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Abstract

Feldstein (1977) questioned the classical proposition which attributed to Ricardo (call the *Ricardian Classical Proposition*), that *an increase in the land rent tax lowers the price of land by the capitalized value of the tax and thus the landowners bear the entire burden of the tax*, in an overlapping generations economy with *fixed* land but with *flexible* supply of capital, whereas Calvo, Kotlikoff, and Rodriguez (1979) pointed out that Feldstein's finding no longer remains valid in the equivalent economy except that an intergenerational *altruistic* bequest motive is perfectly operative. The purpose of this paper is to demonstrate that the counterargument of Calvo et al. is *not* necessarily valid *despite the presence of operative bequest motive*, if either a selfish motive to leave bequests, inheritance taxes, or property income taxes is introduced into their model. The key reason for the failure of the *Ricardian Classical Proposition* is the *nonlinearity* of either the utility function with respect to the amount of bequests, or of the tax rate with respect to the size of inheritances or property income.

1. Introduction

In the seminal paper of Feldstein (1977), he investigated tax incidence on land rents in the life-cycle model with a fixed land which is substitutable with capital and labor. He obtained the "surprising" result that with an imposition of tax on land rents, if land is sufficiently substitutable for productive capital, in the long-run a land price goes up and thus the rental rate on capital falls substantially. He therefore claimed that the *Ricardian Classical Proposition* (henceforth *RCP*), that the land rent tax is unshiftable and thus landowners bear its entire burden, is *inappropriate* in a dynamic economy where the supply of capital is *endogenously* determined by a choice between consumption and saving.

On the other hand, under the perfect foresight assumption, Chamley and Write (1987) have developed the analysis by focusing on the transitional dynamics of Feldstein's model. They showed that the land rent tax may initially raise land values but there is an upper bound of one-half of tax revenues on the extent to which land values are raised. Their analysis is a dynamic complement to the conventional comparative statics analysis of Feldstein. Eaton (1988) used their analysis to investigate the effects of an exogenous increase in foreign investment on national welfare as well as domestic capital accumulation. More recently, Ihori (1990) extended it to a monetary economy.

By contrast, Calvo, Kotlikoff, and Rodriguez (1979) (henceforth CKR) reexamined Feldstein's proposition in Barro's bequest (or a single dynastic) model where an altruistic bequest motive is perfectly operative, and showed that a *compensated* land rent tax is not shifted at all and thus the tax is fully capitalized in the price of land. As a result, we once again are back to the Ricardian world despite the presence of a flexible factor.

There have been several extensions of Feldstein's analysis, whereas there are no further researches along the line of CKR's analysis despite their apparent denial of Feldstein's conclusions. Moreover, it seems quite unrealistic that life-cycle consumers leave no bequests to their heirs, which was postulated by Feldstein as well as Chamley and Write, on the grounds that bequests commonly take place in real and probably account for a significant part of individual wealth [see, for example, Kotlikoff and Summers(1981)]. Such an observation and their theoretical analysis may lead CKR to conclude that "both economic theory and empirical evidence support David Ricardo's view of the incidence of a tax on rent" (1979,p.874). If their statement is true, whether or not the landowners will bear the 100 percent burden of the capitalized value of the tax depends crucially on whether or not an *altruistic* bequest motive is strong enough to leave bequests; consequently, only the empirical question as to a quantitative assessment of such a bequest motive remains.

It is well-known that *in the context of government debts policy*, *Ricardian Equivalence*, which says that a redistribution in lump-sum taxes accompanied by the issue of an equal amount of government bonds has no effect on the intertemporal allocation of resources, fails to hold if Barro's altruistic bequest model is relaxed to allow one of the following ingredients, such as corner solutions (because of weak inter-generational altruism) [Weil (1987)], imperfect capital markets (or the borrowing constraints) [Hubbard and Judd (1986)], uncertain future income [Feldstein(1988)], endogenous fertility decisions [Barro and Becker (1989)], or the presence of distortionary taxation [Barsky, Mankiw, and Zeldes(1986) and Abel(1986)]. In the light of these considerations we shall reexamine CKR's analysis, which is based on Barro's altruistic bequest model, in more realistic situations.

The purpose of this paper is to accomplish this task. In doing so, we shall maintain the following two assumptions made by CKR. First, the revenue from the land rent tax is refunded in a lump-sum fashion to taxpayers *period by period* (called a *compensated* tax).[1] Since such a compensated tax leaves the consumer's budget constraint unchanged (i.e., no income effects), we can concentrate exclusively on the intertemporal relative price effects (or the effects on the intergenerational terms of trade) of the tax, which have not been addressed by Feldstein, as well as Chamley and Write. Since in their models the tax would affect an economy mainly through income effects rather than substitution effects, it generates neither distortions nor deadweight loss. This is solely because their analysis is based on a saving function, which makes it impossible to separate out the substitution effect from the tax induced impacts on saving behavior; instead, we shall analyze directly the Euler equation for consumption derived from individual's intertemporal decisions, which serves to sharpen analytical results. Second, there are always *positive* bequests motivated by intergenerational altruism as in the Barro-type overlapping generations model, thereby ruling out the possibility of corner solutions. If allowing corner solutions (i.e., no bequests are left to heirs), there is no qualitative difference between CKR's and Feldstein's models so that the *RCP* necessarily fails. Therefore, the proper question to be considered is that *even if an altruistic bequest motive is perfectly operative*, under what circumstances the *RCP* may fail. In addition, these two assumptions together would serve not only to keep our model paralleling CKR's so as to examine the robustness of CKR's results, but also to confine our analysis to the following three extensions rather than all possibilities appeared in the literature of government debt policy.

The first modification is to introduce *impure* altruism into the model (Section 3). Parents would experience a warm glow from making bequests to their children [e.g.,Andreoni (1989)]. The parents may also feel obligation to leave bequests due to family heirlooms, custom, or social status; alternatively, the risk averse parents may prefer to hold positive bequests (or life-cycle saving) in order to avoid the depletion of their wealth at too early an age [called *accidental bequests*, see Abel (1985)]. Recently Bernheim et al.(1985) have suggested that parents may *strategically* use bequests either to control their children or to receive more attention services provided by them. These causal observations and deeper theoretical considerations would indicate that such seemingly *selfish* bequest motives are quite common in a real world. To incorporate those motives into the model and to avoid unnecessary complications, we assume simply that the parent's utility function depends directly on the size of bequests in addition to the heir's utility function. It is shown that the *RCP* fails to hold *despite the presence of perfectly operative altruistic bequests, when utility is nonlinear with respect to the size of bequests*.

The second is to introduce a *nonlinear bequest* tax (it may be called either an *inheritance*, *estate*, or *wealth* tax) (Section 4). Most developed countries have a progressive tax on bequests. The presence of such a tax invalidates the *RCP* because *owing to the nonlinearity of the tax rate with respect to the size of bequests*, changes in the land rent tax will affect the price of heir's consumption in terms of the price of parents' current consumption through changes in the marginal tax rate of bequests, thereby distorting the Euler condition. For the similar reason, the RCP fails in the presence of a *nonlinear property income tax* as well, while the long-run price of land may fall or rise depending on whether or not the marginal tax rate of property income may rise with the tax (Section 5).

In Section 2, as a prerequisite and for comparison purposes, the basic model is presented, which is a simplified version of CKR's model. The plans and results of Section 3, 4, and 5 have been already stated. Section 5 gives brief concluding remarks and possible extensions. Mathematical details are found in appendixes.

2. The Basic model

Consider technology which requires land, labor, and capital to produce output which can be used for either consumption or capital. This technology is represented by a time-invariant, twice-differentiable production function, $Y_t=F(K_t,N_t,L_t)$, which exhibits positive and diminishing marginal products with respect to each input, $\lim_{i\to\infty} F_i=0$ and $\lim_{i\to 0} F_i=\infty$ $(i=K,N,L)$, and constant-returns to scale, where Y is output, K capital, N labor, L land, and F_i the partial derivative with respect to the corresponding argument i, respectively. In intensive form it can be expressed by

$$y_t = F(K_t/N_t, 1, L_t/N_t) = f(k_t, l_t), \tag{1}$$

where $y \equiv Y/N$, $k \equiv K/N$, and $l \equiv L/N$. We assume that the supplies of labor and land are both *fixed* over time, and moreover that $N_t=L_t=1$ for all t.[2]

Assuming a competitive setting and that capital does not depreciate, the real wage w_t, the real rate of return on capital r_t, and the real rate of rent on land m_t, respectively, are given by

$$r_t = r(k_t) = f_k(k_t, 1), \tag{2a}$$

$$w_t = w(k_t) = f(k_t, 1) - k_t f_k(k_t, 1) - f_l(k_t, 1), \tag{2b}$$

$$m_t = m(k_t) = f_l(k_t, 1), \tag{2c}$$

where f_j $(j=k, l)$ represents the partial derivative with respect to the corresponding argument. Note also that $r'<0$, but the signs of w' $(= -kf_{kk}-f_{lk})$ and $m'(=f_{lk})$ are ambiguous in general. For analytical convenience, we assume that f_{kl} is positive (i.e., capital and land are complementary) but *not very large*, so their signs are both positive.[3]

Given perfect foresight without uncertainty, capital and land are perfect portfolio substitutes, and hence the return to holding one unit of land must equal the return to holding one unit of capital:

$$1 + r_{t+1} = \frac{(1-\theta)m_{t+1} + P_{t+1}}{P_t}, \tag{3}$$

where P_t is the price of land in period t and θ is the tax rate on pure land rent.

Each consumer lives for only one period.[4] Thank to the assumption of constant population, each consumer has one child, and this child is only his heir. A representative generation t consumer derives utility from his own consumption c_t and from the welfare of his heir W_{t+1}. His preference is expressed by the additively separable utility function:

$$W_t = u(c_t) + \beta W_{t+1}, \tag{4}$$

with $u'>0$, $u''<0$, $\lim_{c\to\infty} u'(c)=0$, and $\lim_{c\to 0} u'(c)=\infty$, where $\beta\in(0,1)$ is the *constant* weight assigned to the utility of the heir.[5] He supplies one unit of labor inelastically in exchange for receiving a wage w_t, receives inheritances b_t from the previous generation and lump-sum transfers z_t from the government, and earns property income accrued to b_t. He allocates all of his available resources between consumption and altruistic bequests. Thus his flow budget constraint is

$$c_t + b_{t+1} = (1+r_t)b_t + w_t + z_t. \tag{5}$$

The generation t consumer with perfect foresight maximizes his intertemporal utility function (4) subject to (5) and a given amount of inheritances b_t. Since bequests are the only form of savings in this economy, the bequest motive is always operating for all periods (i.e., $b_t>0$ for all t).[6] Thus the above decision problem for generation t will be equivalent to maximizing altruistic infinite-horizon dynasty's welfare subject to the corresponding infinite-horizon budget constraint, because generation t is linked to all future descendants through positive bequests. Letting λ_t be the Lagrangian shadow price of (5), we have the following first-order necessary conditions for all t:

$$u'(c_t) = \lambda_t, \tag{6a}$$

$$\beta\lambda_{t+1}(1+r_{t+1}) = \lambda_t. \tag{6b}$$

In addition, optimality imposes the following transversality condition:

$$\lim_{t\to\infty} \beta^t \lambda_t b_t = 0. \tag{7}$$

The asset market clearing condition is

$$b_{t+1} = k_{t+1} + P_t. \tag{8}$$

We assume that the government rebates the tax revenue from land rent taxes in a lump-sum way to the consumer *period by period*:

$$z_t = \theta m_t. \tag{9}$$

Substituting (8) and (9) into (5) and making use of (2a)-(2c), we get the goods market clearing condition:

$$f(k_t, 1) - k_{t+1} + k_t - c_t = 0. \tag{10}$$

Substituting (6a) into (6b), we obtain the dynamics of c, k, and P for all t which are characterized by (3), (10), and

$$\beta u'(c_{t+1})[1+r_{t+1}] = u'(c_t), \tag{11}$$

together with the transversality condition (7). (11) implies that at the optimum the consumer equates the utility from consuming an extra unit in period t with the utility from bequeathing an extra unit, thus increasing his heir's consumption by $\beta[1+r_{t+1}]$ units in period $t+1$.

In the stationary equilibrium, setting $c_t=c_{t+1}=c^*$, $k_t=k_{t+1}=k^*$ and $P_t=P_{t+1}=P^*$ for all t yields

$$\beta[1+r(k^*)] = 1, \tag{12a}$$

$$r(k^*) = \frac{(1-\theta)m(k^*)}{P^*}, \tag{12b}$$

$$f(k^*,1) = c^*. \tag{12c}$$

It is easy to prove the following proposition (the proof is found in appendix A):

Proposition 1: *There is a unique stationary equilibrium, which is saddle-point stable in the locally linearized system* of (3), (10), and (11).

Furthermore, from (12a)-(12c) it is straightforward to show that

$$\frac{dk^*}{d\theta} = 0, \tag{13a}$$

$$\frac{dP^*}{d\theta} = -\frac{m^*}{r^*} < 0, \tag{13b}$$

$$\frac{d(1-\theta)m^*}{d\theta} = -m^* < 0. \tag{13c}$$

These results coincide with those of CKR (1979); namely, the land rent tax has no impact on the long-run capital stock and is fully capitalized in the price of land, so that the *RCP* holds. This is mainly because (12a) solely determines the stationary-state capital stock, being independent of changes in either P or θ. In words, since the long-run supply curve of capital is *infinitely elastic* (or horizontal) with respect to the interest rate at rate $\beta^{-1}-1$ and the long-run demand curve of capital is given by $f'(k)$, the intersection of both curves, which determines the long-run capital stock, is invariant to changes in P or θ. Thus the interest rate and the *gross* rent on land both remain unchanged as a consequence of the *fixed* capital stock in conjunction with the fixed supplies of land and labor.[7] Hence, the after-tax rent on land, $(1-\theta)m$, has to fall immediately just by the tax, so does the price of land by exactly the full capitalization of the tax so as to keep equilibrium in the no-arbitrage condition (12b). Consequently, capital, consumption, and the welfare of all succeeding generations are unaffected in the long run, while the only initial landlords bear the *entire* burden of the tax resulting from the fall in the land price.[7] To sum up:

Proposition 2: *When the economy is initially in a stationary equilibrium, an increase in the land rent tax affects neither the capital stock nor consumption but causes the price of land to immediately jump to its new stationary-state level.*

Two important remarks are stated. CKR have assumed the recursive utility function $W_t=u(c_t,W_{t+1})$, unlike the present model with the *constant* intercohort discount factor. With this preference, the modified golden-rule condition (12a) will be amended to

$$\frac{\partial W_{t+1}}{\partial W_t}[1+r(k^*)] = 1, \qquad (14)$$

where $W_t=u(c^*, W_{t+1}(c^*, W_{t+1}(c^*, \cdots)))$. Accordingly, the constant parameter

β in (12a) is replaced by the marginal rate of substitution between the utilities of two succeeding generations, which gives an *elastic* long-run supply curve of capital [see Epstein and Hynes (1983)]. Although this intergenerational MRS depends on the entire path of future consumption, in the stationary state it depends only on the constant path of c^* and thus that of k^* through (12c). This implies that (14) can be solved for the constant path of k^* in a recursive manner independently of the rest of the system. As a result, the stationary-state capital stock still remains unaffected in response to changes in the price of land and/or the land rent tax. Therefore, the infinitely elastic supply curve of capital is neither sufficient nor necessary to establish the *RCP* ("*not sufficiency*" will be proven later). Second, according to CKR, the full capitalization of the *compensated* land rent tax continues to hold for economies which have not yet converged to a stationary state. It is easy to show that this is true of our basic model also. When there is initially no tax, the price of land is determined according to $P_t = \sum_{i=0}^{\infty}(1+r_{t+i})^{-1}m_{t+i}$. Suppose now that when the land rent tax θ is introduced, the paths of k_t and c_t remain unchanged and the after-tax price of land is given by $\tilde{P}_t = \sum_{i=0}^{\infty}(1+r_{t+i})^{-1}(1-\theta)m_{t+i}$. It is immediately seen that the dynamic equations (10) and (11) are unchanged, so does (3) by substitution of the after-tax land price $\tilde{P}_t$ instead of P. Thus these paths are solutions of the dynamics system (3), (10), and (11) in the presence of θ, implying that the *RCP* holds along the transition path as well.

Chamley and Write (1987) have shown that in the overlapping generations model without intergenerational bequests, an *uncompensated* increase in the land rent tax not only lowers the price of land immediately but also induces accumulation of capital. The two reasons of this difference can be identified. The first is that the lifetime saving in their model is *not* infinitely elastic with respect to the interest rate even in the stationary state, due to the absence of an *altruistic* bequest motive. The second is that, instead of the golden-rule condition such as (12a), in their model the long-run capital stock is determined by the asset market equilibrium condition whose demand side contains the price of land. Since the lower price of land caused by the higher land rent tax implies simply a reduction in the demand for assets, *with the upward-sloping* (i.e., *finite elastic*) *supply curve of saving* the interest rate has to be decreased so as to clear this market. This fall in turn stimulates capital accumulation.

3. Modification introduced by impure altruism

As mentioned in the introduction, we assume that the amount of bequests directly enters the parents' utility function as a separate argument, in addition to the heir's utility as well as his own consumption. For analytical convenience, a generation t consumer is assumed to maximize the additively separable utility function:[8]

$$W_t = u(c_t) + v(b_{t+1}) + \beta W_{t+1}, \tag{15}$$

subject to (5) and a given amount of b_t, where $v(b_{t+1})$ represents the utility from bequeathing b_{t+1} to the heir, with $v'>0$, $v''<0$, $\lim_{b\to\infty} v'(b)=0$, and $\lim_{b\to 0} v'(b)=\infty$.

Solving the infinite horizon optimization problem associated with (15) in the same way as in the previous section yields the following first-order conditions for all t:

$$\beta u'(c_{t+1})[1+r_{t+1}(k_{t+1})] + v'(k_{t+1}+P_{t+1}) = u'(c_t), \tag{16}$$

which states that the consumer equates the utility from consuming an extra unit in period t with the sum of the utility from bequeathing an extra unit, which increases his heir's consumption by $\beta[1+r_{t+1}]$ units in period $t+1$, and the utility from enjoying "warm glow".

In the stationary equilibrium, we have (12b), (12c), and

$$\beta[1+r(k^*)] + [v'(k^*+P^*)/u'(c^*)] = 1. \tag{17}$$

As can be seen from (17), the land rent tax can affect the stationary-state capital stock through changes in P. The stationary-state effects of the land rent tax are given by

$$\frac{dk^*}{d\theta} = |D|^{-1} m^* v'' > 0, \tag{18a}$$

$$\frac{dP^*}{d\theta} = |D|^{-1}\{v''+\beta u'r'+u''r^*[\beta(1+r^*)-1]\}(-m^*) \gtreqless 0, \tag{18b}$$

$$\frac{d(1-\theta)m^*}{d\theta} = -m^* + (1-\theta)m'\frac{dk^*}{d\theta}$$

$$= m^*\{-1 + |D|^{-1}(1-\theta)m'v''\} \gtreqless 0, \tag{18c}$$

where all variables are evaluated at their stationary-state values and $|D| \equiv r^*\{v''+\beta u'r'+u''r^*[\beta(1+r^*)-1]\}-v''\{P^*r'-(1-\theta)m'\}$. As shown in appendix B, the *negative* sign of $|D|$ is a sufficient (and necessary) condition for the stationary equilibrium to be a saddlepoint. On the other hand, if $|D|>0$, the stationary equilibrium is completely unstable (called a *source*), while if $|D|=0$, the path is "stable" in the sense that it stays at the initial point; however, the implicit function theorem fails at the stationary equilibrium because it is *non-hyperbolic*. Therefore, it is reasonable to assume that $|D|<0$ in order to perform economically meaningful comparative statics analysis. Putting differently, because all the roots cannot be less than unity in modulus, the system never has the case of price indeterminacy and thus the only structure of a saddlepoint can be compatible with the assumption of perfect foresight. This stands in a sharp contrast to the model of Chamley and Write (1987) in which the indeterminacy of price may well arise.

Furthermore, the impact of the land rent tax on the long-run capital stock is positive if and only if the stationary equilibrium is saddlepoint stable. By contrast, in the model of Chamley and Write the assumption of saddlepoint stability does not suffice to determine the long-run effect on capital. Instead, they have assumed the so-called Hicksian stability condition in that the slope of the demand curve is less than that of the supply curve in the asset market.[9] On the other hand, it is seen from (18b) and (18c) that the effects on the price of land and the after-tax return on land are still ambiguous in our model. In order to sign these effects, we have to strengthen the saddlepoint stable condition by assuming that $v''+\beta u'r'+u''r^{*}[\beta(1+r^{*})-1]<0$; consequently, we have $dP^{*}/d\theta<0$ and $d(1-\theta)m^{*}/d\theta<0$. Our *strengthened* assumption is quite similar to the Hicksian stability condition used by the authors above in the sense that our assumption can be depicted as a situation where the long-run supply curve of capital, $\beta^{-1}-1-\beta^{-1}(v'/u')$, be less steeper than the long-run demand curve of capital, $f'(k)$, as shown in figure 1. Moreover, by comparing (13b) and (18b) one can see that the *negative* effect of the tax on the price of land is smaller in absolute value than the full capitalization of the tax, because a higher capital stock reduces the interest rate at which land rents are capitalized, thus offsetting the full capitalization effect to some extent. Hence, the initial landowners do not bear the full burden of the present value of all of the tax revenue. Part of the tax is shifted from land to capital in the long run.

On the other hand, changes in the land rent tax has two *immediate* impacts on the price of land through changes in the after-tax rent on land and through changes in the future price of land [see (3)]. The former effect is negative at an instantaneous moment (recall that capital and thus the gross rent on land are fixed instantaneously), while the latter effect hinges on the entire path of land prices and hence largely on the long-run effect on the price of land. Since under our strengthened assumption in the long run the land price falls in response to the increase in the land rent tax, so does it immediately [see (C4) in appendix C]. Since this fall is simply a reduction in the size of bequests, the marginal utility of bequests, $v'(k+P)$, is increased, thereby enhancing an incentive of the current generation to bequeath to their heirs. As a result, current consumption also falls immediately [see (C3) in appendix C], thus causing capital accumulation to begin. The resulting higher capital stock in turn induces the *gross* land rent to rise but the interest rate to fall. Accordingly, the price of land is rising through time, but in the new stationary equilibrium it has to be lower than in the old one; if it were back to the original level, under our strengthened assumption the opportunity cost of holding capital, $\beta^{-1}-1-\beta^{-1}(v'/u')$, is higher than the return on capital as a result of the higher capital stock (see figure 1). To restore equilibrium in the long-run capital market [i.e., to satisfy the equality of (17)], the price of land has to be decreased, leading to a reduction in the opportunity cost and thus shifting the long-run supply curve downward as depicted in figure 1.

Proposition 3: *A stationary equilibrium is saddlepoint stable and an increase in the land rent tax raises the long-run capital stock if and only if* $r^*\{v''+\beta u'r'+u''r^*[\beta(1+r^*)-1]\}-v''\{P^*r'-(1-\theta)m'\}<0$. *Furthermore, both the price of land and the after-tax rate of return on land fall in the long run if and only if* $v''+\beta u'r'+u''r^*[\beta(1+r^*)-1]<0$.

However, Feldstein (1977) also pointed out the possibility that the price of land and the after-tax rent on land *may rise* in the long run, *if the elasticity of substitution between capital and land is sufficiently low, and/or if the elasticity of saving with respect to the interest rate is negatively large*. In contrast, under the saddlepoint stability condition, the long-run land price rises in our model if and only if $v''+\beta u'r'+u''r^{*}[\beta(1+r^{*})-1]>0$, or equivalently, using (17)

$$v''+\beta u'r'+(-c^{*}u''/u')(r^{*}/c^{*})v'>0; \tag{19}$$

namely, the long-run supply curve of capital is steeper than the long-run demand curve of capital, as shown in figure 2. Observation of (19) reveals that the higher the elasticity of marginal utility with respect to consumption, $-cu''/u'$, the smaller the *initial* capital stock, and/or the higher the marginal utility of bequests, the more likely (19) is satisfied. A rise in land prices is more likely to occur in, for example, economies with small capital stocks such as developing countries. Note, however, that the larger the magnitude of (19), the more likely will be the system completely unstable, so that (19) has to be bounded above to meet the saddlepoint stability condition.

When the long-run land price rises, the immediate impact on the price of land is qualitatively ambiguous [see (C5) in appendix C]. If this impact is *negative*, the same argument as in proposition 3 can be applied, except that the long-run price of land in the new equilibrium is higher than in the old equilibrium. On the other hand, if the price of land rises immediately, the marginal utility of bequests is decreased, thus discouraging an incentive to bequeath. Nevertheless, consumption falls immediately, thus causing accumulation of capital. The reason can be explained as follows; both the increased land rent tax and the higher land price reduce the after-tax rent on land instantaneously, thus inducing the consumer to substitute land for capital; since the desired level of bequests is smaller, positive capital accumulation can occur only through the fall in consumption. In the long run, the higher price of land is consistent with more accumulation of capital regardless of whether the initial impact on land prices is positive or negative. If the price of land were unchanged, under (19) the opportunity cost of holding capital is lower than the return on capital due to the increased capital stock (see figure 2). To restore equilibrium in the long-run capital market the price of land has to be increased, causing a upward shift in the long-run supply curve of capital as indicated in figure 2.

It should be stressed that the *mere* presence of impure altruism is not sufficient to destroy the *RCP*. Indeed, if the utility function of bequests is linear, that is, $v''=0$, (18) reduces to (13) and thus we again get the *RCP*. However, if it is nonlinear, the marginal utility of bequests varies according to changes in the price of land. Such variations would violate the equity of (17), thereby affecting the stationary-state capital stock.

4. Modification introduced by nonlinear inheritance taxes

Consider the case where an inheritance (bequest or estate) tax τ is levied on the bequest b. Assume that it is nonlinear with respect to the level of bequests, and moreover that its marginal tax rate is non-negative, less than one and is nondecreasing in b (i.e., $0 \le \tau' < 1$ and $\tau'' \ge 0$). Then the flow budget constraint of generation t will be replaced with

$$c_t + b_{t+1} = (1+r_t)\{b_t - \tau(b_t)\} + w_t + z_t^i,$$

where $z_t^i \equiv \theta m_t + (1+r_t)\tau(b_t)$.

As before, we obtain the following first-order conditions:

$$\beta u'(c_{t+1})(1+r_{t+1})\{1-\tau'(b_{t+1})\} = u'(c_t). \tag{20}$$

In the stationary equilibrium, we have (12b), (12c), and

$$\beta(1+r^*)\{1-\tau'(b^*)\} = 1. \tag{21}$$

The stationary-state effects of the land rent tax are given by

$$\frac{dk^*}{d\theta} = |Q|^{-1}\{-(1+r^*)\tau''m^*\}>0, \tag{22a}$$

$$\frac{dP^*}{d\theta} = |Q|^{-1}\{r'(1-\tau')-(1+r^*)\tau''\}(-m^*)<0, \tag{22b}$$

$$\frac{d(1-\theta)m^*}{d\theta} = -m^*\{1 + |Q|^{-1}(1-\theta)m'(1+r^*)\tau''\}<0, \tag{22c}$$

where $|Q|\equiv\{r'(1-\tau')-(1+r^*)\tau''\}r^*+(1+r^*)\tau''\{P^*r'-(1-\theta)m'\}<0$. It should be noted that the stationary equilibrium is a saddlepoint without further restrictions (see appendix D). Therefore, in the long run the capital stock rises; moreover, the price of land *unambiguously* falls by a lesser amount of the full capitalization of the tax; and the after-tax rent on land *unambiguously* falls less by the tax.

The intuition behind these results is straightforward. Suppose that the economy is initially in a stationary equilibrium. By applying the same argument as in appendix C, it can be demonstrated that an increase in the land rent tax causes an immediate fall in the price of land. This fall and hence a reduction in bequests will decrease the marginal tax rate (recall that $\tau''\geq 0$), thereby lowering the price of the heir's consumption relative to that of the parents' own consumption and thus encouraging an incentive to bequeath. As a result, current consumption falls immediately and thus positive capital accumulation takes place. The price of land is rising through time as capital is accumulating. Nevertheless, the price of land in the new stationary equilibrium is lower than in the initial equilibrium, because as a consequence of the higher capital stock the marginal after-tax return on capital, $r(1-\tau')-\tau' r$, is lower than the opportunity cost of holding capital, $\beta^{-1}-1$ (see figure 3); to restore the equality of (21) the price of land has to be lower than the original level. The resulting lower price of land shifts the long-run supply curve of capital upward as shown in figure 3.

However, when the tax rate is linear ($\tau''=0$), its marginal tax rate remains the same in response to changes in P, thereby leaving (21) unaffected; hence, (22) reduces to (13). As in the previous section, this implies that the failure of the *RCP* is due not only to the presence of the inheritance tax rate in the modified golden-rule condition (21), but also to the nonlinearity of the tax rate with respect to the size of bequests.

5. Modification introduced by nonlinear property income taxes

Consider the case where a property income tax τ is levied on property income accruing to the bequest b. Assume that the tax rate is nonlinear with respect to the size of property income, and that $0\le\tau'<1$ and $\tau''\ge 0$. With this tax, the flow budget constraint of generation t becomes

$$c_t + b_{t+1} = (1+r_t)b_t - \tau(r_t b_t) + w_t + z_t^p, \tag{23}$$

where $z_t^p \equiv \theta m_t + \tau(r_t b_t)$. Similarly, we obtain the following first-order condition:

$$\beta u'(c_{t+1})[1+\{1-\tau'(r_{t+1}b_{t+1})\}r_{t+1}] = u'(c_t). \tag{24}$$

In the stationary equilibrium, we have (12b), (12c), and

$$\beta[1+r^*\{1-\tau'(r^* b^*)\}] = 1. \tag{25}$$

The stationary-state effects of the land rent tax are given by

$$\frac{dk^*}{d\theta} = |R|^{-1}\{-(r^*)^2\tau''m^*\} > 0, \tag{26a}$$

$$\frac{dP^*}{d\theta} = |R|^{-1}\{r'(1-\tau')-r^*\tau''(r^*+b^*r')\}(-m^*) \gtreqless 0, \tag{26b}$$

$$\frac{d(1-\theta)m^*}{d\theta} = -m^*\{1 + |R|^{-1}(1-\theta)m'r^*\tau''\} \gtreqless 0, \tag{26c}$$

provided $|R|\equiv\{r'(1-\tau')-r^*\tau''(r^*+b^*r')\}r^*+(r^*)^2\tau''\{P^*r'-(1-\theta)m'\}<0$. Although it appears that the modified golden-rule condition (25) is quite similar to (21), its dynamics is slightly more complicated; that is, the stationary equilibrium is a saddlepoint if $|R|<0$; no linearized path converges to the stationary equilibrium otherwise (the proof can be done in an analogous way as in appendix B). Moreover, under the saddlepoint stability condition, an increase in the property income tax unambiguously raises the long-run capital stock, but has ambiguous effects on the land price as well as the net rate of rent on land in the long run. These effects hinge on the relative slope of the long-run supply and demand curves of capital, as in section 3.

If we strengthen the saddlepoint stability condition by assuming that $r'(1-\tau')-r^*\tau''(r^*+b^*r')<0$ (i.e., the long-run demand curve of capital is steeper than the supply curve as shown in figure 4), $dP^*/d\theta<0$ and $d(1-\theta)m^*/d\theta<0$. These long-run effects can be explained in a similar way as in the case of inheritance taxes. By contrast, if $r'(1-\tau')-r^*\tau''(r^*+b^*r')>0$ (i.e., the long-run demand curve of capital is *less* steeper than the supply curve as shown in figure 5), the land price rises in the long run, and thus the initial impact on the price of land is uncertain; hence, we have to consider the two cases separately, as in section 3.

Finally, three remarks are in order. First, when the property income tax rate is linear (i.e., $\tau''=0$), the *RCP* will emerge once again; hence, whether the *RCP* may or may not hold rests on whether the tax rate on property income is *linear or nonlinear*, as in the *case of* inheritance taxes. Second, since the tax payment depends on the interest rate as well as the size of bequests, so does the marginal tax rate. These arguments move in opposite directions to changes in the level of capital, thus resulting in the ambiguous effect on the marginal tax rate. This ambiguity creates several possibilities of the dynamics as well as comparative statics, unlike the case of inheritance taxes. Third, it is not necessarily true that the presence of *any* type of nonlinear taxes always undermines the *RCP*. To see this, consider, for instance, the case of a labor income tax. In this case, the *RCP* continues to hold regardless of whether the labor income tax is linear or nonlinear. Indeed, one can see that neither the tax function nor the land price appears in the Euler condition (24) or (25).[10] In other words, these variables emerge in the Euler condition only if the tax function depends *nonlinearly* on the size of bequests.

6. Concluding Remarks

We have demonstrated that even if an altruistic bequest motive is perfectly operative, the *RCP* ceases to be valid provided that either impure altruism utility or a nonlinear tax on inheritances or on property income is introduced. These findings indicates that the existence of a nonlinear function that depends directly on the size of bequests is crucial to lead the failure of the *RCP*. Therefore, with such nonlinearities the changes in the price of land resulting from changes in the land rent tax can affect the Euler condition determining the intergenerational terms of trade. Under fairly general conditions together with the assumption of saddlepoint stability, moreover, a higher rate of land rent taxes would promote capital accumulation and thus improve the social welfare of succeeding generations, *regardless of whether the long-run land price may or may not rise.* The reasoning is that since our modified golden-rule stationary equilibrium is *dynamically efficient*, any tax reform toward operations that induce capital accumulation leads to higher social welfare. Consequently, the succeeding generations are always benefited from the introduction of land rent taxes, while the landowners (i.e., the initial generation) gain only when the initial price of land goes up but their welfare falls otherwise (recall that the reduction in their income resulting from the tax increase is compensated by lump-sum transfers).

The most natural extension is to introduce an altruistic *gift* motive in which consumers derive utility from raising the utility of parents as well as from their own consumption, and thus are motivated to give resources to their parents. However, it should be clear that the arguments established for operative bequests will hold symmetrically for operative gifts, although the sole presence of a gift motive does not suffice to invalidate the *RCP*. It is straightforward to show that the failure of the *RCP* occurs only if the following two conditions are satisfied simultaneously: (i) a gift motive toward parents is *impure* in the sense that the heirs enjoy the utility from the act of giving itself: (ii) the utility of heirs is *nonlinear* with respect to the size of gifts. The second extension is to introduce *heterogeneous* consumers. Heterogeneity of consumers may be defined with respect to preferences, discount factors, initial holdings of wealth, or labor income. The unequal distribution of the tax burden among such consumers may be generated by different tax payments proportional to different size of land holdings, unless the resulting negative income effects are canceled out by lump-sum transfers. However, so long as the consumers are assumed to be at an interior solution with respect to either bequests or gifts, such redistribution has no effect on the intergenerational allocation of resources. This is because the redistributed income effects will be neutralized by *offsetting* changes in the size of bequests or of gifts, so that the consumption level of each consumer remains unchanged. However, if there are some consumers who leave neither bequests nor gifts because of, for instance, insufficient labor income or insufficient initial wealth, the *RCP* no longer be valid. The third extension is to introduce human capital investment as alternative form of bequests, in addition to nonhuman bequests (i.e.,physical capital and land). However, even though this extension adds another choice variable to the model, it would not alter our conclusion since the Euler condition still remains the same.[11]

The main message from the present paper is that even if the possibility of corner solutions is not allowed (or even if intergenerational altruism is strong enough), CKR's conclusion (i.e., the *RCP*) would be extreme on theoretical grounds as well.

Appendix A

Using the Inada conditions imposed on the production function and from (12a), it is easy to show that there is a unique stationary-state capital stock, that is, a unique stationary equilibrium.

To ascertain that the stationary equilibrium is saddlepoint stable, we take a linear approximation of (3), (10), and (11) around the stationary equilibrium (c^*, k^*, P^*):

$$\begin{bmatrix} \beta u''(1+r^*) & \beta u' r' & 0 \\ 0 & -\{P^* r-(1-\theta)m'\} & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} c_{t+1}-c^* \\ k_{t+1}-k^* \\ P_{t+1}-P^* \end{bmatrix} =$$

$$\begin{bmatrix} u'' & 0 & 0 \\ 0 & 1 & 1+r^* \\ -1 & 1+r^* & 0 \end{bmatrix} \begin{bmatrix} c_t - c^* \\ k_t - k^* \\ P_t - P^* \end{bmatrix}. \qquad \text{(A1)}$$

We shall calculate the product of the following two matrices:

$$[J_1] \equiv \begin{bmatrix} \beta u''(1+r^*) & \beta u'r' & 0 \\ 0 & -\{P^* r-(1-\theta)m'\} & 1 \\ 0 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} u'' & 0 & 0 \\ 0 & 1 & 1+r^* \\ -1 & 1+r^* & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \{\beta u''(1+r^*)\}^{-1}(u''+\beta u'r') & -u'r'(u'')^{-1} & 0 \\ -1 & 1+r^* & 0 \\ -\{P^* r'-(1-\theta)m'\} & \{P^* r'-(1-\theta)m'\}(1+r^*) & 1+r^* \end{bmatrix}.$$

The characteristic equation of the matrix $[J_1]$ is:

$$F_1(\mu) = \mu^3 - \{(u'')^{-1}(u''+\beta u'r')+2(1+r^*)\}\mu^2 +$$

$$\{2\beta^{-1}+u'r'(u'')^{-1}+(1+r^*)^2\}\mu - \beta^{-1}(1+r^*) = 0, \qquad \text{(A2)}$$

noting that $\beta(1+r^*)^{-1}=1$.

As can be checked by straightforward substitution, a root is $1+r^*$. $F_1(\mu)$ may then be divided by the factor $\mu-(1+r^*)$, and so

$$F_1(\mu) = \{\mu-(1+r^*)\}G(\mu), \qquad \text{(A3)}$$

where $G(\mu)\equiv\mu^2-\{(u'')^{-1}(u''+\beta u'r')+(1+r^*)\}\mu+\beta^{-1}$. The discriminant of $G(\mu)$ is given by

$$D = \{(u'')^{-1}\beta u'r'+r^*\}^2 + 4\beta u'r'(u'')^{-1} >0,$$

which implies that the two roots of $G(\mu)$ are real. Using the relationship between the roots and coefficients of $G(\mu)$, and letting its roots as μ_1 and μ_2, we have

$$\mu_1 + \mu_2 = (u'')^{-1}(u''+\beta u'r') + (1+r^*) > 0,$$

$$\mu_1\mu_2 = \beta^{-1}>0.$$

These facts imply that the two roots are positive. Moreover, because

$$G(1) = (1-\mu_1)(1-\mu_2) = -(u'')^{-1}\beta u'r'<0,$$

one root is greater than unity, while the other is less than unity.

To sum up, all the three roots are real and positive; moreover, the two roots are greater than unity and the remaining root is less than unity. Therefore, the stationary equilibrium is a saddlepoint.

Appendix B

Proceeding in the same way as in appendix A, we have the following linear approximation of (3), (10), and (16) around the stationary equilibrium (c^*, k^*, P^*):

$$\begin{bmatrix} c_{t+1} - c^* \\ k_{t+1} - k^* \\ P_{t+1} - P^* \end{bmatrix} = [J_2] \begin{bmatrix} c_t - c^* \\ k_t - k^* \\ P_t - P^* \end{bmatrix}, \qquad \text{(B1)}$$

where

$$[J_2] \equiv \begin{bmatrix} \{\beta u''(1+r^*)\}^{-1}(u''+\beta u'r'+v'') & -\{\beta u''(1+r^*)\}^{-1}(\beta u'r'+v'')(1+r^*) & -\{\beta u''(1+r^*)\}^{-1}v'' \\ -1 & 1+r^* & 0 \\ -\{P^*r'-(1-\theta)m'\} & \{P^*r'-(1-\theta)m'\}(1+r^*) & 1+r^* \end{bmatrix}.$$

The characteristic equation of the matrix $[J_2]$ is

$$F_2(\mu) = \mu^3 - [\{\beta u''(1+r^*)\}^{-1}(u''+\beta u'r'+v'')+2(1+r^*)]\mu^2+[2\beta^{-1}+ u'r'(u'')^{-1}+$$

$$(1+r^*)^2 + \{\beta u''(1+r^*)\}^{-1}v''\{-P^*r'+(1-\theta)m'+(1+r^*)\}]\mu - \beta^{-1}(1+r^*) = 0. \quad \text{(B2)}$$

Since the determinant of $[J_2]$, which equals $\beta^{-1}(1+r^*)$, is positive, there are either one or three positive roots. Furthermore, since it is easily checked that

$$F_2(0) = -\beta^{-1}(1+r^*) < 0, \quad \text{(B3)}$$

$$F_2(1) = \{\beta u''(1+r^*)\}^{-1}[\{v''+\beta u'r'+u''r^*[\beta(1+r^*)-1]\}r^* - v''\{P^*r'-(1-\theta)m'\}]>0,$$
$$\text{if } \{v''+\beta u'r'+r^*u''[\beta(1+r^*)-1]\}r^* - v''\{P^*r'-(1-\theta)m'\}<0, \quad \text{(B4)}$$

$$F_2(\mu) < 0 \quad \text{for any } \mu<0, \quad \text{(B5)}$$

there is at least one real root in the interval (0,1), provided that $\{v''+\beta u'r'+u''r^*[\beta(1+r^*)-1]\}r^* - v''\{P^*r'-(1-\theta)m'\}<0$. Assuming this condition, we further suppose that the remaining two roots are real and positive [note that when the two roots are real, they have to be positive because of (B5)]. In this case, at least one of the remaining two roots must exceed unity, because the product of all three roots [i.e., $\beta^{-1}(1+r^*)$] is greater than unity. Moreover, the other root cannot be less than unity also, because if this were a case, $F_2(1)$ must be negative, which contradicts (B4). Thus, the remaining two roots have to be greater than unity. If the remaining two roots are complex conjugates, the determinant equals the product of the positive real root and the square of the modulus of the complex conjugate roots. Since the determinant is greater than unity and since that positive real root is less than unity, the square of that modulus is greater than unity, so does the modulus itself. Therefore, in either case the stationary equilibrium is a saddlepoint.

If $\{v''+\beta u'r'+r^*u''[\beta(1+r^*)-1]\}r^* - v''\{P^*r'-(1-\theta)m'\}=0$, then $F_2(1)=0$, which implies that unity is a root. To examine the remaining two roots, we shall consider the characteristic equation of the matrix $[J_2-I]$:

$$\tilde{F}_2(\mu) = \mu^3 - a_2\mu^2 + a_1\mu - a_0 = 0, \tag{B6}$$

where $a_2 \equiv \{\beta u''(1+r^*)\}^{-1}\{u''[1-\beta(1+r^*)]+\beta u'r'+v''\}+2r^*>0$, $a_1 \equiv \{\beta u''(1+r^*)\}^{-1}\cdot[2u''r^*\{1-(1+r)\}-v''\{P^*r'-(1-\theta)m'\}]+(r^*)^2>0$, and $a_0 \equiv \{\beta u''(1+r^*)\}^{-1}[r^*\{u''r^*\cdot(1-\beta(1+r^*))-(\beta u'r'+v'')\}+v''\{P^*r'-(1-\theta)m'\}]$.

By assumption, $a_0=0$ so (B6) reduces to

$$\tilde{F}_2(\mu) = \mu^2 - a_2\mu + a_1 = 0. \tag{B7}$$

Denote the two roots of (B7) as $\tilde{\mu}_2$ and $\tilde{\mu}_3$. From the relation between the roots and coefficients of (B7), we see that $\tilde{\mu}_2+\tilde{\mu}_3=a_2>0$ and $\tilde{\mu}_2\tilde{\mu}_3=a_1>0$. These facts imply that the two roots of (B7) have positive real parts; hence, the two roots of (B7) and thus of (B2) exceed unity in modulus, so that the only stable root is unity. As a consequence, the capital stock stays at any initial point; that is, any initial value of the capital stock and the associated price of land are a stable equilibrium.

On the other hand, if $\{v''+\beta u'r'+r^*u''[\beta(1+r^*)-1]\}r^*-v''\{P^*r'-(1-\theta)m'\}>0$, the stationary equilibrium is completely unstable. To prove this, we shall make use of the following theorem which is an application of the Routh theorem to our special case:

Theorem 1: The number of roots of the polynomial in (B6) *with positive real parts is equal to the number of variations of sign in the scheme*

$$1,\ -a_2,\ a_1+(a_0/a_2),\ -a_0.$$

Since a_0 is positive due to the assumption, there are three changes of sign in the scheme stated in Theorem 1, which indicates that all the three roots of (B6) have positive real parts (hence, at least one root is real and positive). From the construction of (B6) the three roots of (B2) are greater than unity in modulus.

Appendix C

Suppose that the land rent tax is increased in period 0. Since the dynamic system (B1) has one stable and two unstable roots provided that $\{v''+\beta u'r'+u''r^*[\beta(1+r^*)-1]\}r^*-v''\{P^*r'-(1-\theta)m'\}<0$ and since the unstable roots are eliminated by invoking the transversality condition, the solutions for c, k, and P must be of the form:

$$c_t = (c(0) - c^*)\mu_1^t + c^*,$$

$$k_t = (k_0 - k^*)\mu_1^t + k^*,$$

$$P_t = (P(0) - P^*)\mu_1^t + P^*,$$

where k_0 is the capital stock in period 0 and μ_1 is the stable root (i.e., $0<\mu_1<1$). Omitting details, the initial jumps in $P(0)$ and $c(0)$ required to ensure that P and c follow the stable path are given by the expressions:

$$c(0) = c^* + (1+r^*-\mu_1)(k_0 - k^*), \tag{C1}$$

$$P(0) = P^* + \phi(k_0 - k^*), \tag{C2}$$

where $\phi\equiv\{Pr'-(1-\theta)m'\}\mu_1\{\mu_1-(1+r)\}^{-1}>0$. Differentiating (C1) and (C2) with respect to θ, respectively, yields

$$\frac{dc(0)}{d\theta} = \frac{dc^*}{d\theta} + (1+r^*-\mu_1)\left\{-\frac{dk^*}{d\theta}\right\} = (-1+\mu_1)\frac{dk^*}{d\theta} < 0, \tag{C3}$$

$$\frac{dP(0)}{d\theta} = \frac{dP^*}{d\theta} + \phi\left\{-\frac{dk^*}{d\theta}\right\} <0, \quad \text{if } \frac{dP^*}{d\theta} <0, \tag{C4}$$

$$\frac{dP(0)}{d\theta} = \frac{dP^*}{d\theta} + \phi\left\{-\frac{dk^*}{d\theta}\right\} \gtreqless 0, \quad \text{if } \frac{dP^*}{d\theta} >0, \tag{C5}$$

noting that $c^*=f(k^*)$ and $dk^*/d\theta>0$.

Appendix D

Similarly, we have the following linear approximation of (3), (10), and (20) around the stationary equilibrium (c^*, k^*, P^*):

$$\begin{bmatrix} c_{t+1}-c^* \\ k_{t+1}-k^* \\ P_{t+1}-P^* \end{bmatrix} = [J_3] \begin{bmatrix} c_t - c^* \\ k_t - k^* \\ P_t - P^* \end{bmatrix}, \tag{D1}$$

where

$$[J_3] \equiv \begin{bmatrix} 1+(u'')^{-1}\beta u'\{r'(1-\tau')-(1+r^*)\tau''\} & (-u'')^{-1}\beta u'\{r'(1-\tau')-(1+r^*)\tau''\}(1+r^*) & (u'')^{-1}\beta u'(1+r^*)\tau'' \\ -1 & 1+r^* & 0 \\ -\{P^* r'+(1-\theta)m'\} & \{P^* r'-(1-\theta)m'\}(1+r^*) & 1+r^* \end{bmatrix}.$$

The characteristic equation of the matrix $[J_3]$ is

$$F_3(\mu) = \mu^3 - [1+(u'')^{-1}\beta u'\{r'(1-\tau')-(1+r^*)\tau''\}+2(1+r^*)]\mu^2 +$$
$$[2(1+r^*)+(1+r^*)(u'')^{-1}\beta u'[\{r'(1-\tau')-(1+r^*)\tau''\}+\tau''\{P^* r'-(1-\theta)m'\}]+$$
$$(1+r^*)^2]\mu - (1+r^*)^2 = 0. \tag{D2}$$

Similarly, we have

$$F_3(0) = -(1+r^*)^2 < 0, \text{ and}$$

$$F_3(1) = (u'')^{-1}\beta u'[\{r'(1-\tau')-(1+r^*)\tau''\}r^*+(1+r^*)\tau''\{P^* r'-(1-\theta)m'\}]>0,$$

there is at least one real root in the interval (0,1). By applying the same argument as in appendix B, it follows that the remaining two roots are greater than unity in modulus so that the stationary equilibrium is a saddlepoint.

Footnotes

1. According to Fane(1984), a *fully compensated tax* is defined as one that

the government issues perpetual bonds at the same time as it levied the tax, uses the proceeds of the bonds to pay the interest on the bonds, and uses the proceeds of the bonds to make lump-sum transfers to the owners of land at the time the tax was introduced. To keep the analysis paralleling CKR's, I do not follow his *fully compensated tax*.

2. Allowing for *positive* population growth, the long-run equilibrium may not be well defined because the marginal products of capital, labor, and land may be changing through time.

3. Alternatively, if $m'<0$ is assumed, then $w'>0$ due to both the homogeneity of F and $r'<0$, which may lead to different comparative statics results. However, we shall not conduct this exercise here, for most of the literature has adopted the assumption we made.

4. This assumption may not be restrictive. To show this, we borrow the following explanation made by Bernheim (1989). Let $<\hat{c}_t^y, \hat{c}_t^o, \hat{b}_t>_{t=0}^{\infty}$ be a dynastic equilibrium if it maximizes the utility function $\sum_{t=0}^{\infty}\beta^t u(c_t^y, c_t^o)$ subject to $c_t^y+(1+r_{t+1})^{-1}(c_t^o+b_{t+1})=b_t+w_t$ and the nonnegative constraints, $c_t^y, c_t^o, b_{t+1}\geq 0$ for all t, where c_t^y and c_t^o are consumption when young and old of generation t, respectively. In a dynastic equilibrium $<\hat{c}_t^y, \hat{c}_t^o>$ maximizes $u(c_t^y, c_t^o)$ subject to $c_t^y+(1+r_t)^{-1}c_t^o= \hat{c}_t^y+(1+r_t)^{-1}\hat{c}_t^o$ for each t. This can be rewritten as $V(C_t)=\max_{C_t\geq c_t\geq 0} u(c_t, (1+r_t)(C_t-c_t))$ for each t, where $C_t\equiv c_t^y+(1+r_t)^{-1}c_t^o$. Using this function, we set up the following problem:

$$\max_{<C_t, b_{t+1}>_{t=0}^{\infty}} \sum_{t=0}^{\infty}\beta^t V(C_t),$$

subject to $C_t+(1+r_t)^{-1}b_{t+1}=w_t+b_t$ and C_t, $b_t\geq 0$.

The solution of this problem, denoted by $<\hat{C}_t, \hat{b}_t>_{t=0}^{\infty}$, corresponds exactly to the dynastic equilibrium defined above. The inclusion of an additional choice of the life-cycle pattern of consumption for any individual would add clutter without altering any of the central implications of the present paper. See also Weil (1987) in which a dynastic decision with a separable utility function is decomposed into distinct inter-generational and intragenerational decisions in a slightly different way.

5. The altruistic overlapping generations economy continues to have well-defined dynastic equilibria even if $\beta>1$, that is, if the rate of time preference, $\beta^{-1}-1$, is negative. If this is a case, such dynastic equilibria in this economy may not be equivalent to the solutions of the corresponding optimal growth model.

6. More precisely, the absence of such a transfer would lead to a zero capital stock, and hence no output (i.e., no consumption) for this economy. This gives rise to unbounded marginal utility of consumption.

7. It should be noted that the fixed supply of labor is not necessary to yield our result. Allowing an *endogenous* leisure/labor choice, a generation t consumer is to maximize the utility function $W_t=u(c_t, n_t)+\beta W_{t+1}$ subject to $c_t+b_{t+1}=(1+r_t)b_t+w_t n_t+z_t$ and given b_t, where n_t stands for *endogenous* labor supply. Even though the rental rate, the wage rate, and the rent on land all depend on the capital stock as well as labor supply, the first-order conditions for utility maximization (6a) and (6b) are not basically changed, and there will be one additional equation that

says that the marginal rate of substitution between labor and consumption equals the wage rate. The stationary equilibrium conditions are given by

$$\beta[1+r(k^*,n^*)] = 1,$$

$$u_n(c^*,n^*)/u_c(c^*,n^*)=w(k^*,n^*),$$

$$f(k^*,n^*) = c^*,$$

$$r(k^*,n^*) = (1-\theta)m(k^*,n^*)/P^*,$$

where u_j $(j=c,n)$ represents partial derivatives. The first three equations together determine the stationary-state levels of capital, consumption, and labor supply independently of the last equation. Hence, both k^* and n^* are unaffected by changes in either P or θ, so we once again obtain (13). Namely, the *RCP* will prevail regardless of whether labor supply may or may not be a choice variable.

8. Even with the slightly general utility function:

$$W_t=u(c_t,b_{t+1}) + \beta W_{t+1},$$

we can establish similar comparative statics results as in (18) although additional restrictions on the utility function are needed. However, if preferences are recursive with bequests in utility, that is, $W_t=u(c_t,b_{t+1},W_{t+1})$, the analysis becomes extremely complicated.

9. More precisely, Chamley and Write (1987) have assumed the property of "crowding out (*CO*)" that is sufficient and necessary for the positive effect on the stationary-state capital stock of the land rent tax, and have shown that the Hicksian stability condition is sufficient for the property of *CO*. Note however that the property of *CO* is neither sufficient nor necessary for saddlepoint stability in their model.

10. To show this, we shall use the model given in footnote 7, except that the budget equation is replaced by $c_t+b_{t+1}=(1+r_t)b_t+w_tn_t-\tau(w_tn_t)+z_t^l$, where τ is regarded as a *nonlinear* labor income tax ($0\le\tau'<1$ and $\tau''\ge0$) and $z_t^l\equiv\theta m_t+\tau(w_tn_t)$. In the stationary equilibrium, we have

$$u_n(f(k^*),n^*)/u_c(f(k^*),n^*)=w(k^*,n^*)[1-\tau'(w(k^*,n^*)n^*)],$$

and the other conditions are identical to those in footnote 7. It is immediately seen that the stationary-state levels of capital and labor supply are determined by the condition above together with the modified golden-rule condition independently of the non-arbitrage condition between capital and land, so that those real variables are unaffected by changes in P and thus in θ.

11. To show this, consider the same problem confronting a generation t consumer as in section 2, except that the budget equation is replaced by $c_t+b_{t+1}+x_t=(1+r_t)b_t+h(x_{t-1})w_t+z_t$, where x_t is human capital investment and $h(.)$ is the stock of human capital that is a function of x_t. Some manipulation of the resulting first-order necessary conditions yields (11) and

$$\beta h'(x_t)w(k_{t+1})u'(c_{t+1})=u'(c_t).$$

Hence, the stationary equilibrium is characterized by (12a), (12b), (12c), and

$$\beta h'(x^*)w(k^*)=1,$$

the last equation determines the optimal level of human capital investment, given k^*. The level of k^* is determined solely by (12a), so that the stationary-state capital stock is still unaffected by changes in the land rent tax; consequently the *RCP* follows.

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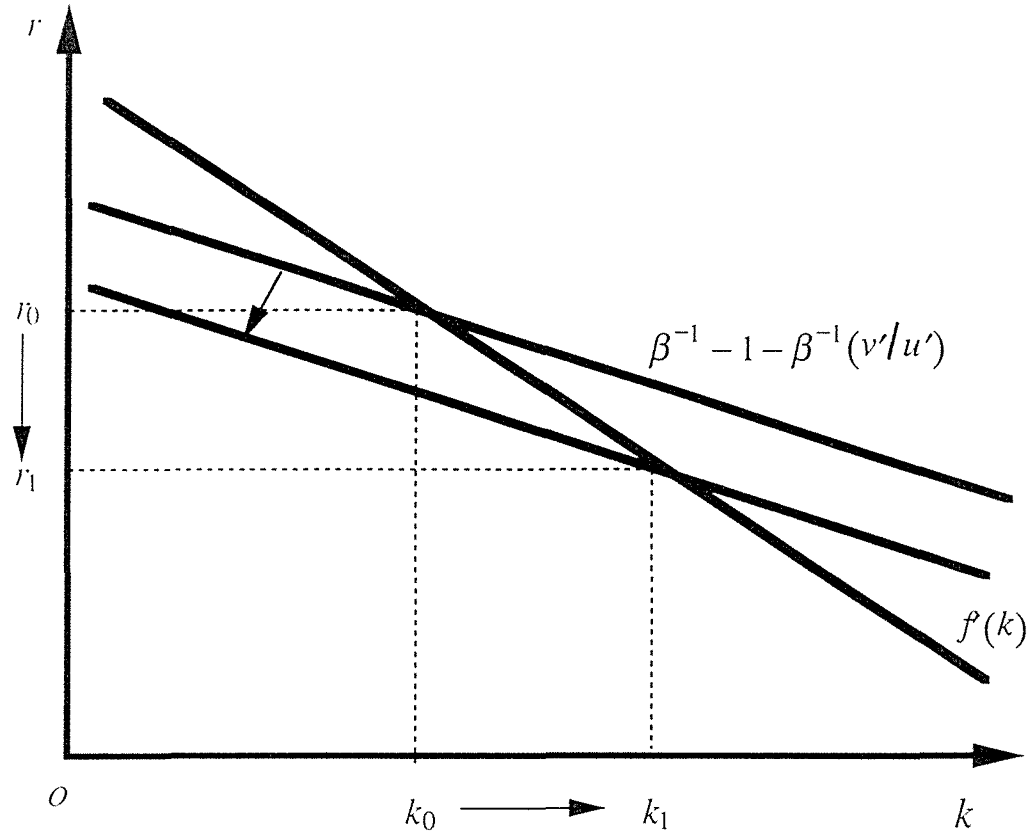

Figure 1. When the long-run supply curve of capital is less steeper than that of the long-run demand curve of capital, an increase in the land rent tax raises the long-run capital stock but lowers the long-run price of land.

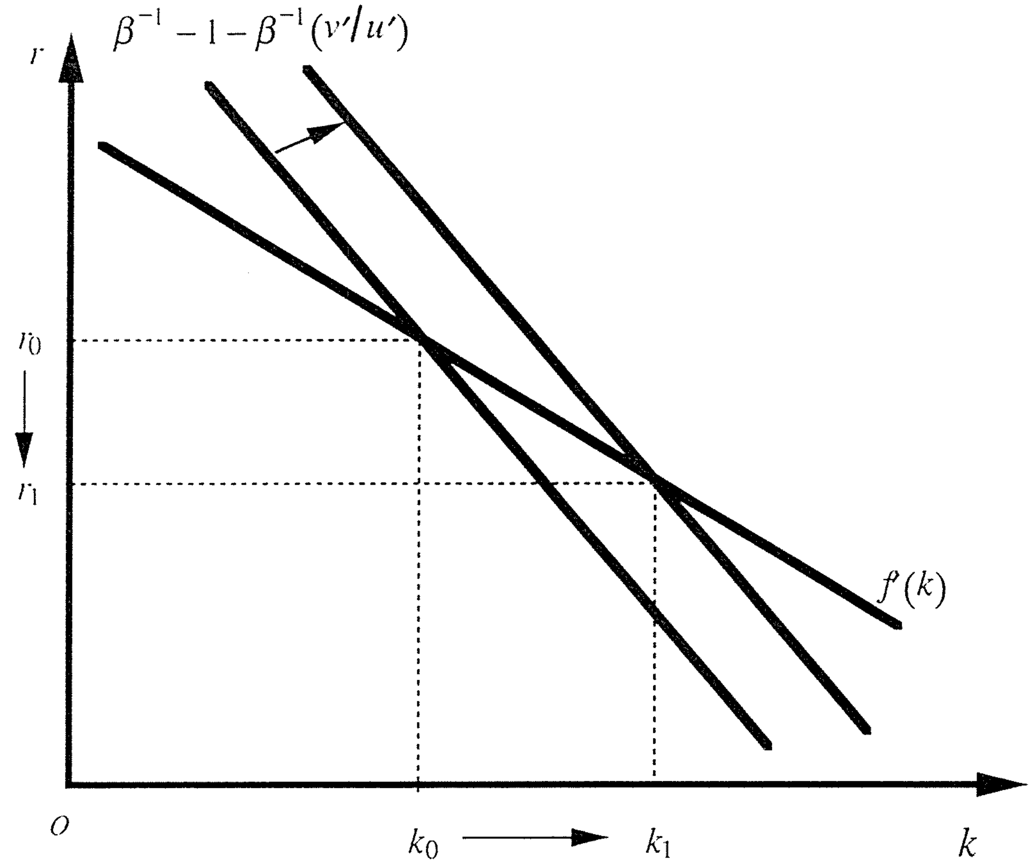

Figure 2. When the long-run supply curve of capital is steeper than that of the long-run demand curve of capital, an increase in the land rent tax raises the long-run capital stock and the long-run price of land.

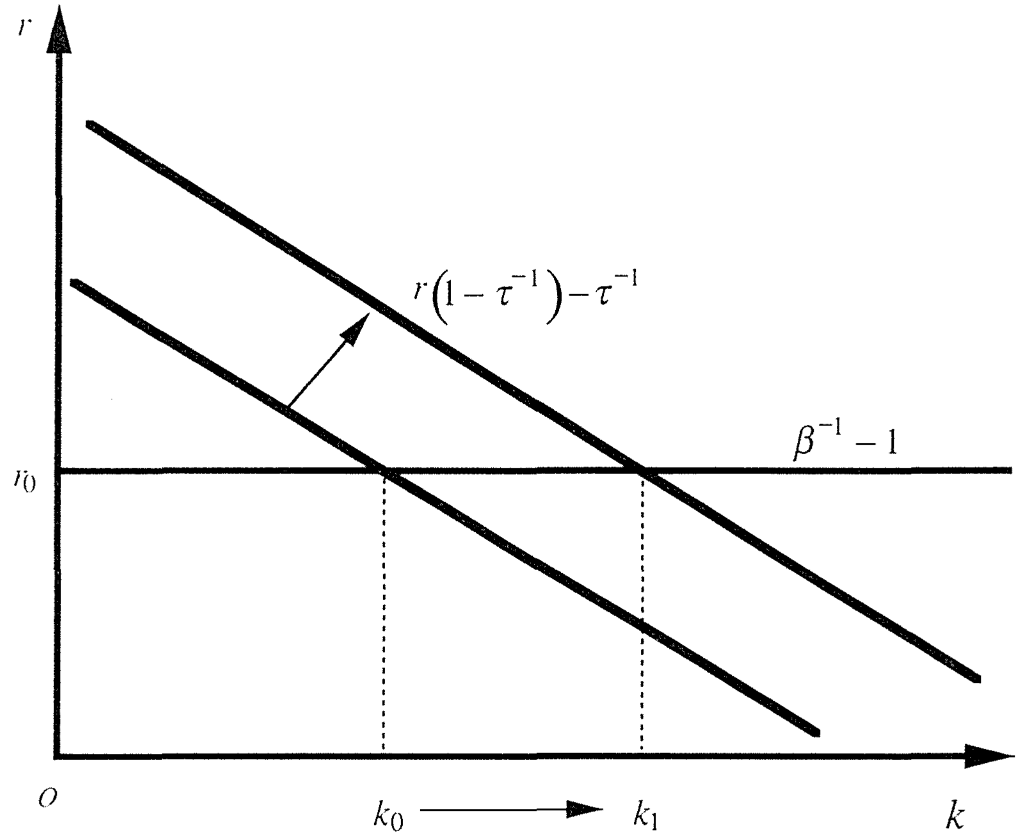

Figure 3. An increase in the land rent tax raises the long-run capital stock but lowers the long-run price of land.

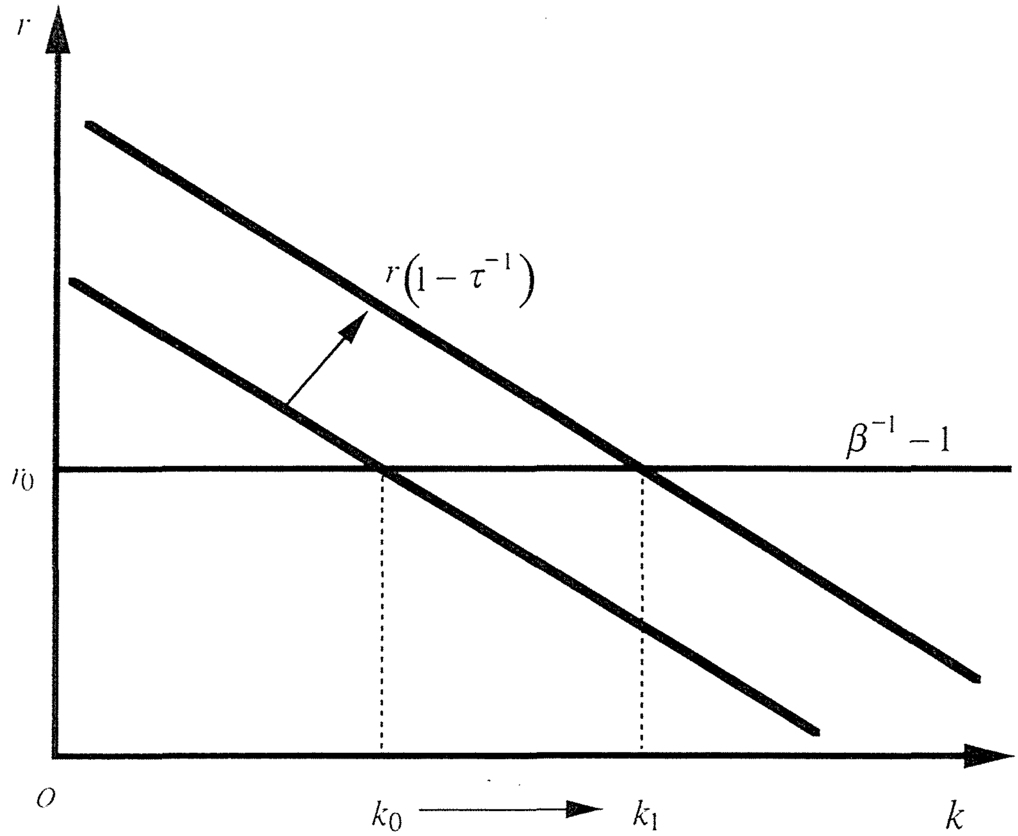

Figure 4. When the slope of the long-run demand curve of capital is smaller than that of the long-run supply curve of capital, an increase in the land rent tax raises the long-run capital stock but loweres the long-run price of land.

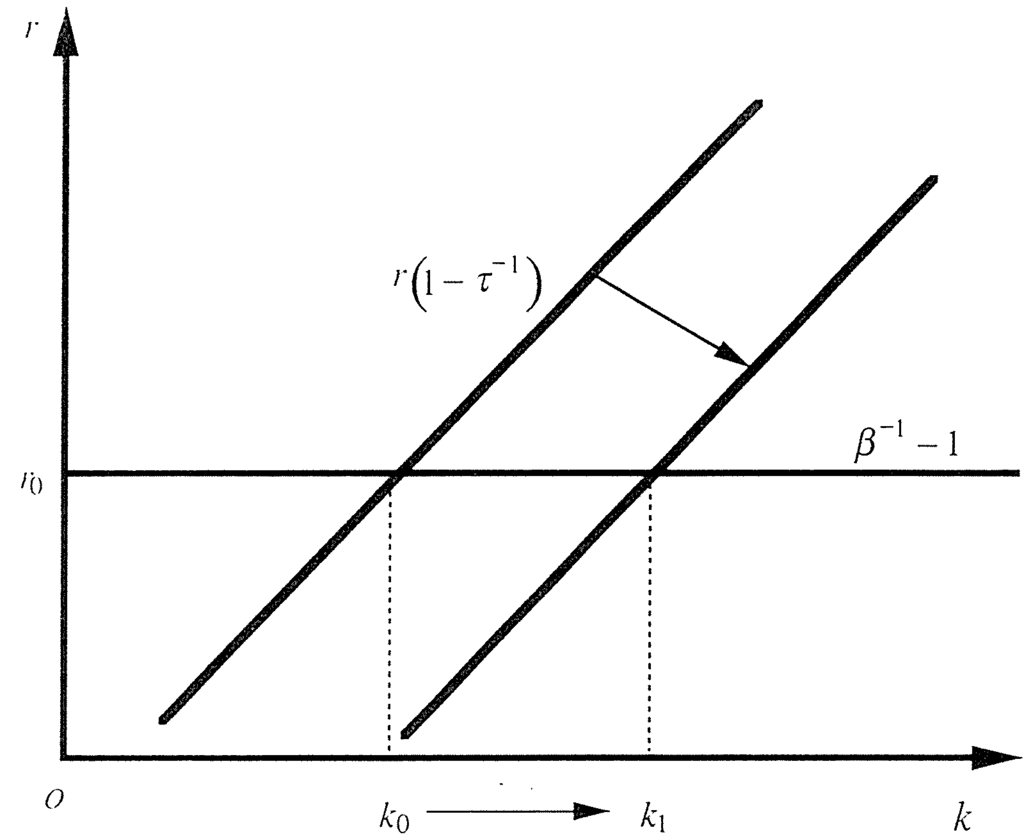

Figure 5. When the slope of the long-run demand curve of capital is greater than that of the long-run curve of capital, an increase in the land rent tax raises the long-run capital stock and the long-run price of land.

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