Emergence of Cooperation in Demographic Prisoner's Dilemma Game
- Tags and Connections - [1]

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Abstract: We extend Demographic Prisoner's Dilemma game introduced by Epstein. He shows emergence of cooperation where Cooperator and Defector are initially randomly distributed in a square lattice of cells. In each period, players move to random cell in von Neumann neighbors if unoccupied and play PD game against neighboring player. If wealth (accumulated payoff) of player becomes negative or his age becomes greater than the lifetime, he dies. If his wealth becomes greater than some threshold and there is unoccupied cell in neighbors, he has an offspring. We introduce global movement, global interaction, and player (we call Referential) who uses tag-based Tit for Tat. We also introduce connections of player which is a list of limited number of recent opponents against whom the player used C and each opponent also used C at the same time. Referential uses C against the opponent in his connections when they meet again. We consider the case where a player can take one unit of wealth from every living member in his connections if his wealth becomes negative.

We show, by using Agent-Based Simulation, the parameter settings where cooperation emerges in some frequency between Referential and Defector, while it is almost impossible between Cooperator and Defector.

Keywords: Prisoner’s Dilemma game, emergence of cooperation, Agent-Based Simulation

1 Introduction

Emergence of cooperation in repeated Prisoner's Dilemma (PD) game is a very important topic in Game Theory. Since it is a very complicated social phenomenon, simulation is a useful tool to gain basic insight and understanding about it.

One stream of this study is the Epstein's model [2]. He shows the emergence of cooperation where Cooperator and Defector are initially randomly distributed in a square lattice of cells. In each period, players move locally (that is, to random cell within the neighboring 4 cells, that is, north, west, south, and east cells; von Neumann neighbors, if unoccupied) and play PD game against local (neighboring) player(s). If wealth (accumulated payoff) of a player becomes negative or his age becomes greater than the lifetime, he dies. If his wealth becomes greater than some threshold and there is unoccupied cell in von Neumann neighbors, he has an offspring and gives the offspring some amount from his wealth. Hales [3] considers the emergence of cooperation in the situation where a

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player (Cooperator or Defector) has a tag and is paired with (globally) randomly selected opponent. If both players have the same tag, they play PD game. If not, other opponents are selected one by one until paired players have the same tag or the number of pairing reaches pre specified number. The frequency of every player in the next period is proportional to his payoff earned by PD game(s) in the current period. Cohen et al. [1] investigate the emergence of cooperation for general situations: The number of players is fixed through periods. They consider two types of player's strategy space, one of which includes Tit for Tat (TFT). Six interaction processes are dealt and one of them uses a tag which takes a real value and is distributed at random in interval [0,1]. A player tries to play PD game against an opponent with the similar tag. After the play of PD games in each period, player changes his strategy by three adaptive processes; one of which basically does (global) imitation of best strategy. In the last two studies, tags are introduced and used to make players play PD game against the opponent with the same (or similar) tag. In our model tags are used differently from them.

We are interested in Epstein's model. While players in his model move locally and interact with local player, we want to examine the situation where a player may move globally and interact randomly with an opponent located in the whole lattice. We introduce a player (we call Referential) who uses tag-based TFT with connections (explained later in the paper), and search the parameter settings where cooperation emerges (in some frequency) between Referential and Defector while it is almost impossible between Cooperator and Defector.

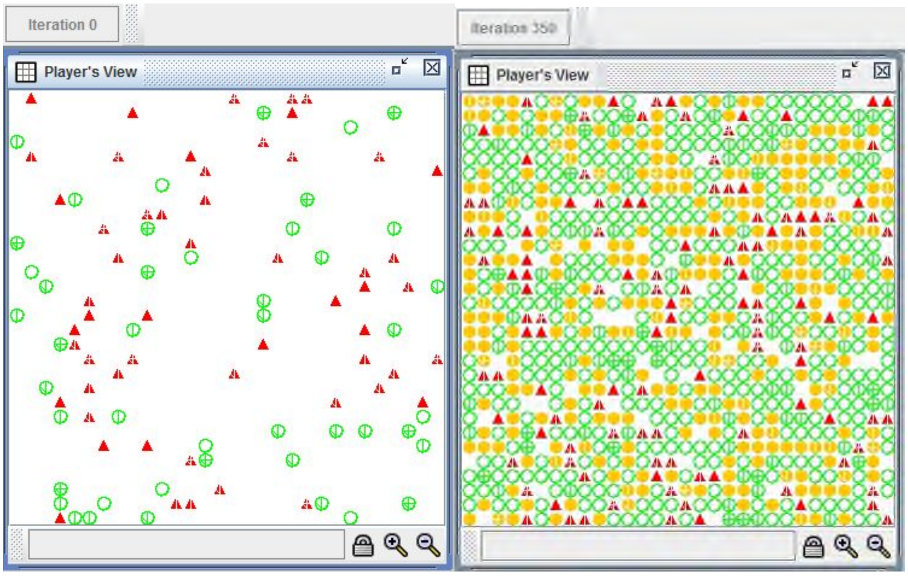
In real life people move and act not only locally but also globally, and distinguish others by some observable feature and act accordingly. He is basically cooperative but retaliates opponent’s defection. Thus he uses TFT based on opponent's tag, that is, uses Cooperate (C) against an opponent for the first encounter of a tag and then uses the same move that the previous opponent with the same tag used against him. We also introduce connections in our model. Connections of a player is a list of limited number of recent opponents against whom the player plays PD game and uses C, and each opponent also uses C at the same time. Our incentive to incorporate connections to a player is as follows. A player acts by himself but thinks that he is not so isolated from others. A player may feel known players, especially who have similar value judgment for important matters, as himself. He tries to keep this feeling during his life. And farther he may think it is essential to his life to maintain this feeling through mutual confidence. We implement this feeling as connections. A player in our model adds the opponent to his connections list if both he and the opponent use C. Since a player thinks his connection as himself, he can take wealth from his connections if necessary, and farther he may give his wealth to his connections when he dies by the expiration of the lifetime.

We have just introduced a player who uses tag-based TFT with connections. Since he includes the things that happen between players, that is, relations in himself, we call him Referential subject, in short *Referential*. Since Defector uses Defect (D), only Referential and Cooperate keep their connections. Referential uses C against an opponent in his connections when they meet again. If the length of connections list is 0, then Referential is exactly the same as tag-based TFT. A player of our model dies if his wealth becomes negative as in Epstein's model. We farther consider the case where a player can take one unit of wealth from every living member in his connections if his wealth becomes negative (and he does not die if the total amount he takes is enough for his negative wealth). Even farther a player can give his some wealth to all living members in his connections almost evenly when he dies because of the expiration of the lifetime.

In Section 2, we explain our model in detail. In Section 3, results of simulation are discussed. And Section 4 concludes the paper.

2 Model

In period 0, N (= 100) players are randomly located in 30-by-30 lattice of cells (see Fig 1 left). The left and right border of the lattice are connected. If a player moves outside, for example, from



Typical example of simulation: The left figure shows the state at period 0 and the right at period 350. Circles or triangles represent players. Their shape shows type (Ref. or Def.), tag (0, 1, 2, or 3), move (C or D) at that period as the right table indicates.

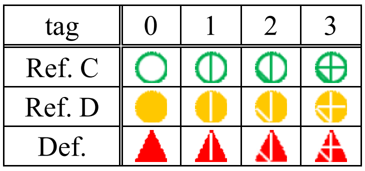


Fig 1.

the right border, then he comes inside from the left border. So are the upper and lower border. A player is one of three types, Referential, Cooperator, or Defector. Defector always uses D in Prisoner's Dilemma game. Cooperator always uses C. Players, specifically Referential, can discriminate other players by their tag. Tag is integer valued and takes 0,..., 3 or 9, that is, the number of tags is numberOfTags (=4 or 10) in our simulation, and we also examine one tag case. Strategy of Referential is a bit complicated. Referential uses tag-based TFT, that is, he uses C for the first encounter of an opponent with a tag and then uses the same move that the previous opponent with the same tag used against him. And Referential uses C if the opponent is in his connections list. We describe in the next paragraph how Referential and Cooperator construct their connections list. Initial wealth of every player is initialWealth (=6). Their initial (integer valued) age is randomly distributed between 0 and deathAge (=50).

In each period, each player (1) moves, and (2) plays Prisoner's Dilemma game given by Table 1 against another player. (The detailed description of (1) movement and (2) interaction is given in Table 2.) If both a player and his opponent use C, then the opponent is added to the player's connections list in case of positive length of connections list. The length of connections list is 0 or 10 in our model. The payoff of the game is added to his wealth. If the resultant wealth is greater than fissionWealth (= 10 or 60) and there is unoccupied cell in von Neumann neighbors, the player has an offspring and give the offspring 6 units from his wealth. If the resultant wealth becomes negative and the player cannot be helped by his connections (this procedure is described at (3) in Table 2), then he dies. If the resultant wealth is nonnegative, his age is increased by one. If his age is greater than deathAge (= 50), he dies (this exact procedure is described at (3) in Table 2). Then next period starts.

Table 1.

	C	D
C	R,R	S,T
D	T,S	P,P

In this paper, we set T=6, R=5, P=–5, S=–6.

In our simulation we use synchronous updating, that is, in each period, all players move, then all players interact, and then all players have an offspring if possible. Among properties of a player, type, tag, rateOfGlobalMoveToLocal, and rateOfGlobalInteractionToLocal are inherited from parent to offspring. But there is a small mutationRate (= 0.05) with which they are not inherited. Initial distribution of these properties is given in Table 3 and this distribution is also used when mutation occurs. Note that if there exists no Cooperator (or no Referential) at period 0, then there exists no Cooperator (or no Referential) thereafter respectively even if mutation occurs.

Table 2. Detailed description

(1)	With probability rateOfGlobalMoveToLocal, player moves to random unoccupied cell in the whole lattice. If there is no such cell, he stays at the current cell. Or with probability 1−rateOfGlobalMoveToLocal, player moves to random cell in von Neumann neighbors if it is unoccupied. If there is no such cell, he stays at the current cell.
(2)	With probability rateOfGlobalInteractionToLocal, the opponent with whom a player interacts is selected at random from all players (except himself) in the whole lattice. Or with probability 1−rateOfGlobalInteractionToLocal, the opponent is selected at random from von Neumann neighbors (no interaction if none in the neighbors).
(3)	Note that only Referential and Cooperator have their connections. In case of positive length of connections list, we use connections in three ways, (TakeWealthFromConnections, GiveWealthWhenDie) = (false, false), (true, false), or (true, true) as follows: In case of TakeWealthFromConnections = true: When wealth of a player becomes negative after the play of PD game, we check whether he can be helped by his connections, that is, whether his resultant wealth becomes nonnegative or not if he takes one unit of wealth from *every* living member in his connections who has positive wealth. If it does, then he does take one unit from *every* living member in his connections who has positive wealth and he does not die in this period. If it does not, then he dies. In case of GiveWealthWhenDie = true in addition to TakeWealthFromConnections = true: When a player dies because of the expiration of the lifetime and has positive wealth, (integer valued) almost equal share of his wealth (up to 20) is given to every living member in his connections.

Table 3. Initial distribution of inheriting properties

property	initial distribution
type	Takes one of {Referential, Defector} with equal probability (no Cooperator). Or takes one of {Cooperator, Defector} with equal probability (no Referential).
tag	Takes one of {0,..., numberOfTags−1} with equal probability.
rateOfGlobalMoveToLocal	Uniformly distributed at interval [lowRateOfGlobalMoveToLocal, hightRateOfGlobalMoveToLocal].
rateOfGlobalInteractionToLocal	Uniformly distributed at interval [lowRateOfGlobalInteractionToLocal, highRateOfGlobalInteractionToLocal].

If there is no Referential, lowRateOfGlobalMoveToLocal = highRateOfGlobalMoveToLocal = 0, lowRateOfGlobalInteractionToLocal = highRateOfGlobalInteractionToLocal = 0, and the length of connections list is 0, then our model is similar to that of Epstein [2]. His model uses asynchronous updating while our model uses synchronous updating.

3 Simulation and Result

Our purpose to simulate our model is to search parameter settings where the cooperation emerges in some frequency between Referential and Defector, while it almost never emerges between Cooperator and Defector. We use Ascape (http://sourceforge.net/projects/ascape/) to simulate our model.

We summarize our results in the following tables. We execute 30 runs of simulations in each parameter setting. If there exist some Cooperators or some Referentials using C at the end of period 500, we evaluate that the cooperation emerges in this run. In tables, the entity of the first row and the second column indicates the number of tags, that of the second row and the second column is the interval where initial value of rateOfGlobalMoveToLocal is distributed, that of the third row and the second column is the interval where initial value of rateOfGlobalInteractionToLocal is distributed, that of the first row and fifth to eighth column means the length of connections list is 0, (takeWealthFromConnections, giveWealthWhenDie) = (false, false), (true, false), and (true, true), respectively and corresponding second and third row shows the frequency with which the cooperation emerges in Cooperator and Defector case and Referential and Defector case, respectively. For example, Table 4 shows that the frequency with which the cooperation emerges is 8/30 when the number of tags is 4, rateOfGlobalMoveToLocal is initially distributed in [0.4, 1.0], rateOfGlobalInteractionToLocal in [0.1, 0.7], and (takeWealthFromConnections, giveWealthWhenDie) = (true, false). We use the notation, for example, Rtf to indicate the frequency with which the cooperation emerges when players consist of only Referential and Defector and (takeWealthFromConnections, giveWealthWhenDie) = (true, false).

Table 4. Setting 1

#tag	4			0	ff	tf	tt
move	[0.4, 1.0]		Coop.	0/30		0/30	0/30
int.	[0.1, 0.7]		Ref.	0/30	5/30	8/30	18/30

Table 5. Setting 2

#tag	10			0	ff	tf	tt
move	[0.4, 1.0]		Coop.	0/30		0/30	0/30
int.	[0.1, 0.7]		Ref.	7/30	12/30	20/30	18/30

Table 6. Setting 3

#tag	4			0	ff	tf	tt
move	[0.4, 1.0]		Coop.	0/30		0/30	0/30
int.	[0.4, 1.0]		Ref.	0/30	0/30	0/30	4/30

fissionWealth is 60 instead of 10.

Table 7. Setting 4

#tag	10			0	ff	tf	tt
move	[0.4, 1.0]		Coop.	0/30		0/30	0/30
int.	[0.4, 1.0]		Ref.	0/30	1/30	1/30	14/30

fissionWealth is 60 instead of 10.

Table 8. Setting 5

#tag	1			0	ff	tf	tt
move	[0.6, 0.9]		Coop.	0/30		0/30	1/30
int.	[0.1, 0.4]		Ref.	6/30	11/30	11/30	14/30

Initial Connections is given by fictitious 20 plays.

We observe that the cooperation almost never emerges between Cooperator and Defector since the frequencies in all tables between Cooperator and Defector but Table 8 are 0/30. Let us compare Table 4 and Table 5 the difference of which is the number of tags. The number of tags increases from 4 to 10. The frequency Rtf increases from 8/30 to 20/30, but Rtt remains the same. Shifting interval of rateOfGlobalInteractionToLocal by 0.3 to the right makes the emergence of cooperation more difficult as Table 6 and Table 7 show. In Table 6 and Table 7, we set fissionWealth 60 instead of 10, which means that a player needs more wealth to have his offspring. They also indicate that increase in the number of tags takes more effect on Rtt than on Rtf. In order to see the effect of connections without tags, we set the number of tags to be 1 in Table 8. Note that the interval of rateOfGlobalMoveToLocal should be shrunken and rateOfGlobalInteractionToLocal should not be so large in Table 8 compared to Table 4 and Table 5 and also that initial connections is given by fictitious 20 plays, that is, at period 0 players are fictitiously paired to play PD game for 20 times but do not pay any payoff in order to form connections. Without tags, that is, players cannot distinguish others, Rff, Rtf, and Rtt are 11/30, 11/30, and 14/30, respectively, because of connections.

We emphasize that thanks to Referential, the cooperation emerges two thirds (20/30) in Table 5 and more than one third (11/30) in Table 8, especially when connections of a player is used in case of taking one unit of wealth from its members for his negative wealth, but not of giving his wealth at his expiration of lifetime. And also that the cooperation emerges about two thirds (18/30) in Table 4 and Table 5, or around half (14/30) in Table 7 and Table 8, especially when connections of a player is used both for his negative wealth and at his expiration of lifetime.

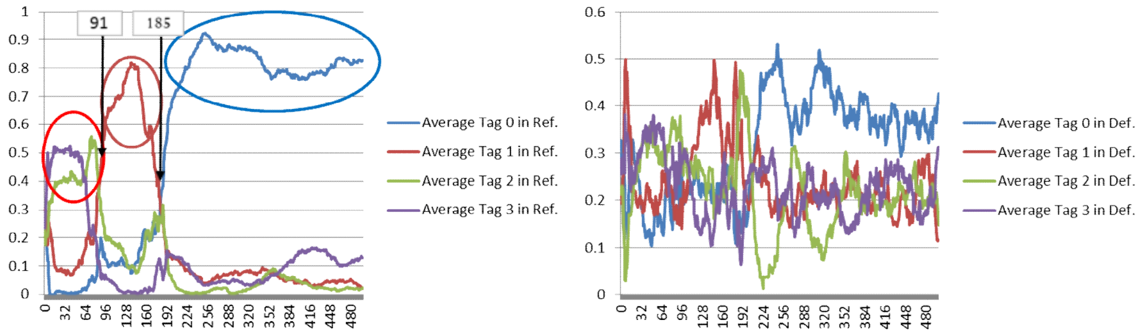


Fig 2. Average rate of each tag of Referential (left) and Defector (right)

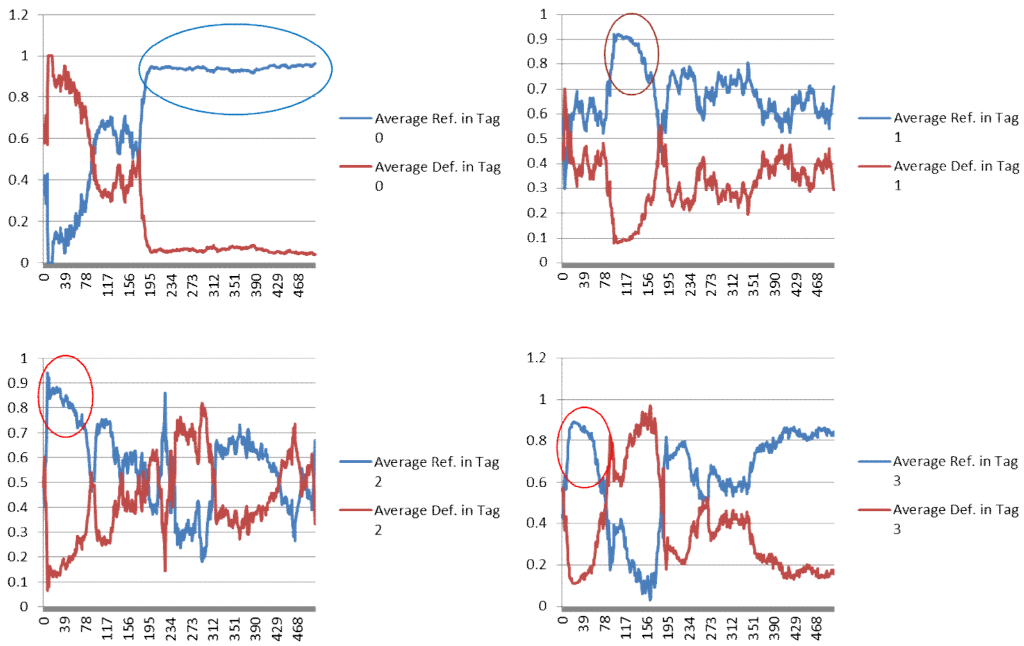


Fig 3. Average rate of Ref. and Def. in each tag

Next we examine the role of tags for the emergence of cooperation in our model. Fig 2 – Fig 4 show one typical run of tt case in Table 4. The left graph in Fig 2 shows that the portion of ○ circle indicates that most of Referential are of tag 2 or tag 3, the portion of ◯ indicates that those are of tag 1, and the portion of ◯ indicates that those are of tag 0. Each corresponding part in Fig 3 shows that the type of most players in each tag is Referential. Thus we conclude that Referential is identified with tag 2 or 3 in portion ◯, with tag 1 in portion ◯, and with tag 0 in portion ◯. Tags have no meaning initially at period 0 and are inheriting properties in our model. We may conclude that Referential is identified with certain tag and it is replaced with another tag over periods by evolution process, and that thus tags play an important role for the emergence of cooperation in our model. Note also (by Fig 4) that the number of players in the lattice is very small around period 91 and 185, where the tag with which Referential is identified is replaced.

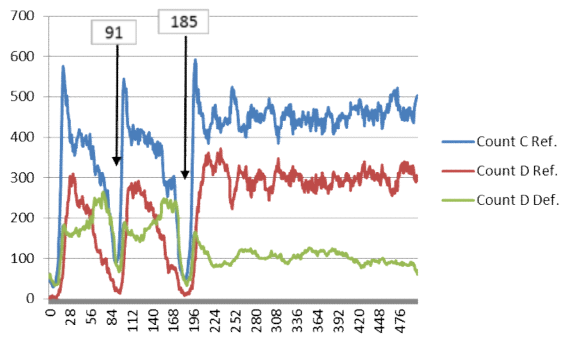


Fig 4. Number of Ref. and Def.

We also collect statistical data about Connections of the run in Fig 2 – Fig 4. Table 9 shows these statistics; Average length of Connections, Average one period amount that player takes from Connections, Average lifetime amount that player takes from Connections, Average lifetime amount that player gives to Connections (excluding the next amount), and Average amount that player gives if he dies because of his lifetime. On average Referential or Cooperator has about 5 members in his Connections, he takes one unit of wealth from about 4 members in his Connections. He takes about 11 units of wealth from Connections and gives about 9 units to Connections through his lifetime, that is, he takes about 2 (=11−9) units from Connections in total. If he is happy enough to live until his lifetime is over, he give about 18 units of wealth to his Connections. The value 18 units is fairly large amount since 18 is close to the maximum amount 20. Thus Connections works well and supports the emergence of cooperation.

Table 9. Some statistics about Connections

statistics	value
Average length of Connections	5.43
Average one period amount that player takes from Connections	3.55
Average lifetime amount that player takes from Connections	11.04
Average lifetime amount that player gives to Connections (excluding the next amount)	9.20
Average amount that player gives if he dies because of his lifetime	17.78

4 Conclusion

We extend Epstein's Demographic Prisoner's Dilemma game [2] by introducing global movement, global interaction, and a player called Referential who uses tag-based TFT. We also introduce connections of a player which is a list of limited number of recent opponents against whom the player uses C and each opponent also uses C at the same time. A player uses C against the opponent in his connections when they meet again. We consider the situation where a player can take one unit of wealth from every living member in his connections who has positive wealth if his wealth becomes negative, and farther the situation where a player can give his some wealth to all living member in his connections at his expiration of lifetime.

We show the parameter settings where the cooperation emerges in some frequency between Referential and Defector, while it is almost impossible between Cooperator and Defector. We also examine the role of tags; tags have no meaning initially at period 0, but evolution process makes a certain tag identify Referential more or less and this tag is replaced with another over periods.

In summary, we show through simulation that Referential, that is, tags and connections are useful for the emergence of cooperation where players may move and interact globally in Demographic Prisoner's Dilemma game.

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