the discount rate; consequently, the lesser the rate of change of durables through (10), creating the possibility that \dot{k} may rise. On the other hand, the smaller the value of α , the less sensitive the marginal of substitution between nondurable and durable consumption, and the less the rise in durable consumption to the response of an increase in τ_c . In consequence, the induced reduction in productive capital causes output to fall by a lesser proportion in nondurable consumption, so that the rate of capital accumulation may be increased.

In contrast, if nondurable and durable consumption are perfect complements, increasing the consumption tax makes the speed of capital formation faster. To ascertain this result, we assume that $u(c,h) \equiv u(\xi)$, where $\xi = \min[(1/\nu)c, h]$. In this case, the linearized system is given by

$$\begin{bmatrix} \dot{\lambda} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} 0 & -\lambda^* f'' \\ -h_\lambda (\nu + \delta) & \rho + \lambda^* h_\lambda f'' - h_k (\nu + \delta) \end{bmatrix} \begin{bmatrix} \lambda - \lambda^* \\ k - k^* \end{bmatrix}, \quad (13)$$

where $h_\lambda = (u'')^{-1} \{(\nu + \rho)(1 + \tau_c) + \delta\} < 0$ and $h_k = (u'')^{-1} f'' \lambda^* (1 + \tau_c) > 0$.

Differentiating both the trace, Tr , and the determinant, Det , of the matrix on the R.H. S. of (13) with respect to τ_c leads to

$$\frac{\partial(Tr)}{\partial \tau_c} < 0 \quad \text{and} \quad \frac{\partial(Det)}{\partial \tau_c} = 0.$$

That is, an increase in τ_c makes the negative root larger in absolute value. There is no substitution effect between nondurable and durable consumption, while the negative income effect causes both consumption levels to fall. Although the redistributed transfer income offsets to some extent this negative effect, the net effect is to decrease the demand for durables, leading to a rise in that for productive capital and hence in output. Consequently, the rate of capital accumulation

unambiguously goes up.

On the other hand, if we assume the form of quasilinear utility, that is, $u(c,h) \equiv u(c)+h$, we have the corresponding linearized system:

$$\begin{bmatrix} \dot{k} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} (u')^{-1} & 0 \\ f' - (u')^{-1} - c_k - \delta & -\delta \end{bmatrix} \begin{bmatrix} k - k^* \\ h - h^* \end{bmatrix}, \quad (14)$$

where c_k is the derivative of c with respect to k . In this case, we can easily calculate the eigenvalues of this system. The unique negative root is $-\delta$, which is unaffected by changes in τ_c . This means that changes in the consumption tax do not affect the speed of capital accumulation. An increase in the consumption tax reduces the demand for nondurables, but raises that for durables owing to the substitution effect, thus leading to a fall in that for productive capital. On the other hand, the redistributed transfer income goes entirely to durable consumption, which implies a rise in the demand for durables but a fall in that for productive capital. The induced reduction in output is just offset by the fall in nondurable consumption, thus leaving \dot{k} unchanged.

4. Dynamic Welfare Analysis

In this section we shall make an assessment of the welfare implications of a permanent increase in the consumption tax in the presence of consumer durables. For simplicity, suppose that the economy begins at the steady state, so we focus on the comparison of the initial steady state with the entire path of adjustment to the new steady state following the tax increase.

The dynamic welfare loss (or gain) is defined as the present discounted sum of the difference between utility flows on the new equilibrium path and utility flows on the initial (steady state)

equilibrium path. It can be written as

$$\Delta U = \int_0^{\infty} [u(c(t), h(t)) - u(c_0, h_0)] e^{-\rho t} dt, \quad (15)$$

where c_0 and h_0 are the initial steady state levels of consumption and durables, respectively. Substituting the first-order approximation of the instantaneous utility function:

$$u(c(t), h(t)) = u(c_0, h_0) + u_c(c_0, h_0)[c(t) - c_0] + u_h(c_0, h_0)[h(t) - h_0],$$

into (15) yields

$$\Delta U = u_c(c_0, h_0) \int_0^{\infty} [c(t) - c_0] e^{-\rho t} dt + u_h(c_0, h_0) \int_0^{\infty} [h(t) - h_0] e^{-\rho t} dt. \quad (16)$$

By inserting $c(t) - c_0 = c(t) - c^* + c^* - c_0$ and $h(t) - h_0 = h(t) - h^* + h^* - h_0$ into the integrals in (16), respectively, and noting that $\int_0^{\infty} [c(t) - c^*] e^{-\rho t} dt = 0$ [see Fig.2], (16) can be rewritten as

$$\begin{aligned} \Delta U = & u_c(c_0, h_0) \int_0^{\infty} [c^* - c_0] e^{-\rho t} dt + u_h(c_0, h_0) \int_0^{\infty} [h^* - h_0] e^{-\rho t} dt \\ & - u_h(c_0, h_0) \int_0^{\infty} [h^* - h(t)] e^{-\rho t} dt. \end{aligned} \quad (17)$$

The first and second components of each welfare effect represent respectively the steady state effects given by the difference between the two steady state values of nondurable and durable consumption, while the third term represents the transitional effect accrued during adjustment. Note also that the transitional effect adversely affects the overall welfare. Furthermore, substituting $h^* - h(t) = (h^* - h_0) e^{-\delta t}$ into (17) and rearranging, we have

$$\Delta U = u_c(c_0, h_0) \frac{c^* - c_0}{\rho} \left[1 - \left(\rho + \frac{\delta}{1 + \tau_c} \right) (\rho + \delta)^{-1} \right] < 0. \quad (18)$$

When an increase in the consumption tax takes place, the steady state level of capital remains unchanged, but nondurable consumption

drops instantaneously to the steady state level and durable consumption gradually approaches the steady state level, as illustrated in Figs.1 and 2.¹⁰ As can be seen in these figures, it turns out that $dc(0)/d\tau_c < 0$. It follows from (18) that $\Delta U < 0$, regardless of whether $\rho + \frac{\delta}{1+\tau_c} - \delta < 0$ or $\rho + \frac{\delta}{1+\tau_c} - \delta > 0$. In other words, irrespective of the response of steady state welfare, the overall welfare which is composed of the steady state as well as transitional welfares unambiguously falls with τ_c , except the case where its initial tax rate equals zero. Moreover, the higher the initial tax rate, the larger the welfare loss, while the higher the depreciation rate, the faster the adjustment speed of durables, and the less the welfare loss because the (negative) transitional effect in absolute value becomes smaller as a result of the shorter adjustment period.

5. Consumption Taxation versus Income Taxation

In this section we modify the original model presented in section 2 to incorporate (comprehensive) income taxation instead of lump-sum taxation, since lump-sum transfers (or taxes) are not available in actual economies. The government is assumed to substitute consumption for income taxes, keeping the government budget balanced. Without loss of generality, we assume that government expenditures equal zero, so its budget constraint can be expressed by

$$\tau_y f(k) = \tau_c (c + \dot{h}), \quad (19)$$

where τ_y is the income tax rate. The individual's budget constraint then becomes

$$(1+\tau_c)(c + \dot{h}) + \dot{k} + \delta(k + h) = (1-\tau_y)f(k). \quad (20)$$

With these modifications, the optimization conditions are the same as

(3a)-(3g) except for (3b). In the steady state, we have (4b), (4c), and

$$(1-\tau_y)f'(k^*) = \rho + \delta. \quad (21a)$$

$$\tau_y f(k) = \tau_c c, \quad (21b)$$

Straightforward comparative statics exercises on (4b), (4c), (21a), and (21b) lead to

$$\frac{dc^*}{d\tau_c} = \hat{J}^{-1} \left[f'_c(\delta-f') \frac{u_{hh}^u c^u - u_{hh}^u c^u}{u_c} - \frac{\delta^2 u_c}{(1+\tau_c)^2} M \right] < 0, \quad (22a)$$

$$\frac{dh^*}{d\tau_c} = \hat{J}^{-1} \left[-f'_c(\delta-f') \frac{u_{ch}^u c^u - u_{ch}^u c^u}{u_c} + \frac{\delta u_c}{(1+\tau_c)^2} \left\{ M + f' \tau_c (\delta-f') \right\} \right] \geq 0, \quad (22b)$$

$$\frac{dk^*}{d\tau_c} = \hat{J}^{-1} \left[f'_c \left\{ \frac{u_{ch}^u c^u - u_{ch}^u c^u}{u_c} - \frac{u_{hh}^u c^u - u_{hh}^u c^u}{u_c} \right\} - \frac{\delta^2 u_c}{(1+\tau_c)^2} f' \tau_c \right] \geq 0, \quad (22c)$$

where $\hat{J} \equiv M \{ \delta (u_{ch}^u c^u - u_{ch}^u c^u) / u_c - (u_{hh}^u c^u - u_{hh}^u c^u) / u_c \} - (\delta-f') f' \tau_c (u_{hh}^u c^u - u_{hh}^u c^u) / u_c < 0$ due to the normality and saddlepoint assumptions, $M \equiv (1-\tau_y) f f'' + \tau_y (f')^2 < 0$.¹¹ Moreover, it follows from (22a) and (22b) that the effect on steady state welfare is ambiguous. Under the saddlepoint assumption, nondurable consumption falls by a greater proportion than does that in the case where lump-sum taxation is available, while the effects on durable consumption as well as on steady state capital are indeterminate. The source of this ambiguity is that the induced fall in nondurable consumption leads to a fall in the tax revenues, thus creating the possibility that the income tax rate may be increased in order to keep the government budget balanced, for an increase in the income tax has a detrimental effect on all real variables. In short, an increase in the consumption tax accompanied by compensated changes in the income tax may not encourage savings in the steady state.

6. Concluding Comments

The main results obtained in this paper are summarized as follows. First, in the durables-in-the-utility-function model, we obtain the neutrality of consumption taxation in terms of the long-run capital intensity *even if labor supply is fixed and if the tax revenues are returned to consumers as a lump-sum transfer*. This neutrality, however, breaks down with any of the following conditions: (1) labor supply is flexible, (2) the discount rate depends on the stock of durables, (3) the stock of productive capital enters the utility function. In this sense, the steady state neutrality of consumption taxation is not robust in more general models. The consumption tax, on the other hand, affects nondurable and durable consumption in opposite directions. Neutrality holds either if all expenditures on consumer durables (including depreciation) are levied, or if certain fraction of true economic depreciation is deductible from the tax base.

Secondly, the neutrality in terms of capital accumulation during the transitional path no longer holds except for quasilinear utility. Accordingly, in the presence of durable goods, the neutrality with respect to the consumption/savings decisions is generally invalid despite the fact that the price of future in terms of today's (nondurable) consumption is unaffected by the tax rates. How the consumption tax affects capital accumulation depends on the form of utility functions. Nevertheless, using the specific model with the CRRA utility function of the composite Cobb-Douglas bundle of nondurables and durables and with reasonable parameter values, it is highly likely that the lower speed of capital accumulation is associated with the higher consumption tax rate.

Thirdly, when consumption may induce non-separabilities over time because of habit formation, the history (or the stock) of past consumption will affect the marginal utility of current consumption. In

this case, however, neutrality holds because neither additions to the stock of accumulated past consumption nor the stock itself is subject to the consumption tax.¹² In other words, the introduction of a stock variable into the utility function is not sufficient to destroy neutrality. In addition, the *asymmetric* tax treatment across different goods or consumptions in the sense described in section 2 is needed to produce the distortions in intertemporal consumption choices.

Fourth, the overall welfare defined along the transitional path as well as in the steady state unambiguously falls, even if steady state welfare is improved. In the presence of durable goods, both the failure of neutrality in intertemporal consumption choices and the *adverse* effect on the overall welfare would provide counter justification for the tax reform towards consumption taxation. In addition, our simulation result provides a counter example to the conventional wisdom which suggests that the consumption tax would promote capital accumulation. Therefore, the case for preferring consumption to income taxation is more fragile than one might infer from reading the modern tax literature *in the presence of durables*.

The important extension is to introduce transaction/adjustment costs in durable purchases due to its illiquidity or indivisibility, instead of our *frictionless* intertemporal optimizing model. As a result, the dynamic path of durables may be characterized by frequency and amplitude of purchases, which may be affected by changes in the consumption tax.

Footnotes

1. Seidman (1982) showed that incorporating a bequest motive into Summer's model undermines such neutrality. Menchik and David (1982) further pointed out that in the bequest-as-consumption model, where the bequest itself enters the parent's concave utility function as a separate argument, the consumption tax with the same rate over time is neutral in the sense that the tax does not distort any of the marginal rate of substitution conditions, *as long as bequests as well as lifetime consumption are*

levied at the same rate. In addition, if the tax revenues are fully rebated in a lump-sum way to consumers so as to get rid of the income effect, neutrality obtains in the general equilibrium's sense also. On the other hand, in Barro's altruistic bequest model, where the offspring's utility function enters the parent's utility function as a separate argument, the consumption tax together with fully rebated transfers is neutral in either sense. On the other hand, Batina(1987) extended the Becker-Barro altruistic model to contain both cash bequests and human capital investments and to allow for endogenous fertility decisions, and concluded that neutrality *almost always* fails in spite of fixed labor supply. Further arguments are found in Batina and Ihuri (1991) using the overlapping generations model of an open economy.

2. The meaning of neutrality in this paper is slightly different from that of Brennan and Nellor (1982), Grieson and Musgrave (1985), and Menchik and David(1982), in which changes in the consumption tax do not affect any of the individual's marginal rate of substitution conditions. Our neutrality implies that its change affects neither those MRS conditions nor all real variables that are determined by market-clearing conditions.

3. Note that, in the absence of labor-leisure choice, the lump-sum tax can be regarded as the tax on labor income, and therefore an increase in the consumption tax is equivalent to a switch from the non-distorting wage tax to the consumption one.

4. Our conclusion is not affected in any essential way but calculations becomes complicated, if we adopt the heterogeneous rates of depreciation between capital and durables.

5. However, in the case of progressive consumption taxation, the consumption tax ceases to be neutral along the transitional path even if the stock of durables neither depreciates at all nor enters the utility function, although steady state capital remains unaffected.

6. Suppose that a representative infinitely-lived consumer with recursive preferences depending on consumption and consumer durables maximizes

$$- \int_0^{\infty} e^{-\phi} dt,$$

subject to (2a), (2b),

$$\dot{\phi} = \rho(c, h),$$

where ρ is twice continuously differentiable, with $\rho_c > 0$, $\rho_h > 0$, $\rho_{cc} < 0$, and $\rho_{hh} < 0$. The steady state conditions are given by (4c),

$$f'(k^*) = \delta + \rho(c^*, h^*),$$

$$\rho_h(c^*, h^*) / \rho_c(c^*, h^*) = \rho(c^*, h^*) + \delta / (1 + \tau_c).$$

Under the normality assumption, steady state comparative statics excises result in:

$$\frac{dc^*}{d\tau_c} < 0 \quad \text{and} \quad \frac{dh^*}{d\tau_c} > 0.$$

Moreover, it is shown that if $\rho(c^*, h^*) + \frac{\delta}{1 + \tau_c} \geq \delta$,

$$\frac{dk^*}{d\tau_c} \leq 0 \quad \text{and} \quad \frac{d\rho(c^*, h^*)}{d\tau_c} = f'' \frac{dk^*}{d\tau_c} \geq 0.$$

7. In this case, the representative consumer maximizes

$$\int_0^{\infty} u(c, W) e^{-\rho t} dt,$$

subject to (2a) and (2b), where $W=h+k$. This formulation may be regarded as an infinite version of Brennan and Nellor's model(1982). We can solve this optimization problem in a similar fashion outlined in the text. In the steady state, we have (4c),

$$f'(k^*) + (1+\tau_c)u_w(c^*, W^*)/u_c(c^*, W^*) = \rho + \delta,$$

$$u_w(c^*, W^*)/u_c(c^*, W^*) = \rho + \delta/(1+\tau_c).$$

These conditions clearly imply that the steady state capital stock is affected by changes in the consumption tax.

8. Consider the following simple two-period overlapping generations model with no bequest motive. In the steady state, each agent when young maximizes a lifetime utility function given by:

$$W^* \equiv u^y(c_1^*, h^*) + \frac{1}{1+\rho} u^o(c_2^*),$$

subject to

$$(1+\tau_c)c_1^* = w - k^* - h^* + x_1,$$

$$(1+\tau_c)c_2^* = (1+r)k^* + x_2,$$

where u^y and u^o are the first and the second period's utility function, respectively, r is the real interest rate, c_i is period i 's consumption, and $x_i = \tau_c c_i$, $i=1,2$.

The first order necessary conditions for an interior solution are:

$$u_c^y(c_1^*, h^*) = \frac{1+r}{1+\rho} u_c^o(c_2^*),$$

$$(1+\tau_c)u_h^y(c_1^*, h^*) = \left[1 - \frac{1}{1+r}\right] u_c^o(c_1^*, h^*).$$

Solving the first order conditions for c^* and h^* , and noting that $w(k^*) = f(k^*) - k^* f'(k^*)$, we have the steady state savings function:

$$k^* = w(k^*) - c_1^*(k, \tau_c) - h^*(k, \tau_c).$$

Differentiating this equation with respect to τ_c to obtain:

$$\frac{dk^*}{d\tau_c} = - \frac{(\partial c_1^*/\partial \tau_c) + (\partial h^*/\partial \tau_c)}{1 + (\partial c_1^*/\partial k^*) + (\partial h^*/\partial k^*) + k^* f''} < 0,$$

where the denominator and the numerator are positive due to the stability condition in the Hicksian sense and the normality assumption, respectively. Intuitively, an increase in τ_c reduces the opportunity cost of holding durables, thereby raising the demand for durables. Therefore, the pure substitution effect induces the young to substitute durables for capital. Since the income effect on capital holdings is *negative*, these two effects work in the same direction, and thus unambiguously depress the young's demand for capital. Moreover, differentiating the steady state welfare W^* with respect to τ_c results in:

$$\frac{dW^*}{d\tau_c} = u_c \left[\frac{1}{1+r} \left(\frac{\partial h^*}{\partial \tau_c} + \frac{\partial h^*}{\partial k} \frac{\partial k^*}{\partial \tau_c} \right) - \frac{r}{1+r} k^* f''(k^*) \frac{\partial k^*}{\partial \tau_c} \right] < 0.$$

9. Since the growth rate of consumption is governed by

$$\dot{c} = (1/\gamma)[\rho + \delta - f'(k) - \{(1-\alpha)(1-\gamma)\dot{h}/h\}]c,$$

the instantaneous reductions in c and k cause \dot{c} to fall, while the instantaneous increase in h causes \dot{c} to rise, so that the effect of increasing τ_c on \dot{c} and hence on \dot{h} through (10) are ambiguous.

10. Since the path of consumption is given by

$$c(t) = c^* + (\rho - \omega)(k(t) - k^*),$$

differentiating this equation with respect to τ_c at time 0 and noting that $dk(0)/d\tau_c = 0$ and $dk^*/d\tau_c = 0$ yields

$$\frac{dc(0)}{d\tau_c} = \frac{dc^*}{d\tau_c}.$$

11. This is a sufficient condition for the steady state to be saddlepoint stable. The saddlepoint stable assumption may be defended on the grounds that a meaningful comparative statics analysis should be limited on stable steady state equilibria, and moreover that the possibility of multiple stable equilibrium paths (or the indeterminacy of equilibrium) should be ruled out.

12. In the presence of habit formation, the consumer maximizes

$$\int_0^{\infty} u[c(t), z(t)] e^{-\rho t} dt,$$

subject to

$$(1+\tau_c)c(t) + \dot{k}(t) + \delta k(t) = f[k(t)] + x(t),$$

$$\dot{z}(t) = \sigma[c(t) - z(t)],$$

$$z(0) = z_0 > 0, \text{ and } k(0) = k_0 > 0,$$

where $z(t) = \sigma \int_{-\infty}^t c(s) e^{\rho(s-t)} ds$ is a weighted average of past consumption levels [see Ryder and Heal (1973)]. In this case, it is easy to show that the consumption tax is intertemporally neutral, that is, the consumption/savings decisions are unaffected by changes in the consumption tax.

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Table 1: Absolute Values of Negative Roots
When $\delta=0.1$

α	τ_c	$1/\gamma$						
		.1	.5	.8	1.0	2.0	5.0	10.0
.6	.05	.0791089	.106572	.12234	.131334	.164825	.214901	.247808
	.1	.0818374	.106944	.121724	.130238	.16232	.210883	.242911
	.2	.0868807	.107696	.120638	.128262	.157763	.203668	.234206
	.3	.091419	.108443	.119715	.126532	.153727	.197379	.226708
	.4	.095508	.109169	.118925	.125006	.15013	.19185	.220182
.7	.05	.0991999	.109866	.118242	.123652	.146905	.186952	.214453
	.05	.0626358	.0762744	.131051	.143374	.190492	.262927	.314694
	.1	.064888	.0777547	.129752	.142185	.187865	.258211	.308385
	.2	.0691233	.0805898	.128387	.140011	.183023	.249638	.29706
	.3	.0730087	.0832447	.127196	.138074	.178672	.242055	.287192
.8	.4	.0765648	.085716	.126148	.13634	.174745	.235308	.278524
	.5	.0798172	.0880084	.146903	.134781	.171187	.229271	.270854
	.05	.0488512	.110991	.141242	.158006	.220635	.325699	.412632
	.1	.0503982	.110809	.140424	.156851	.218221	.320889	.405331
	.2	.0534158	.110531	.138922	.154702	.213689	.311951	.391954
.8	.3	.0563016	.110343	.137574	.152747	.209522	.303838	.380019
	.4	.0590356	.110222	.136361	.150962	.205686	.296458	.369322
	.5	.0616098	.117855	.135264	.149328	.202149	.289726	.359694

Table 2: Absolute Values of Negative Roots
When $\delta=0.15$

α	τ_c	$1/\gamma$						
		.1	.5	.8	1.0	2.0	5.0	10.0
.6	.05	.12387	.154969	.174	.185167	.228391	.296743	.343502
	.1	.128065	.155712	.173221	.183645	.224698	.290794	.33625
	.2	.135871	.157206	.171866	.180897	.21794	.280071	.323327
	.3	.14295	.15868	.170735	.178489	.211915	.270676	.31216
	.4	.149368	.161011	.169784	.176367	.206513	.262381	.302418
	.5	.155195	.161482	.16898	.174487	.201644	.255005	.293844
.7	.05	.100272	.154745	.18413	.20067	.262363	.360102	.43195
	.1	.103744	.155044	.183123	.198999	.258501	.353141	.422635
	.2	.110314	.155721	.181316	.195935	.25134	.340421	.405853
	.3	.116385	.156465	.179748	.193197	.244855	.329103	.391171
	.4	.121975	.157244	.178379	.188528	.238964	.318981	.378229
	.5	.127113	.158034	.177176	.203513	.233597	.309884	.366744
.8	.05	.12387	.132405	.174	.185167	.228391	.296743	.343502
	.1	.128065	.135505	.173221	.183645	.224698	.290794	.33625
	.2	.135871	.141343	.171866	.180897	.21794	.280071	.323327
	.3	.14295	.146709	.170735	.178489	.211915	.270676	.31216
	.4	.149368	.151629	.169784	.176367	.206513	.262381	.302418
	.5	.155195	.156135	.16898	.174487	.201644	.255005	.293844

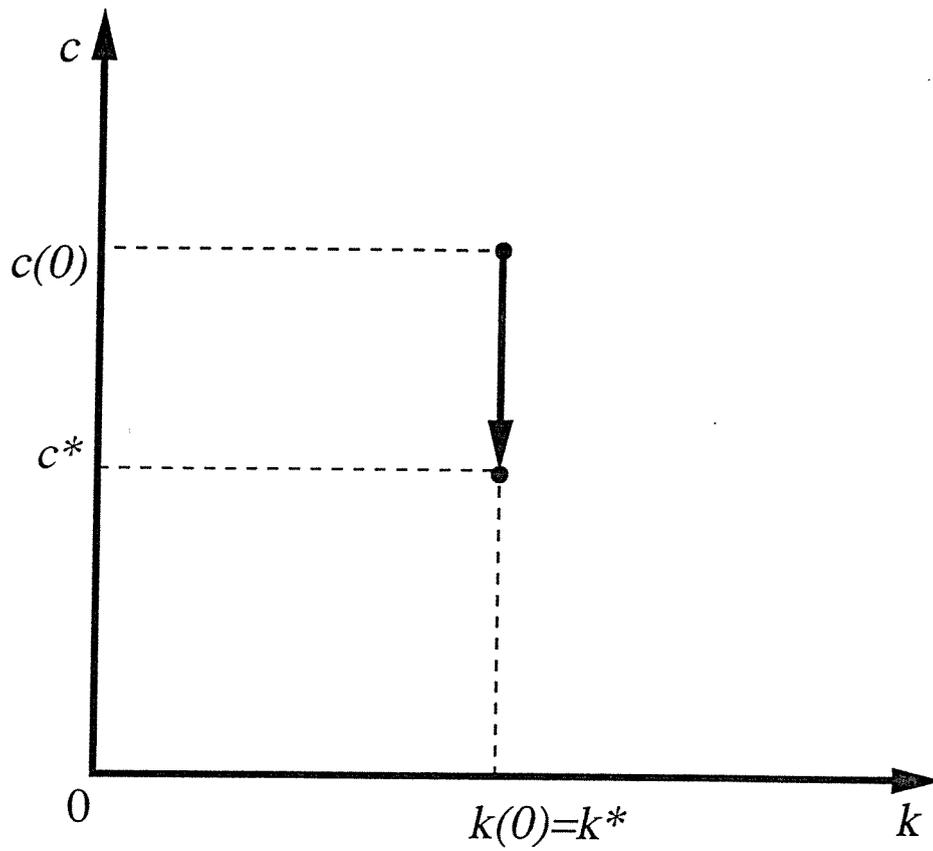


Figure 1 When the economy is initially in the steady state, an increase in the consumption tax reduces nondurable consumption

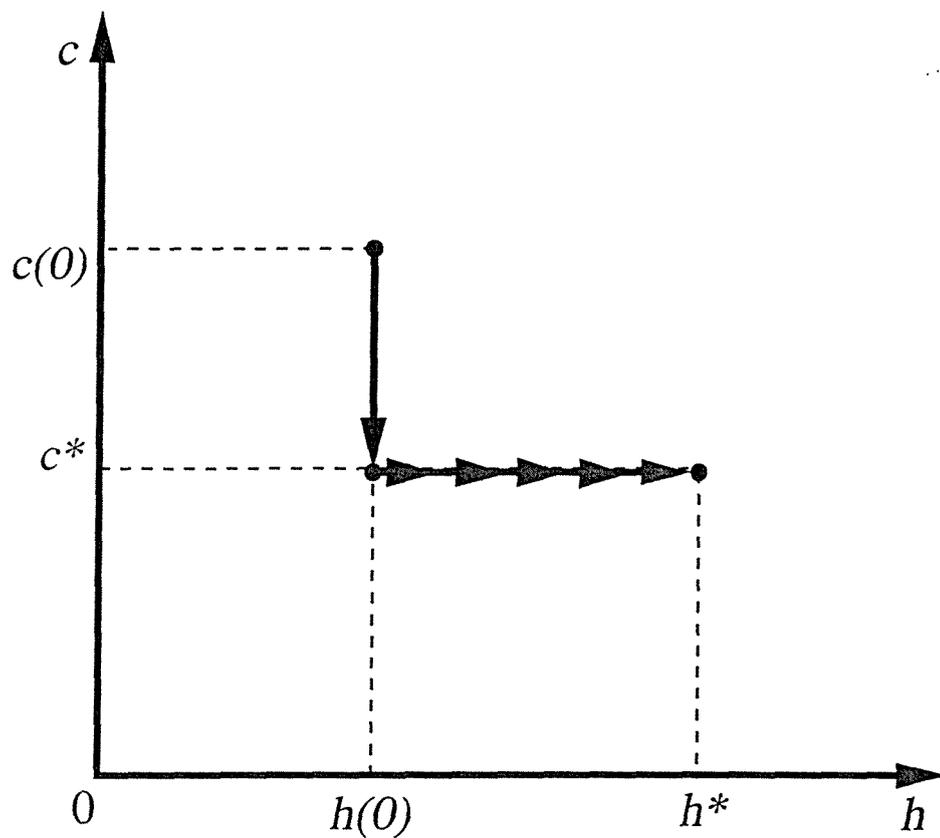


Figure 2 When the economy is initially in the steady state, nondurable consumption immediately falls, but the stock of durables gradually reaches another steady state

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