Effect of Variable-Threshold Strategies in Demographic Donor-Recipient and Prisoner's Dilemma Games[1]

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Abstract. We consider effect of variable-threshold strategies on emergence of cooperation in demographic dilemma game, Donor-Recipient or Prisoner's Dilemma game.

Tit for Tat changes his move at each period depending on the previous opponent's move. In real life, people further change their tendency toward cooperation or defection. We want to incorporate this change into our model. We introduce variable-threshold strategies whose components are extended forms of TFT. We interpret that TFT uses Defect if the state is 0, Cooperate if it is 1 and the smallest state number that prescribes using Cooperate as a *threshold*. AllC has zero threshold in this interpretation. We allow up to three states. Variable-threshold strategy changes its threshold at most once at some age (once in his lifetime) depending on its experience until then. Thus variable-threshold TFT who was born as TFT may change to AllC or AllD. Also variable-threshold AllC who was born as AllC may change to TFT.

Players are initially randomly distributed in square lattice of cells. In each period, players move locally to random cell in von Neumann neighbors if unoccupied or globally to random unoccupied cell in the whole lattice, and play dilemma game against local neighboring player or against randomly selected player from the whole lattice. If wealth (accumulated payoff) of player becomes negative or his age becomes greater than his lifetime, he dies. If his wealth becomes greater than some amount and there is unoccupied cell in neighbors, he has an offspring.

A stage game is Donor-Recipient or Prisoner's Dilemma game. Donor-Recipient game is a two-person game where one player is randomly selected as Donor and the other as Recipient. Donor has two moves, Cooperate and Defect. Cooperate means Donor pays cost c in order for Recipient to receive benefit b ($b>c>0$). Defect means Donor does nothing. Note that Recipient has no move. Prisoner's Dilemma game which we use here is a two-person simultaneous move game where both players have two moves, Cooperate and Defect, whose meanings are the same as in Donor-Recipient game. For convenience sake we shift the

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original payoffs in order for the new payoffs of player to be positive if the opponent uses Cooperate and negative if Defect.
We investigate, by Agent-Based Simulation, emergence of cooperation where there are variable-threshold strategies and the difference between Donor-Recipient and Prisoner's Dilemma game, and show some cases where cooperation emerges more frequently with variable-threshold strategies than without them.

Keywords: Donor-Recipient game, Prisoner's Dilemma game, emergence of cooperation, generalized reciprocity, Agent-Based Simulation

1 Introduction

Emergence of cooperation in repeated dilemma game is a very fascinating and important topic. People change their patterns of behavior through their experience. This paper investigates this change by introducing variable-threshold strategies and their effect on the emergence of cooperation in demographic dilemma games. How does pattern of behavior of a player vary? We consider two ways of varying his pattern of behavior; one is by the comparison between his experienced cooperation rate at which the opponents cooperated with him and his tendency toward cooperation, and the other between his experienced cooperation rate and his subjective idea (vague image commonly shared among people including him) on the social cooperation rate. It is natural for us to assume that the tendency toward cooperation affects both his moves and his pattern of behavior. We do assume that the subjective idea affects not his moves directly but his pattern of behavior.

Many types of strategies are considered in the literature on repeated dilemma games, for example, AllC, AllD, and Tit for Tat (TFT). AllC and AllD are constant strategies since they never change their move although the opponent's move may change, whereas TFT changes his observable move at each period depending on the previous opponent's move. Suppose that a player was born as AllC and has experienced Cooperation and Defection almost equally until now. Is it natural for him to be still AllC from now on or to become TFT since the society is not full of cooperation? We think there is the case that he becomes TFT, that is, he changes his tendency toward cooperation. We want to incorporate this change into our model. We introduce variable-threshold strategies whose components are extended forms of TFT. We interpret TFT as two-state automaton, where TFT uses Defect if the state is 0, Cooperate if it is 1 and the smallest state number that prescribes using Cooperate as a *threshold*. AllC has zero threshold in this interpretation. We allow up to three states. Variable-threshold strategy changes its threshold at most once at some age (once in his lifetime) depending on its experience until then. Thus variable-threshold TFT who was born as TFT may change to AllC or AllD. Also variable-threshold AllC who was born as AllC may change to TFT. We deal with two ways of varying threshold; one is based on player's *cooperation tendency*, and the other on a subjective idea of a player, *expected cooperation rate* of the society, that is an inheritable property from his parent. The former is the case, for example, where a player who was born as AllC becomes TFT if he experiences cooperation and defection equally since cooperation tendency 1/2 of TFT is nearer to his experienced cooperation rate 50% than cooperation tendency 1 of AllC. The latter corresponds to the case, for example, where a player who has 40% expected cooperation rate tries to decrease his threshold (in order to become more cooperative) in the same situation since the society is more cooperative (50%) than he expected (40%) subjectively. We show cases where cooperation emerges more frequently with variable-threshold strategies than without them in demographic Donor-Recipient (DR) or Prisoner's Dilemma (PD) game.

Epstein (2006) introduces demographic model. He shows the emergence of cooperation where AllC's and AllD's are initially randomly distributed in a square lattice of cells. In each period, players move locally (that is, to random cell within the neighboring 4 cells, that is, north, west, south, and east cells; von Neumann neighbors, if unoccupied) and play PD game against local (neighboring) player(s). If wealth (accumulated payoff) of a player becomes negative or his age becomes greater than his lifetime, he dies. If his wealth becomes greater than some amount and there is an unoccupied cell in von Neumann neighbors, he has an offspring and gives the offspring some amount from his wealth. Namekata and Namekata (2010) extend Epstein's original model discussed above by introducing global move, global play, and a player called Referential who uses tag-based TFT with connections. They show cases where the cooperation emerges in some frequency between Referential and AllD, while it is almost impossible between AllC and AllD. Also Namekata and Namekata (2011) introduce Reluctant players, who delay replying to changes and use extended forms of TFT, into demographic PD game and consider the effect of Reluctant players on the emergence of cooperation.

Nowak and Sigmund (1998) consider the emergence of cooperation in different setting where two players are randomly matched, one is selected as Donor and the other as Recipient at random, and play DR game at each period. Frequency of a strategy at the next period is proportional to the payoff of the strategy earned at the current period, which is different from that in our demographic model. The chance that the same two players meet again over periods is very small. Every player has his own image score that takes on some range, is initially zero, and increases or decreases by one if he cooperates or defects, respectively. Donor decides his move (Cooperate or Defect) depending on the opponent's image score. Riolo et al. (2001) deal with similar repeated DR game setting where, instead of image score, every player has his own tag and tolerance and Donor cooperates only if the difference between his tag and the opponent's is smaller than his tolerance.

In general, reciprocity explains the emergence of cooperation in several situations (Nowak and Sigmund 2005): Direct reciprocity assumes that a player plays games with the same opponent repeatedly and he determines his move depending on moves of the same opponent. If a player plays games repeatedly and the opponents may not be the same one, indirect (downstream) reciprocity assumes that the player determines his move to the current opponent depending on the previous moves of this current opponent, or indirect upstream reciprocity, or generalized reciprocity, assumes that the player determines his move to the current opponent depending on the previous experience of his own. Since a player in our model and Namekata and Namekata (2010, 2011) determines his move depending on his own previous experience, we deal with generalized reciprocity. Nowak and Sigmund (1998) deal with indirect (downstream) reciprocity because Donor determines his move to his opponent Recipient depending on the image score of the Recipient that relates to the previous moves of the Recipient. There is no reciprocity, either direct or indirect in the model of Riolo et al. (2001) because Donor's move does not depend on the opponent's previous moves as well as his own previous experience.

Nowak and Sigmund (1994) investigate the emergence of cooperation (by direct reciprocity) in infinitely repeated alternating PD game. In repeated alternating PD game, players do not take actions simultaneously at every period but alternately. DR game is a special case of alternating PD game. Two DR games produce one corresponding usual PD game, but not vice versa. Nowak and Sigmund (1994) show the winning strategy in repeated alternating PD game is different from that in usual repeated simultaneous PD game. Frean (1994) considers the difference between alternating move and simultaneous move in different setting, where two players take one action at every two successive PD games alternately and the payoff of the game at each period is determined by the current or previous moves, if available, of the two players. Thus we should be careful that alternating move game and simultaneous move game are very different situations.

In Section 2, we explain our model in detail. In Section 3, results of simulation are discussed. And Section 4 concludes the paper.

2 Model

We start with extending TFT as follows in order to introduce variable-threshold strategy: Let m=0,1,2; t=0,…,m+1; s=0,…,m. Strategy component (m,t;s) is illustrated in Fig 1. It has m+1 inner states. The inner states are numbered 0, 1,…, m; thus m is the largest state number. State i is labeled D_i if $i<t$ or C_i if not. If current state is labeled C or D, then the strategy component prescribes using C or D, respectively. In other words, the strategy component prescribes using D if current state $i<t$ and using C if not; thus the value t is the *threshold* which determines the move of a player. Initial state in period 0 is state s; its label is D_s if $s<t$ or C_s if not. If current state is i, then the next state is min{i+1,m} or max{i−1,0} given that the opponent uses C or D, respectively, in this period. How to vary threshold is given shortly in this section. Note that TFT is expressed as (1,1;1) in this notation if

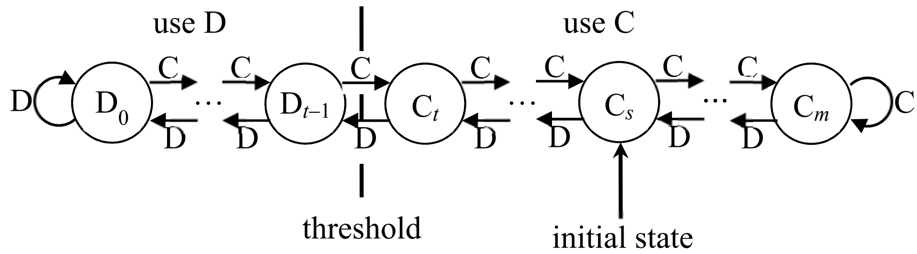

Fig. 1. Strategy component (m,t;s) in case of $t<s<m$. Circles denote inner states. Initial state is the state pointed by arrow labeled "initial state". Threshold divides states into two subclasses; one prescribes using D and the other using C. The transition between states occurs along the arrow labeled C or D if the opponent uses C or D, respectively.

threshold is fixed and we regard the strategy component as a strategy. We abbreviate fixed threshold case as fTh. Thus strategy component (m,t;s) is an extended form of TFT. To sum up, our strategy components are expressed as (m,t;s); m is the largest state number, t is the threshold, and s is the initial state number. We omit the initial state like (m,t;*) if it is determined randomly. We also omit the initial state like (m,t) if we have no need to specify it.

We now explain how to vary threshold in detail. A player has, as his inheritable property, ageOfChange (abbreviated as ageCh) at which he may vary his threshold in accordance with his experience in encounter with others if he is still alive at his ageOfChange. We define *experienced cooperation rate* (abbreviated as erCr) of a player as the number of move C used by the opponents divided by the total number of games played by him (as Recipient if the game is DR) until his ageOfChange. If the denominator of erCr is 0 and thus it is not defined, then nothing does happen. How does a player vary his threshold in accordance with this objective erCr if it is defined? We deal with two ways of varying threshold; one is based on player's cooperation tendency that is firmly related to his strategy component, and the other on a subjective idea of a player, expected cooperation rate of the society, that is

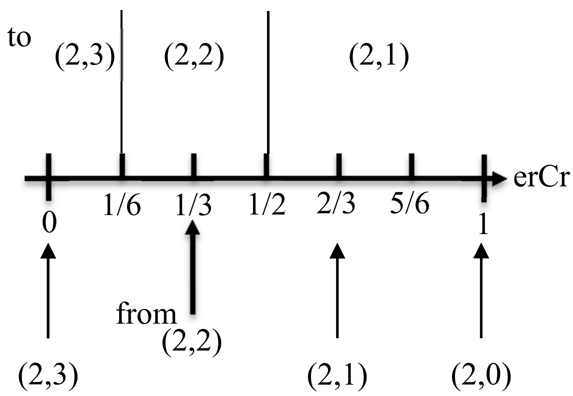

Fig. 2. (vTend): The above figure shows how to vary threshold, for example, of strategy component (2,2). Current threshold 2 increases to 3, remains at 2, or decreases to 1 if erCr<1/6, 1/6≤erCr<1/2, or erCr≥1/2, respectively.

independent of his strategy component.

(vTend): We define *cooperation tendency* of (m,t) as $\mathrm{CT}(m,t) := \frac{m+1-t}{m+1}$, that is, the number of states labeled C divided by the total number of states in Fig 1. This value is interpreted as the tendency toward cooperation. AllC, AllD and TFT have 1, 0 and 1/2 cooperation tendency respectively. If a player with (m,t) experiences erCr at his ageOfChange, then he tries to adjust his cooperation tendency (actually adjust his threshold) to be as near the erCr as possible by at most one increment or decrement (see Fig 2), that is, adjust his threshold to a new threshold t^* which is given by

$$|\mathrm{CT}(m,t^*) - \mathrm{erCr}| = \min \begin{Bmatrix} |\mathrm{CT}(m, \max\{t-1,0\}) - \mathrm{erCr}|, \\ |\mathrm{CT}(m,t) - \mathrm{erCr}|, \\ |\mathrm{CT}(m, \min\{t+1, m+1\}) - \mathrm{erCr}| \end{Bmatrix}$$

where t^* is given to be the smallest t if the minimum of the right hand side of the above equation is attained by multiple values of t. This way of varying threshold is based on two objective values, cooperation tendency and experienced cooperation rate of the player and is called varying threshold by cooperation tendency (abbreviated as vTend). Note that strategy components (1,t) (t=0,1,2) tend to vary to (1,1) through generations if erCr is in (1/4,3/4). Also that strategy components (2,t) (t=0,1,2,3) tend to vary to (2,1) through generations if erCr is in (1/2,5/6).

(vCr): We assume that a player has *expected cooperation rate* (abbreviated as ecCr) as his inheritable property. The ecCr is a subjective rate at which he expects the society is cooperative. If a player with ecCr experiences erCr at his ageOfChange, then he tries to adjust his threshold to erCr irrespective of his cooperation tendency as follows (see Fig 3):

Decrease his threshold by one (try to be more cooperative) if possible

in case of erCr≥ecCr+tolerance (since the society is more cooperative than

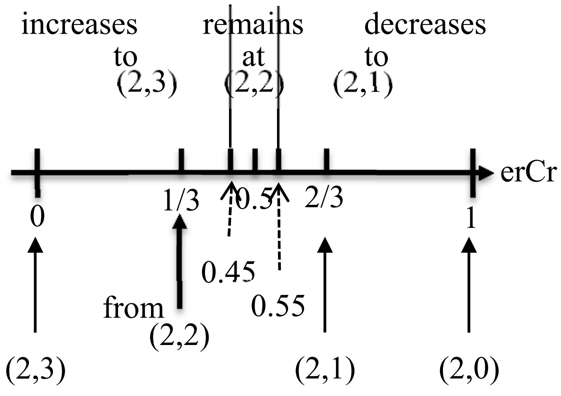

Fig. 3. (vCr): The above figure shows how to vary threshold, for example, of strategy component (2,2) for ecCr=0.5 and tolerance=0.05. Current threshold 2 increases to 3, remains at 2, or decreases to 1 if erCr<0.45, 0.45≤erCr<0.55, or erCr≥0.55, respectively.

expected),

increase his threshold by one (try to be more defective) if possible

in case of erCr<ecCr−tolerance (since the society is more defective than expected),

do not vary his threshold

in other cases,

where tolerance is set to be 0.05 in our simulation. This second way of varying threshold is based on two values; one is objective experienced cooperation rate and the other is subjective expected cooperation rate of the society and is called varying threshold by expected cooperation rate (abbreviated as vCr). This second way of varying threshold is our attempt to incorporate some subjective property of a player which is not directly related to strategy component into decision process; his subjective property affects not his moves directly but his pattern of behavior. Note that threshold does not increase if erCr ≥ 0.5 = 0.55 − 0.05 since we will set ecCr of the society to be rather pessimistic range [0.35,0.55) in Table 4.

We have fully defined a variable-threshold strategy by specifying strategy component ($m,t;s$), way of varying threshold (fTh, vTend or vCr), ageOfChange, and ecCr in case of vCr. Thus strategy component ($m,t;s$) of a variable-threshold strategy of a player may vary to another strategy component (m,t') ($t'=t+1$ or $t'=t-1$) at his ageOfChange. Note that we need not to specify all elements of the latter strategy component (m,t') because they are determined automatically. Thus we can say variable-threshold strategy ($m,t;s$) if way of varying threshold is understood in the context. Usual TFT, AllC, and AllD are (1,1;1), (m,0;s), and (m,m+1;s), respectively if their threshold is fixed. We also use TFT, AllC, and AllD to call strategy components (1,1), (m,0), and (m,m+1), respectively. Strategy and its strategy components are not the same in the strict sense, but we do not distinguish these terms strictly unless there is any confusion. So we say, for example, variable-threshold TFT varies to AllC. Notations ($m,t;s$), (m,t;*), and (m,t) are used to indicate both strategy and strategy component. Note that we deal with indirect upstream reciprocity, that is, generalized reciprocity since moves of the strategy are determined only by the previous experience of the strategy.

We restrict our model to satisfy the following condition:

(AllCneverAllD): If a player (m,0) (AllC) before his ageOfChange belongs to the first

Table 1. Payoff matrix of DR game. Constant x is given by $x=\frac{b-c}{4}$. We set b=8 and c=1 in this paper.

		Recipient
Donor	C	$-c-x, b-x$
	D	$-x, -x$

Table 2. Payoff matrix of PD game. We set $T=\frac{b+c}{2}$, $R=\frac{b-c}{2}$, $P=-R$, $S=-T$, b=8, and c=1 in this paper.

	C	D
C	R,R	S,T
D	T,S	P,P

generation, then his all descendants are never (m,m+1) (AllD). And if a player (m,m+1) (AllD) before his ageOfChange belongs to the first generation, then his all descendants are never (m,0) (AllC).

Note that AllC of the form (0,0) is always AllC and AllD of the form (0,1) is always AllD because of this condition.

We deal with Donor-Recipient (DR) game or Prisoner's Dilemma (PD) game as a stage game. DR game is a two-person game where one player is randomly selected as Donor and the other as Recipient. Donor has two moves, Cooperate (C) and Defect (D). C means Donor pays cost c in order for Recipient to receive benefit b (b=8>c=1>0). Defect means Donor does nothing. Note that Recipient has no move. PD game which we use here is a two-person simultaneous move game where both player has two moves, C and D, whose meanings are the same as in DR game. We call our DR game as *randomly alternating move game* because one player is randomly selected as Donor and the other as Recipient. Since we want to compare the effect of variable-threshold strategies between DR game and PD game, each player plays two games against (possibly different) players at each period if the stage game is DR game. Since Donor is selected at random in each DR game, it is expected that at each period each player plays four DR games as Donor two times and as Recipient two times. On the other hand each player plays one game against another player at each period if the stage game is PD game. It is expected that at each period each player plays two PD games. Since it is common in demographic dilemma game that the payoff of a player to be positive if the opponent uses C and negative if D and the worst payoff of a player is equal to the best payoff in absolute value, we transform the original payoffs to new ones by subtracting

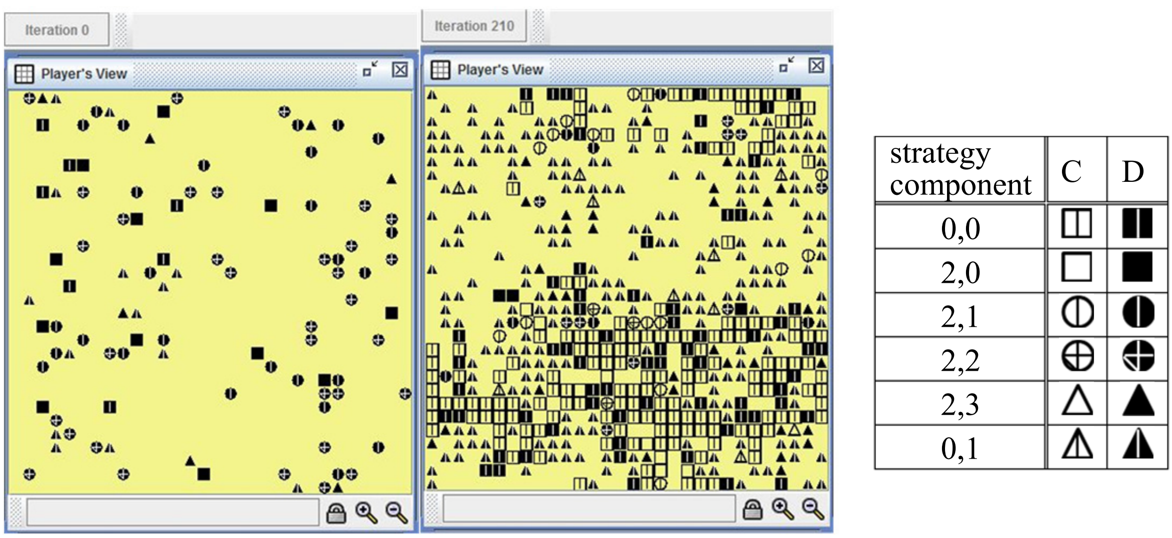

Fig. 4. Typical example of simulation (in Table 5): The left figure shows the state at period 0 and the right at period 210. Shapes represent players. Their shape shows strategy component and move (C or D) at that period as the right table indicates. Here move C means that player's average C rate is larger than or equal to 0.5, and move D means that it is smaller than 0.5. Player's average C rate is defined in Section 3.

Table 3. Detailed description. (1) describes move and (2) describes play in detail.

(1)	With probability rateOfGlobalMoveToLocal (abbreviated as rGML), player moves to random unoccupied cell in the whole lattice. If there is no such cell, he stays at the current cell. Or with probability 1−rGML, player moves to random cell in von Neumann neighbors if it is unoccupied. If there is no such cell, he stays at the current cell.
(2)	With probability rateOfGlobalPlayToLocal (abbreviated as rGPL), the opponent against whom a player plays dilemma game is selected at random from all players (except himself) in the whole lattice. Or with probability 1−rGPL, the opponent is selected at random from von Neumann neighbors (no interaction if none in the neighbors). This process is repeated 2 times if the stage game is DR game. (Two opponents are possibly different.)

constant x. Table 1 and 2 show the transformed payoff matrices of DR game and PD game, respectively. In our simulation, the key difference between DR game and PD game is that between randomly alternating and simultaneous move.

In period 0, N (=100) players are randomly located in 30-by-30 lattice of cells (see Fig 4 left). The left and right borders of the lattice are connected. If a player moves outside, for example, from the right border, then he comes inside from the left border. So are the upper and lower borders. Players use strategies of (m,t;s) form. Initial distribution of strategy components is described in the later paragraph. Initial wealth of every player is 6. Their initial (integer valued) age is randomly distributed between 0 and deathAge (=50).

In each period, each player (1st) moves, and (2nd) plays dilemma game(s) given by Table 1 or Table 2 against another player or other players. Positive payoff needs opponent's C. (The detailed description of (1) move and (2) play is given in Table 3.) The payoff of the game is added to his wealth. If the resultant wealth is greater than fissionWealth (=10) and

Table 4. Initial distribution of inheriting properties.

property	initial distribution
strategy component	We deal with 4 types of populations, 1ALL, 2ASYM, AllCAllD, and TFTAllD with the specified initial distribution as follows: 1ALL:={(1/6)(0,0;0), (1/6)(1,0;*), (1/3)(1,1;*), (1/6)(1,2;*), (1/6)(0,1;0)}, 2ASYM:= {(1/8)(0,0;0), (1/8)(2,0;*), (1/4)(2,1;*), (1/4)(2,2;*), (1/8)(2,3;*), (1/8)(0,1;0)}, AllCAllD:= {(1/2)(0,0;0), (1/2)(0,1;0)} (fixed-threshold case), TFTAllD:= {(1/2)(1,1;1), (1/2)(0,1;0)} (fixed-threshold case). The notation, for example, of 1ALL, means that with probability 1/6 strategy component (0,0;0) (AllC) is selected, with probability 1/3 strategy component (1,1;*) (indicating initial state is selected randomly) is selected, and so on. Note that initially 50% of players use C on the average since both ((0,0;0) or (1,1;1)) and (0,1;0) are included with the same probability and so are both (m,t;*) and (m,m−t+1;*).
rGML	Uniformly distributed at interval [lowRGML, highRGML) (=move).
rGPL	Uniformly distributed at interval [lowRGPL, highRGPL) (=play).
ageCh	Takes one randomly from {15, 16, 17, 18, 19, 20}.
ecCr	Uniformly distributed at interval [0.35,0.55) (=vCr).

there is an unoccupied cell in von Neumann neighbors, the player has an offspring and give the offspring 6 units from his wealth. His age is increased by one. If his age is equal to his ageOfChange, then follow the varying-threshold process discussed above. If the resultant wealth becomes negative or his age is greater than deathAge (=50), then he dies. Then next period starts.

In our simulation we use synchronous updating, that is, in each period, all players move, then all players play, then all players have an offspring if possible, and then each player does the varying-threshold process if he is at his ageOfChange. Among properties of a player, strategy component, rateOfGlobalMoveToLocal (rGML), rateOfGlobalPlayToLocal (rGPL), ageOfChange (ageCh), and ecCr are inherited from parent to offspring. We remark that the strategy component and its initial state of the offspring are set to the current strategy component and the current state of the parent. But there is a small mutationRate (=0.05) with which they are not inherited. Initial distribution of these properties is given in Table 4 and this distribution is also used when mutation occurs. Initial distribution of strategy components is one of four distributions, 1ALL, 2ASYM, AllCAllD, and TFTAllD, listed in Table 4. We comment on their set and distribution in more detail. We first consider them only as a set of strategy components and then their initial distribution over the set in this order. Let nALL:=$\{(m,t;*)|m=0,\ldots,n,\ t=0,\ldots,m+1\}$, nSYM:=$\{(m,t;*)|m$: odd, $1\leq m\leq n,\ t=(m+1)/2\}$, nAllCAllD := $\{(m,t;*)|1\leq m\leq n, t=0, m+1\}$, and nASYM := nALL−nSYM−$(n-1)$AllCAllD. nALL includes all strategy components whose number of inner states is smaller than or equal to $n+1$. nSYM includes all strategy components in nALL which is symmetric between C and D. nAllCAllD includes AllC and AllD that have more than 1 inner states within nALL. nASYM includes nALL but nSYM and $(n-1)$AllCAllD. The key idea that we deal with AllCAllD(=0ALL), 1ALL, and particularly 2ASYM is that we want to construct a strategy component set which has small number of strategy components and has symmetric pattern between using C and D over the set and includes AllC (0,0;0) and AllD (0,1;0) that cannot vary their threshold by AllCneverAllD condition as well as variable-threshold AllC ((1,0;*) or (2,0;*)) and AllD ((1,2;*) or (2,3;*)). Thus we exclude 2SYM and 1AllCAllD from 2All in order to get 2ASYM. Now we comment on distribution over strategy components in the set. For example, 1ALL includes 5 strategy components, (0,0;0), (1,0;*), (1,1;*), (1,2;*), and (0,1;0). But the first two strategy components are AllC and the last two are AllD. 1ALL actually has three strategy components and these three strategy components are selected with the same probability 1/3 which is divided equally between, for example, (0,0;0) and (1,0;*). Thus we obtain 1ALL={(1/6)(0,0;0), (1/6)(1,0;*), (1/3)(1,1;*), (1/6)(1,2;*), (1/6)(0,1;0)}. We include AllCAllD and TFTAllD that are fixed-threshold cases in Table 4 as reference populations. We assume that with errorRate (=0.05) a player makes mistake when he makes his move. Thus AllC may Defect sometime.

If population of strategy components is AllCAllD, move = [0.0, 0.0], and play = [0.0, 0.0], then our model is similar to that of Epstein (2006). His model uses asynchronous updating while our model uses synchronous updating.

3 Simulation and Result

Our purpose to simulate our model is to search parameter settings where the cooperation emerges more frequently with variable-threshold strategies than without them and investigate the effect of variable-threshold strategies on the emergence of cooperation. We use Ascape (http://sourceforge.net/projects/ascape/) to simulate our model.

We consider the following range of parameters: (move, play) = ([0.0, 0.15), [0.5, 0.9)) and ([0.2, 0.6), [0.2, 0.3)). We call these situations lMgP (local move global play) and gMlP (global move local play), respectively, although move = [0.2, 0.6) is larger than [0.0, 0.15) but is not global move literally.

We execute 300 runs of simulations in each parameter setting. We judge that the cooperation emerges in a run if there are more than 100 players and the average C rate ("Cr") is greater than 0.2 at period 500, where the average C rate at a period is the average of the player's average C rate at the period over all players and the player's average C rate at the period is defined as the number of move C used by the player divided by the number of games (as Donor if the game is DR) played at the period. (We interpret 0/0 as 0.) This average C rate is the rate at which we see cooperative move C as an outside observer. Since negative wealth of a player means his death in our model and he has a lifetime, it is necessary for many players to use C in order that the population is not extinct.

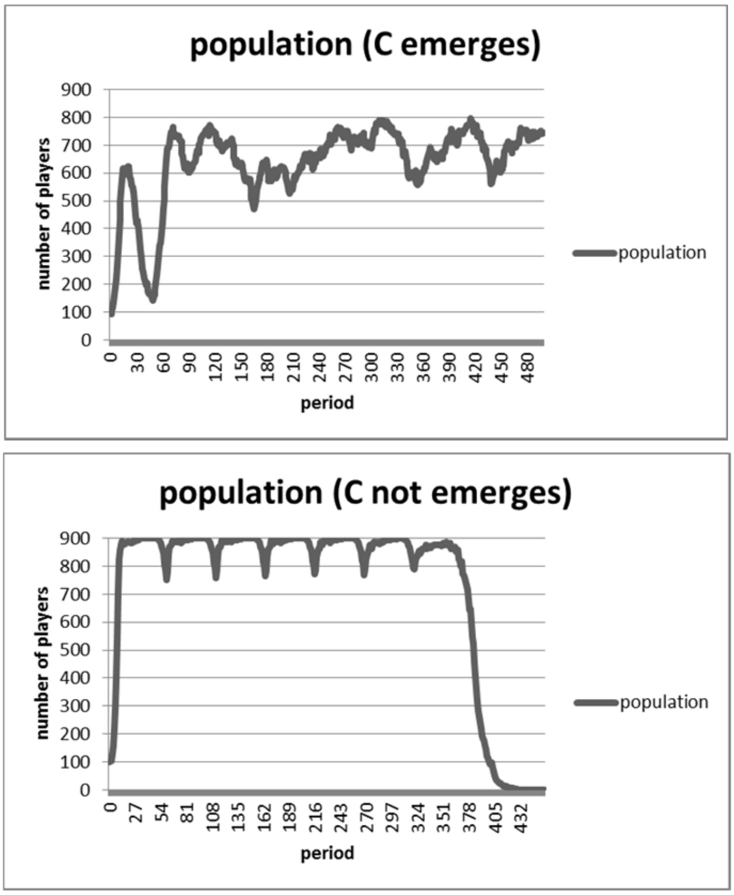

Fig. 5. Two typical examples of simulation. The upper is a case (DR game, 2ASYM (vCr=[0.35,0.55)), lMgP (move=[0.0,0.15), play=[0.5,0.9))) in Table 5. The lower is a case (PD game, 2ASYM (vTend), gMlP (move=[0.2,0.6), play=[0.2,0.3))) in Table 6.

First we show two typical examples in Fig 5; cooperation emerges in one example but it

Table 5. lMgP (move=[0.0,0.15), play= [0.5,0.9)) case

lMgP		TFTAllD	AllCAllD	1ALL			2ASYM			M/m
				fTh	vTend	vCr	fTh	vTend	vCr	
Ce	DR	0.000	0.303	0.360	0.320	0.457	0.510	0.457	0.547	1.80
	PD	0.000	0.180	0.160	0.180	0.213	0.343	0.307	0.453	2.52
Sa	DR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.012	-
	PD	0.000	0.000	0.000	0.000	0.031	0.000	0.207	0.169	-

Table 6. gMlP (move=[0.2,0.6), play=[0.2,0.3)) case

gMlP		AllCAllD	TFTAllD	1ALL			2ASYM			M/m
				fTh	vCr	vTend	fTh	vCr	vTend	
Ce	DR	0.083	0.250	0.467	0.407	0.723	0.810	0.470	0.923	3.69
	PD	0.017	0.120	0.130	0.210	0.360	0.457	0.477	0.710	5.92
Sa	DR	0.000	0.000	0.000	0.016	0.009	0.004	0.014	0.000	-
	PD	0.000	0.000	0.000	0.190	0.037	0.000	0.545	0.263	-

does not in the other. The upper graph shows the number of all players at one successful case (DR game, 2ASYM(vCr=[0.35,0.55)), lMgP (move=[0.0, 0.15), play= [0.5, 0.9))) in Table 5. The lower graph shows that at one unsuccessful case (PD Game, 2ASYM(vTend), gMlP (move=[0.2, 0.6), play= [0.2, 0.3))) in Table 6. Note that in the lower graph players are almost full over the whole lattice but the population becomes extinct around at period 410. We summarize our results in the following tables. Tables 5 and 6 deal with lMgP and gMlP case, respectively. In tables 5 and 6, the entity of the first row and the second to fifth column indicates initial distribution of strategy components. In the second row, fTh, vTend, and vCr indicate fixed-threshold case, variable-threshold by cooperation tendency case, and variable-threshold by expected cooperation rate case, respectively. "Ce" in the first column indicates this row gives the emergence rate of cooperation that is the frequency

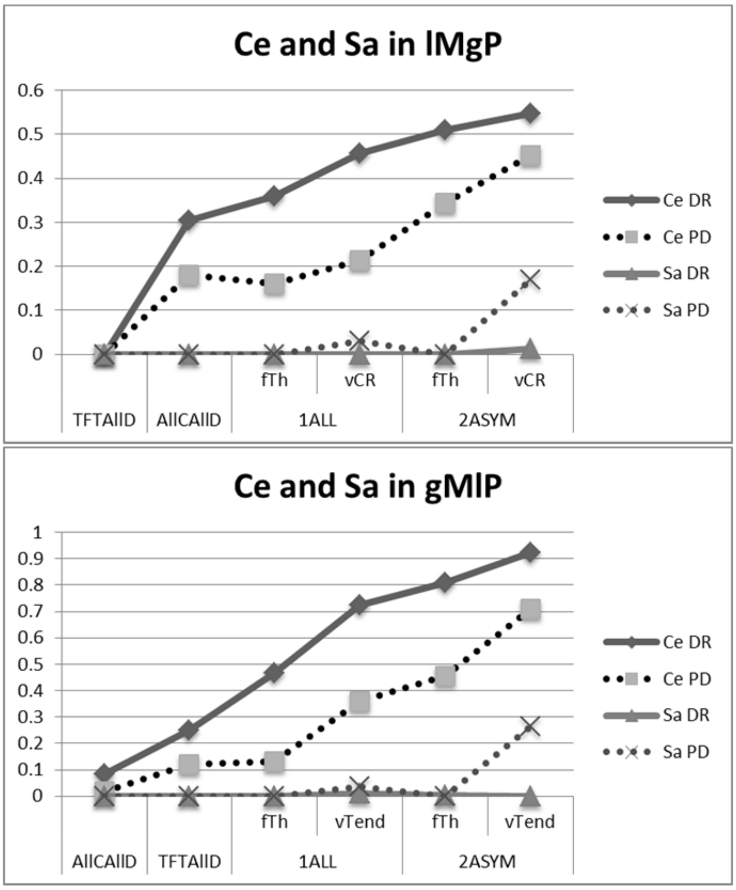

Fig. 6. Graphs Ce and Sa. The upper and lower are for Table 5 and 6, respectively.

with which the cooperation emerges. “Sa” indicates the saturation rate which is defined as the number of runs, where the average C rate is greater than 0.7 and there exist more than 810 players (= 90% of 30-by-30) at the last period 500, divided by the number of runs where the cooperation emerges. This saturation rate measures the rate at which players are almost full over 30-by-30 cells. “DR” and “PD” in the second column indicate the corresponding rows are about DR and PD game, respectively. The last column “M/m” indicates the rate between maximum Ce and the Ce of TFT or AllCAllD (which is the larger of the two), for example, 1.80 = 0.547/0.303.

For example, Table 5 shows that the frequency with which the cooperation emerges is 54.7% and the saturation rate is 1.2% in DR game when the population is 2ASYM (vCr), rateOfGlobalMoveToLocal is initially distributed in [0.0, 0.15), and rateOfGlobalPlayToLocal in [0.5, 0.9). We observe that the cooperation almost never emerges if population is TFTAllD in all cases of Tables 5. Table 5 shows that Ce in PD game is, for example, 45.3% for 2ASYM (vCr), while that is 18.0% for AllCAllD; the former is 2.52 times larger than the latter. Table 6 shows that the cooperation emerges at 25.0% for TFTAllD but Ce is 92.3% for 2ASYM (vTend) in DR game; the latter is 3.69 times larger than the former.

We observe that Ce’s of vCr are larger than those of vTend in lMgP case (Table 5) and the opposite relation, that is, Ce’s of vTend are larger than those of vCr in gMlP case (Table 6). Also that Ce’s of AllCAllD are larger than those of TFTAllD in lMgP case (Table 5) and the opposite relation that Ce’s of TFTAllD are larger than those of AllCAllD in gMlP case (Table 6). From now on we concentrate on favorable way of varying threshold, that is, we deal only with vCr in lMgP case and only with vTend in gMlP case for varying threshold, respectively. Fig 6 shows the graphs of Ce and Sa. We see that cooperation emerges more frequently in variable-threshold (vCr or vTend) case than in fixed-threshold (fTh) case although the net effects vary from 0.547/0.510=1.07 to 0.360/0.130=2.77. Ce’s in DR game are larger than those in PD game. M/m’s in PD game are larger than those in DR game. Sa’s in PD game are larger than those in DR game and the latter are almost zero (smaller than or equal to 1.2% in Fig 6). Thus we conclude that variable-threshold strategies promote emergence of cooperation in Demographic DR and PD games in our simulation. We summarize important results in the following two observations:

Observation (favorable population between AllCAllD and TFTAllD): lMgP case favors AllCAllD over TFTAllD, whereas gMlP case does TFTAllD over AllCAllD with respect to the emergence of cooperation.

Observation (favorable way of varying threshold): lMgP case favors vCr over vTend, whereas gMlP case does vTend over vCr with respect to the emergence of cooperation.

The latter observation will be deduced from other observations and conjectures later in this section.

Next we investigate the behavior of threshold. Tables 7 and 8 summarize further data related to threshold for lMgP and gMlP, respectively. These data are averages of some amounts at period 500 over non saturation runs among all 300 runs since the saturation causes unusual behavior of the amounts. For example, "mature" indicates the average rate at which a player is older than his ageOfChange over players who are dying at period 500. "no", "inc", or "dec" means the average rate at which a player did nothing, increased, or decreased his threshold, respectively, over mature players when he was at his ageOfChange. "aCr" is the average actual C rate at period 500 where the cooperation emerges. aCr is slightly different from Cr and is defined as the average of the player's average C rate at the period over all players who actually play at least one game (as Donor if the game is DR) at the period. aCr is regarded as the rate at which a player receives the opponents' cooperative move C on the average. "erCr" shows the average experienced cooperation rate over players who increase (or decrease) their threshold at period 500. "deathAge" is the average age over the players who die at period 500.

Table 7. Further data in lMgP.

lMgP			1ALL vCr	2ASYM vCr
mature	DR		0.053	0.059
		no	0.922	0.888
		inc	0.004	0.002
		dec	0.074	0.111
mature	PD		0.124	0.096
		no	0.940	0.915
		inc	0.000	0.001
		dec	0.060	0.084
aCr		DR	0.453	0.459
		PD	0.482	0.477
erCr	DR	inc	0.431	0.444
		dec	0.575	0.569
	PD	inc	---	0.429
		dec	0.595	0.608
deathAge		DR	5.30	5.63
		PD	8.51	7.28

Table 8. Further data in gMlP.

gMlP			1ALL vTend	2ASYM vTend
mature	DR		0.054	0.059
		no	0.784	0.760
		inc	0.057	0.074
		dec	0.159	0.165
mature	PD		0.084	0.102
		no	0.751	0.840
		inc	0.097	0.031
		dec	0.152	0.129
aCr		DR	0.453	0.464
		PD	0.477	0.502
erCr	DR	inc	0.629	0.538
		dec	0.584	0.586
	PD	inc	0.668	0.658
		dec	0.591	0.602
deathAge		DR	5.25	5.49
		PD	6.79	7.53

Table 7 shows, for example, that in lMgP and DR game case among players who die at period 500, about 5% of them are older than their ageOfChange; about 90% of whom did nothing, smaller than 0.5% of whom increased their threshold, and about 10% of whom decreased their threshold. Thus in lMgP case (more generally in vCr case although detailed data are not given here) among players who die at period 500, most players do not reach their ageOfChange; and players who have reached that age did not vary their threshold very often, decreased their threshold (to become more cooperative) less often, or almost never increased their threshold (to become more defective). This is true for both DR and PD game although the mature rates are larger in PD game than in DR game. erCr's for inc are smaller than the corresponding aCr's, whereas erCr's for dec are larger than the corresponding aCr's.

Table 8 shows, for example, that in gMlP and DR game case among players who die at period 500, about 5% of them are older than their ageOfChange; about 77% of whom did nothing, about 7% of whom increased their threshold, and about 16% of whom decreased their threshold. Thus in gMlP case (more generally in vTend case although detailed data are not given here) among players who die at period 500, most players do not reach their ageOfChange; and players who have reached that age did not vary their threshold very often, decreased their threshold (to become more cooperative) less often, or increased their threshold (to become more defective) still less often. This is true for both DR and PD game although the mature rates are larger in PD game than in DR game. erCr's for both inc and dec are larger than the corresponding aCr's.

aCr's in Tables 7 and 8 are almost smaller than 0.5, that is, it is expected that a player receives the opponents' cooperative move C smaller than 50% of the time on the average. By the relation between aCr's and erCr's in Tables 7 and 8, the players who decrease their threshold at period 500 experience higher cooperation rate than on the average, whereas the players who increase their threshold at period 500 experience smaller cooperation rate than on the average in lMgP case (Table 7) or higher cooperation rate than on the average in gMlP case (Table 8). This relation between aCr's and erCr's holds in general if we replace lMgP with vCr and gMlP with vTend, which is stated in the following Observation (varying threshold and experienced Cr), although detailed data are not given here. Also deathAge's in Tables 7 and 8 are much smaller than 15 (minimum value of ageOfChange), which is consistent with the low mature rates. Note that deathAge's of PD games are larger than those of DR games.

We summarize important results in the following observations about at period 500 and conjectures about at typical periods:

Observation (average of actual cooperation rate): A player receives the opponents' cooperative move C almost smaller than 50% of the time on the average.

Observation (varying threshold and experienced Cr, erCr): A player who decreases his threshold (to become more cooperative) has experienced the opponents' cooperative move C more often than on the average. A player who increases his threshold (to become more defective) has experienced the opponents' cooperative move C less often than on the average in vCr case and more often than on the average in vTend case.

Observation (behavior of threshold): Most players die before their ageOfChange. Among players who reach their ageOfChange, most of them do nothing, some of them decrease their threshold, and almost none of them (in vCr case) or less of them (in vTend case) increase their threshold. Particularly, a player varies his threshold very rarely in his lifetime (smaller than 2.1%). Furthermore, it almost never holds that erCr < ecCr −

tolerance (0.05) at ageOfChange in vCr case; since ecCr is in [0.35,0.55).

Conjecture (varying threshold in vCr): Transitions between strategy components such as (1,2) => (1,1) => (1,0) and (2,3) => (2,2) => (2,1) => (2,0) tend to happen; by the part of vCr case in Observation (behavior of threshold).

Conjecture (varying threshold in vTend): Transitions between strategy components such as (1,2) => (1,1), (1,0) => (1,1), (2,3) => (2,2) => (2,1), and (2,0) => (2,1) tend to happen; since among players who reach their ageOfChange, some of them decrease their threshold, and less of them (in vTend case) increase their threshold by Observation (behavior of threshold) and erCr's are in (1/4,3/4) for 1ALL(vTend) and in (1/2,5/6) for 2ASYM(vTend) by Table 8 and detailed data not given here.

Fig 7 shows average frequency of strategy components at period 500 over non saturation runs (upper) in 2ASYM (vCr) and DR game of Table 5 (lMgP) and that (lower) in 2ASYM (vTend) and PD game of Table 6 (gMlP). Note that AllD bar and AllC bar have large frequency in upper vCr case but AllD bar and (2,1) bar have large frequency in lower vTend case. The upper frequency of strategy components in Fig 7 follows from Conjecture (varying threshold in vCr). The lower frequency of strategy components in Fig 7 follows from Conjecture (varying threshold in vTend). Let us deduce Observation (favorable way of varying threshold) from other observations and conjectures. 1ALL(vTend) and 2ASYM(vTend) push their strategy components to (1,1) and (2,1), respectively, by Conjecture (varying threshold in vTend), but AllCAllD is favored over TFTAllD in lMgP case by Observation (favorable population between AllCAllD and TFTAllD). Therefore we deduce that the cooperation emerges less frequently at vTend than at vCr in lMgP case. 1ALL(vCr) and 2ASYM(vCr) push their strategy components to (1,0) and (2,0), respectively, by Conjecture (varying threshold in vCr), but TFTAllD is favored over AllCAllD in gMlP case by Observation (favorable population between AllCAllD and TFTAllD). Therefore we deduce that the cooperation emerges less frequently at vCr than at vTend in gMlP case.

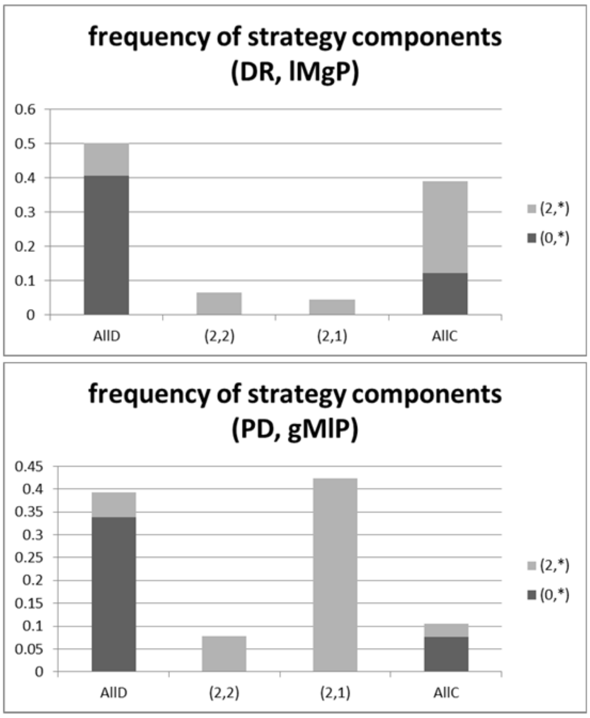

Fig. 7. Average frequency of strategy components at period 500 over non saturation runs in 2ASYM (vCr) in DR game for lMgP case (upper) and in 2ASYM (vTend) in PD game for gMlP case (lower).

4 Conclusion

Our main concern is to devise a proper representation of how people change their pattern of behavior. We introduce variable-threshold strategies whose component is an extended form of TFT. Thus we can say variable-threshold AllC who was born as AllC changes to TFT.

We show two parameter settings where the cooperation emerges more frequently with variable-threshold strategies than without them although each player varies his threshold very rarely in his lifetime (smaller than 2.1%). We deal with two ways of varying threshold, one (vTend) by cooperation tendency and the other (vCr) by expected cooperation rate. vTend is based on the comparison between two objective values; experienced cooperation rate and cooperation tendency. vCr is based on the comparison between two values; objective experienced cooperation rate and subjective expected cooperation rate. We emphasize that the latter subjective expected cooperation rate of a player affects not his moves directly but his pattern of behavior. vTend promotes the emergence of cooperation by pushing threshold in both cooperative and defective directions in one parameter setting where TFTAllD is favored over AllCAllD with respect to the emergence of cooperation. vCr promotes the emergence of cooperation by pushing threshold in one cooperative direction in the other parameter setting where AllCAllD is favored over TFTAllD. Donor-Recipient game is randomly alternating move game and Prisoner’s Dilemma game is simultaneous move game. They are quite similar but their emergence rates of cooperation is different; the former is larger than the latter. It is important to pay attention to which is better to represent real situations in order to model them.

In summary, we show through Agent-Based Simulation that the variable-threshold strategies are useful to enhance the emergence of cooperation where players may move and play globally in Demographic Donor-Recipient and Prisoner's Dilemma games.

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