Collusion Deterrence Mechanisms in Hierarchical Regulatory Contracts

Hiroyuki Sano

This paper studies regulatory contracts in a three-tier hierarchical structure of a principal, a monopoly firm which has private information, and a supervisor who is employed by the principal to bridge the informational gap between the principal and the firm. If the supervisor is selfinterested, then collusion between the firm and the supervisor is possible. This paper derives a collusion-deterrence mechanism which attains the same welfare result as the collusion-free contract, even when collusion is possible. The timing of the supervisor's audit of the firm is found to play a crucial role in this mechanism.

1. Introduction

The purpose of this paper is to study optimal regulation in a natural monopolistic industry which has private information. Baron and Myerson (1982), Laffont and Tirole (1986) and subsequent research analyze the optimal regulatory mechanisms in a two-tier hierarchical relationship of one principal and one or multiple firm(s) whose information is asymmetric. The firm often has private information about its cost structure and its effort level to reduce the cost. The problem arising from such asymmetric information is that the firm may have an incentive to falsify so as to shirk the cost-reducing effort even if the principal can ask the firm to tell the true information. Thus, the principal must give up a costly rent to the firm in order that it reveals its true cost structure. To reduce this costly rent, however, the firm's cost-reducing effort must be distorted downwards from the full information first-best level. This trade-off between the rent extraction and the cost-reducing incentive is the basic issue in the regulatory debate.

Because of this trade-off, the optimal allocation is different from the firstbest level and results in lower social welfare. To mitigate this trade-off problem, the principal can employ a supervisor whose role is to bridge the informational gap between the principal and the firm. The supervisor has the time, resources and expertise to acquire information about the firm's cost structure; and to an extent, is successful in acquiring true information. The principal can make the supervisor report the result. Hence, when the supervisor acquires this true information, the principal has full information and extracts the firm's rent without reducing the incentive. However, if the supervisor is self-interested, this is not the case. For example, a self-interested supervisor may shirk exerting effort to acquire the information, thus the probability of acquiring the truth will become lower. Demski and Sappington (1987) analyzes this moral hazard problem.

A more serious problem in the three-tier hierarchy: principal, supervisor and firm is the possibility of collusion between the firm and the supervisor.¹ That is, the firm tries to capture the supervisor by a side-transfer (e.g., bribing) for hiding information. Tirole (1986) and Laffont and Tirole (1991) showed that the mechanism to deter collusion requires the principal to give up the rent to the supervisor, while extracting the rent from the firm. Further, Kofman and Lawarree (1993, 1996) studied the collusion deterrence mechanism including double-checking by employing two supervisors.

In this paper, we will restrict our attention to a collusion-deterrence mechanism in a three-tier hierarchy and will neglect the moral-hazard problem of the supervisor. The basic framework of the model follows Laffont and Tirole (1991). However, the collusion-deterrence mechanism of Laffont-Tirole model includes a trade-off between the supervisor's rent extraction and the firm's incentive since the principal gives up the rent for the supervisor instead of the firm. Therefore, the possibility of collusion yields an additional social cost as the supervisor receives the rent, and therefore social welfare is lower than the collusion-free (or benevolent supervisor) case. In this paper, we consider a collusion deterrence mechanism which, even given the possibility of collusion, extracts rent from the self-interested supervisor without reducing the firm's incentive and thus does not lower social welfare.

In our model, the principal asks not only the supervisor for a report of the result but also the firm for an announcement of its cost structure. Accordingly, the collusion deterrence mechanism in the Laffont-Tirole model can be interpreted as a two-stage sequential announcement game: at stage 1 the supervisor reports and then at stage 2 the firm announces. We will also consider a three-stage announcement game: at stage 1 the firm announces, at stage 2 the supervisor reports and at stage 3 the firm announces again. In this three-stage game, the key idea is to make the firm announce before the supervisor reports. In doing so, we can find a scheme which makes the firm tell the truth before the supervisor reports. Consequently, this mechanism can deter collusion without leaving the rent for the supervisor, as well as the firm.

The paper is organized as follows. In section 2, we present the basic frame-

¹⁾ If the supervisor is benevolent in the sense that it has the same objective as the principal, then the principal and the supervisor can be regarded as one party.

work of the model. In section 3, we study the case, as a benchmark, that the supervisor is benevolent. Section 4 presents the collusion deterrence mechanisms in the case of the non-benevolent supervisor. Finally, section 5 summarizes our main results.

2. The Model

We will consider a three-tier hierarchy with one principal, one supervisor and one agent in a regulatory contract. Let us call these three parties Congress, the agency and the firm, respectively. We suppose that Congress signs a contract with the firm which has private information about its cost structure. It also signs a contract with the agency whose role is to partially correct the informational asymmetry through auditing the firm's cost structure.

2.1. Characterization of the Three Parties

2.1.1. Firm

Suppose that the firm produces output q at cost, $C = (\theta - e)q$ where θ is a productivity parameter and e is the manager's cost-reducing effort. Cost and output are publicly observable and verifiable ex post. The firm can be one of two types: efficient $(\underline{\theta})$ with probability ρ or in efficient $(\overline{\theta})$ with probability $1-\rho$, where $\underline{\theta} < \overline{\theta}$ and $\Delta \theta \equiv \overline{\theta} - \underline{\theta}$. ρ is a common prior belief of the firm's type. The cost-reducing effort, e, is unobservable. Managers incur a disutility, k(e) by making an effort, assuming k'(e) > 0, k''(e) > 0.

Let the gross consumer's surplus be S(q) (assuming S'>0, S''<0) when the output is q. Therefore, the inverse demand function is p=P(q)=S'(q) where p denotes the price. We suppose that the production cost, C, is reimbursed and the revenue, P(q)q, is received by Congress and a (net) monetary transfer, t, is paid from Congress to the firm. Thus, we assume that the firm's utility or rent is

$$U \equiv t - k(e),$$

(1)

normalizing its reservation utility at 0.

2.1.2. Agency

The agency is employed to audit the firm's productivity and report the result to Congress. It receives a monetary transfer, w, as a reward. We assume that the agency's utility or rent is

$$V \equiv w - w^*,$$

(2)

where w^* is a reservation income of the agency. That is, the agency is selfinterested in the sense that its object is to maximize its own monetary income. It may receive a covert transfer from the firm as well as Congress. This possibility will be discussed in section 4.

2.1.3. Congress

Congress's utility is the sum of the consumer's, agency's and firm's surplus. That is, it is a social welfare function:

$$W \equiv S(q) - P(q)q - (1+\tau) \left[(\theta - e)q - P(q)q + t + w \right] + U + V,$$
(3)

where τ denotes a distortionary cost by raising one unit of public funds. Using (1) and (2), (3) is rewritten as follows.

$$W \equiv S(q) + \tau P(q) q - (1+\tau) [(\theta - e) q + k(e) + w^*] - \tau U - \tau V$$

= $\tilde{S}(q) - T(e,q) - \tau U - \tau V,$ (4)

where $\tilde{S}(q) \equiv S(q) + \tau P(q) q$ and $T(e,q) \equiv (1+\tau) [(\theta-e)q + k(e) + w^*]$. Note that both the firm's rent (U) and the agency's (V) are costly for Congress.

2.2. Informational Structure

The firm learns the realization of its productivity, θ . Congress and the agency learn that belongs to $\{\underline{\theta}, \overline{\theta}\}$ and the probability of it being the efficient type is ρ . The agency acquires a signal $\sigma \in \{\underline{\theta}, \overline{\theta}, \phi\}$ through the audit where $\sigma = \underline{\theta}$ or $\overline{\theta}$ means that it learns the true type of the firm, while ϕ means that it learns nothing. Assume that the agency learns the true type (i.e., $\sigma = \theta$) with probability α and nothing (i.e., $\sigma = \phi$) with probability $1-\alpha$, and α is exogeneous. Congress cannot observe the signal, while the firm can. The agency obtains verifiable information if $\sigma = \theta$. Therefore, it has no discretion of reporting it is $\underline{\theta}$ (respectively, $\overline{\theta}$) rather than $\overline{\theta}$ (resp., $\underline{\theta}$), but can report either $r = \theta$ or ϕ to Congress if $\sigma = \theta$. That is, it can choose whether it reports the truth or not because the signal is not observed by Congress. However, if $\sigma = \phi$, then it just reports ϕ . Thus, when it learns the true type, the agency has discretion as to whether to report the truth or not.

2.3. Incentive Mechanism

Congress can also ask the firm to announce its productivity as well as the agency to report. Let the announcement of the firm be denoted by $a \in \{\underline{\theta}, \overline{\theta}\}$. A strategy set for the firm is denoted by A, while for the agency is Ψ . Let an allocation vector be $X \equiv (t, C, q, w)$.

Definition: A mechanism Γ is a collection of the strategy sets for the firm and agency, $\{A, \Psi\}$, and an outcome function. $g: A \times \Psi \to X$. That is, $\Gamma = \{A, \Psi, g(\cdot)\}$.

Congress designs the incentive mechanisms depending on a pair of strategies of the firm and the agency so as to maximize social welfare. That is, it designs an allocation (t(a, r), C(a, r), q(a, r), w(a, r)) if the firm's and the agency's strategies are $a \in \{\underline{\theta}, \overline{\theta}\} \equiv A$ and $r \in \{\theta, \phi\} \equiv \Psi$ respectively.

2.4. The Timing

Summarizing, the timing is as follows. (i) The firm learns its own type. Congress and the agency learn the probability of it being the efficient type. (ii) Congress designs an incentive mechanism and signs contracts with them. (iii) Congress sends the agency to the firm to audit its cost structure. The agency receives a signal through this audit. (iv) The firm announces and the agency reports. (v) After cost and output are realized, the transfer and the reward are undertaken in accordance with the contract.

3. Benevolent Agency Benchmark

In this section, we consider the case, as a benchmark, that the agency is benevolent or not self-interested. Since its object is not to maximize its own monetary income, its reward can be set at its reservation income level, w^* , so that $V=w-w^*=0$. Further, it is supposed that the benevolent agency always reports truthfully. In other words, it never behaves strategically.

3.1. The Informed Agency Case

We will derive an optimal allocation and a social welfare when the agency learns the true productivity. Suppose that $\sigma = \theta$. The benevolent agency reports $\underline{\theta}$ if $\sigma = \underline{\theta}$, or $\overline{\theta}$ if $\sigma = \overline{\theta}$. This implies that Congress also has full information due to the agency's report. In order to make the firm tell the truth, Congress can set the transfer at $-\infty$ when the firm announces $a \neq r$ which exposes the firm's lie. Thereby, Congress can force the firm into the truth-telling. Thus, when either $r = \underline{\theta}$ or $\overline{\theta}$, the firm's announcement becomes trivial.

For any given $\theta \in \{\underline{\theta}, \overline{\theta}\}$, Congress will choose *e*, *q* and *U* to maximize the social welfare *W* given by (4), where V=0, subject to the Individual Rationality (IR) constraints: $U \ge 0.^2$ Since *U* is costly, the IR constraint is binding at the optimum: U=0. Hence, the Congress's problem is, for each $\theta \in \{\underline{\theta}, \overline{\theta}\}$,

²⁾ Throughout this paper, we assume that an interior solution exists.

Hiroyuki SANO

$$\max_{e,q} W \equiv S(q) + \tau P(q) q - (1+\tau) [(\theta - e) q + k(e) + w^*].$$

Thus, the first order conditions are, for each $\theta \in \{\underline{\theta}, \overline{\theta}\}$,

$$\frac{\partial W}{\partial e} = (1+\tau) (q-k'(e)) = 0,$$

$$\frac{\partial W}{\partial q} = p + \tau p + \tau q (dp/dq) - (1+\tau) (\theta-e) = 0$$

which yields

and

$$k'(e) = q \tag{5a}$$

(5)

$$\frac{p - (\theta - e)}{p} = \frac{\tau}{1 + \tau} \frac{1}{\varepsilon(p)}$$
(5b)

where $\varepsilon(p)$ denotes the elasticity of demand: $\varepsilon(p) = (dq/dp)(p/q)$, and the effort level and the output which satisfy equations in (5a) and (5b) are denoted by e^* and q^* , respectively. Since U=0, the optimal transfer is $t^* = k(e^*)$. From (5b), we can see that price, p, is given by a Ramsey formula, and the Ramsey output, q(e), is defined as the output level which satisfies (5b) for any e.

The allocation which satisfies the conditions given by (5a), (5b) and U=0 is first-best. Let this first-best allocation be denoted by $(\underline{t}^*, \underline{C}^*, \underline{q}^*)$ for the efficient type and $(\overline{t}^*, \overline{C}^*, \overline{q}^*)$ for the inefficient type. Thus, the expected social welfare when $\sigma=\theta$, W^* , is as follows.

$$W^* = \rho(\tilde{S}(q^*) - T(e^*, q^*)) + (1 - \rho)(\tilde{S}(\bar{q}^*) - T(\bar{e}^*, \bar{q}^*)).$$
(6)

3.2. The Uninformed Agency Case

Let us derive an optimal allocation and social welfare when the agency learns nothing. Suppose that $\sigma = \phi$ and therefore the agency reports ϕ . Congress cannot ascertain whether the firm tells a lie or not since it learns nothing from the agency's report. Hence, Congress has asymmetric information and designs an allocation depending on the announcement of the firm. We will denote the allocation when $a = \underline{\theta}$ (respectively, $a = \overline{\theta}$) by $(\underline{t}, \underline{C}, \underline{q})$ (resp., $(\overline{t}, \overline{C}, \overline{q})$). Let \tilde{e} denote an effort level where one type of the firm realizes the other type's cost and output in order to conceal its own productivity. If it tells a lie, then the efficient type requires that $\overline{C}/\overline{q} = \overline{\theta} - \overline{e} = \underline{\theta} - \tilde{e}$ where \overline{e} denotes the effort level induced when $a = \overline{\theta}$. Hence, a "deviated effort" is $\tilde{e} = \overline{e} - \Delta \theta$. Similarly, if it tells a lie, the inefficient type must choose $\tilde{e} = \underline{e} + \Delta \theta$. That is, the firm can falsify its type covertly by choosing \tilde{e} .

In order to make the firm reveal its true type, Congress must satisfy the following Incentive Compatibility (IC) constraints.

$$\underline{U} \equiv \underline{t} - k(\underline{e}) \ge \overline{t} - k(\overline{e} - \Delta\theta) \text{ and } \overline{U} \equiv \overline{t} - k(\overline{e}) \ge \underline{t} - k(\underline{e} + \Delta\theta)$$
(7)

for each type. Since $\overline{t} = \overline{U} - k(\overline{e})$ and $\underline{t} = U - k(\underline{e})$, we can transform the IC constraints into

$$U \ge \overline{U} + R(\overline{e}) \text{ and } \overline{U} \ge U - R(e + \Delta\theta)$$
 (8)

where $R(e) \equiv k(e) - k(e - \Delta \theta) > 0$.

Both equations in (8) imply $R(\overline{e}) - R(e + \Delta \theta) \leq 0$. Using this fact, it can be shown that at the optimum, the IC constraint for the inefficient type (the latter equation in (7) or (8)) is satisfied whenever the IR constraint for the inefficient type $(\overline{U} \ge 0)$ is satisfied. (We will check this later.) Thus, we will ignore the IC constraint for the inefficient type. Ignoring the IR constraint for the efficient type $(\underline{U} \ge 0)$ since $\overline{U} + R(\overline{e}) > 0$, Congress chooses $\underline{e}, \overline{e}, \underline{q}, \overline{q}, \underline{U}$ and \overline{U} to maximize the following expected social welfare

$$EW \equiv \rho\{S(\underline{q}) + \tau P(\underline{q}) \underline{q} - (1+\tau) [(\underline{\theta} - \underline{e}) \underline{q} + k(\underline{e}) + w^*] - \tau \underline{U}\} + (1-\rho)\{S(\overline{q}) + \tau P(\overline{q}) \overline{q} - (1+\tau) [(\overline{\theta} - \overline{e}) \overline{q} + k(\overline{e}) + w^*] - \tau \overline{U}\}$$
(9)

subject to $\overline{U} \ge 0$ and $U \ge \overline{U} + R(\overline{e})$.

Since \underline{U} and \overline{U} are costly, both constraints are binding at the optimum. Therefore, each type's rent is $\overline{U}=0$ and $\underline{U}=R(\overline{e})$ respectively. That is, Congress gives up a positive rent to the efficient type. This implies that a trade-off problem exists as the efficient type's rent cannot be extracted without reducing the inefficient type's incentive. Substituting $\overline{U}=0$ and $\underline{U}=R(\overline{e})$ into EW, the first order conditions are given by (5a) for \underline{e} and (5b) for q and \overline{q} . For \overline{e} ,

 $\partial EW/\partial \bar{e} = -\rho \tau R'(\bar{e}) + (1-\rho)(1+\tau)(\bar{q}-k'(\bar{e})) = 0$

which yields

$$k'(e) = \overline{q} - \frac{\rho \tau R'(\overline{e})}{(1-\rho)(1+\tau)}.$$
(9a)

Note that the output is the Ramsey level: q(e) or $\overline{q}(e)$. Let the optimal allocation which satisfies the above conditions be denoted by $(\underline{t}_0, \underline{C}_0, \underline{q}_0)$ for the efficient type and $(\bar{t}_0, \bar{C}_0, \bar{q}_0)$ for the inefficient type. Thus, we can show that the

inefficient type's effort, e_0 , is distorted downwards: $\overline{e}_0 < \overline{e}^*$.³ Whereas, for the efficient type, $\underline{C}_0 = \underline{C}^*$, $\underline{q}_0 = \underline{q}^*$ and $\underline{e}_0 = \underline{e}^*$, and the optimal transfer is $\underline{t}_0 = R(\overline{e}_0) + k(\underline{e}_0)$. To show that the IC constraint of the inefficient type is not binding it suffices to note that $R(\overline{e}) - R(\underline{e} + \Delta\theta) \leq 0$. That is, the inefficient type has no incentive to imitate the efficient type since it obtains a negative rent :

$$\underline{t}_0 - k(\underline{e}_0 + \Delta\theta) = R(\overline{e}_0) - R(e_0 + \Delta\theta) < 0.$$

Thus, the expected social welfare when $\sigma = \phi$, W° , is as follows.

$$W^{\mathfrak{o}} = \rho\left(\widetilde{S}\left(\underline{q}^{*}\right) - T\left(\underline{e}^{*},\underline{q}^{*}\right)\right) + (1-\rho)\left(\widetilde{S}\left(\overline{q}_{\mathfrak{o}}\right) - T\left(\overline{e}_{\mathfrak{o}},\overline{q}_{\mathfrak{o}}\right)\right) - \rho\tau R\left(\overline{e}_{\mathfrak{o}}\right). \tag{10}$$

Thus, when the agency is benevolent, the expected social welfare before auditing (i.e., before the agency receives the signal) is

$$EW^{B} \equiv \alpha W^{*} + (1 - \alpha) W^{0} \tag{11}$$

where W^* and W^0 are given by (6) and (10) respectively. (11) will be used as a benchmark when we investigate an expected social welfare in the non-benevolent agency case.

4. Non-Benevolent Agency

In this section, we consider an incentive mechanism when collusion between the firm and agency is possible. Suppose that the agent is self-interested or nonbenevolent. The non-benevolent agency can falsify the signal received through the audit since the signal is not observed by Congress. For example, the agent can report $r=\phi$ even though $\sigma=\underline{\theta}$. In section 3, we showed that the efficient type gets a positive rent when $r=\phi$ while its rent is zero when $r=\underline{\theta}$. This implies that the efficient type would prefer $r=\phi$ to $\underline{\theta}$ when $\sigma=\underline{\theta}$. Therefore, the efficient type would try to have the agency report untruthfully by a side-transfer to obtain an informational rent.

4.1. Side-Contract

Let us specify a side-contract between the firm and the agency, and the resulting rents of both parties. Assume that the side-transfer is a fixed fraction of the firm's rent: $\tilde{w} = bU$ ($b \le 1$ is a positive constant) and the firm incurs the cost of v by raising the side-transfer by one unit. Therefore, the firm's utility or rent incorporating the side-transfer is $\tilde{U} \equiv U - (1+v) \tilde{w} = [1 - (1+v) b] U$. Since

³⁾ This statement is verified by a revealed preference argument. See Laffont and Tirole (1991).

the firm will never carry out this side-transfer if it leads to a negative rent, \tilde{U} must be positive. Hence, $b \le 1/(1+v)$ if U > 0. Thus, the agency's rent incorporating the side-transfer is

$$\widetilde{V} \equiv w + \widetilde{w} - w^* = w + bU - w^* \le w + U/(1+v) - w^* \equiv \widetilde{V}_{\max}.$$

(12)

 \widetilde{V}_{\max} represents the agency's maximal rent with the side-transfer.

4.2. The Two-Stage Announcement Game

Consider a mechanism which can deter collusion as mentioned above. It will be shown that this mechanism yields a lower social welfare than the benevolent agency case.

We suppose that Congress sets up the following two-stage announcement game.

- Stage 1: Congress asks the agency for the report. The agency chooses either $r = \theta$ or ϕ if $\sigma = \theta$, while $r = \phi$ if $\sigma = \phi$. If it chooses $r = \theta$, then the game ends. If it chooses $r = \phi$, then the game proceeds to stage 2.
- Stage 2: Congress asks the firm for the announcement. The firm chooses either $a = \theta$ or $\overline{\theta}$.



Fig. 1. The Two-stage Announcement Game When the Firm is Efficient

The extensive-form game in Figure 1 represents the two-stage announcement game when the firm is efficient. First, "nature" chooses $\sigma = \underline{\theta}$ with probability α

and $\sigma = \phi$ with probability $1-\alpha$. At stage 1, the agency chooses either $r = \underline{\theta}$ or ϕ if $\sigma = \underline{\theta}$, while $r = \phi$ if $\sigma = \phi$. If $r = \phi$, the game proceeds to stage 2 and the efficient type chooses either $a = \underline{\theta}$ or $a = \overline{\theta}$ at stage 2.

4.2.1. Implementable Allocation

We will characterize an implementable allocation which elicits truth-telling at each stage. To do so, we show that the incentive compatibility constraints hold at each stage.

At stage 2, Congress has asymmetric information because at stage 1, it receives $r = \phi$. Hence, at stage 2, the firm tells the truth if the allocation satisfies the IC constraints in (8): $\underline{U} \ge \overline{U} + R(\overline{e})$ and $\overline{U} \ge \underline{U} - R(\underline{e} + \Delta\theta)$ which imply $R(\overline{e}) - R(\underline{e} + \Delta\theta) \le 0$. Therefore, Congress gives up a positive rent to the efficient type since $\underline{U} = R(\overline{e}) > 0$, while it extracts a rent from the inefficient type since $\overline{U} = 0 \ge R(\overline{e}) - R(\underline{e} + \Delta\theta)$. This implies that the efficient type has incentive to capture the informed agency by the side-transfer since it obtains a positive rent owing to $r = \phi$.

Assume that the agency's reward depends on the report: w(r). Let us define $\underline{w} \equiv w(\underline{\theta})$, $\overline{w} \equiv w(\overline{\theta})$ and $w_0 \equiv w(\phi)$. Since the truth-telling of the agency is not an issue (i.e., there is no side-transfer) when $\sigma = \overline{\theta}$ or $\sigma = \phi$, Congress sets $\overline{w} = w^*$ and $w_0 = w^*$. When $\sigma = \underline{\theta}$, the agency obtains the rent of $\widetilde{V}_{\max} = \underline{U}/(1+v)$ at most by reporting ϕ rather than $\underline{\theta}$ at stage 1. Whereas, the agency obtains $\underline{V} \equiv \underline{w} - w^*$ when it chooses $r = \underline{\theta}$ at stage 1. Hence, the agency tells the truth if $\underline{V} \ge \widetilde{V}_{\max}$ or

$$\underline{V} \ge \underline{U}/(1+v). \tag{13}$$

(13) is called the Coalition Incentive Compatibility (CIC) constraint. That is, in order for the agency to tell the truth, Congress has only to give a reward more than or equal to the agency's maximum rent with the side-transfer.

4.2.2. Welfare Comparison with the Benevolent Agency Case

We shall now examine the social welfare level under an optimal allocation, and compare it with the benevolent agency case. To do so, we will first discuss the difference between the benevolent and non-benevolent agency case.

When $\sigma = \phi$, Congress has asymmetric information and extracts a rent from the inefficient type, that is, $\overline{U}=0$. At the same time, it gives up the positive rent of $\underline{U}=R(\overline{e})$ to the efficient type since the IC constraint is binding at the optimum. Whereas, when $\sigma = \underline{\theta}$, Congress elicits the truth-telling from the agency and has full information if the CIC constraint of (13) is satisfied. Since the rent is costly, the CIC constraint is binding at the optimum. Hence, using $\underline{U} = R(\overline{e})$ which is the efficient type's rent when $\sigma = \phi$,

$$\underline{V} \equiv \underline{w} - w^* = R(\overline{e})/(1+v). \tag{14}$$

Thus, the only difference with the benevolent agency case is that \underline{V} is positive. The point is that the agency's rent, \underline{V} , depends on an effort level of the inefficient type under asymmetric information, \overline{e} .⁴ Therefore, except for allocations for the inefficient type when $\sigma = \phi$ and for the agency when $\sigma = \underline{\theta}$, allocations are the same as the benevolent agency case.

Let us derive the social welfare level at the optimum. When $\sigma = \phi$, the inefficient type's output is the Ramsey level, $\bar{q}(\bar{e})$, for any effort level \bar{e} . Since $\bar{q}_0 = \bar{q}(\bar{e}_0)$ is the Ramsey optimal output, substituting $\bar{q}(\bar{e})$ into \bar{q}_0 in (10), we obtain, for any \bar{e} ,

$$W^{0}(\overline{e}) = \rho(\widetilde{S}(\underline{q}^{*}) - T(\underline{e}^{*}, \underline{q}^{*})) + (1 - \rho)(\widetilde{S}(\overline{q}(\overline{e})) - T(\overline{e}, \overline{q}(\overline{e}))) - \rho\tau R(\overline{e}).$$
(15)

This represents the expected social welfare when $\sigma = \phi$ conditional on \overline{e} . We assume that $W^{0}(\overline{e})$ is strictly concave.

When $\sigma = \theta$, the allocation is the same as the first-best except for $w = R(\bar{e})/(1+v) - w^*$. Hence, the expected social welfare is $W^* - \rho \tau R(\bar{e})/(1+v)$. Thus, the maximum social welfare with the non-benevolent agency is

$$EW^{NB} \equiv \max \{ \alpha [W^* - \rho \tau R(\overline{e}) / (1+\nu)] + (1-\alpha) W^{\circ}(\overline{e}) \}.$$
 (16)

The first order condition is

$$(1-\alpha) dW^{0}(\overline{e})/d\overline{e} - \alpha \rho \tau R'(\overline{e})/(1+v) = 0$$

which yields $dW^{0}(\bar{e})/d\bar{e} = \alpha \rho \tau R'(\bar{e})/(1-\alpha)(1+\nu) > 0$. Since $W^{0}(\bar{e})$ is strictly concave, $\bar{e} < \bar{e}_{0}$ and $W^{0}(\bar{e}) < W^{0}(\bar{e}_{0}) = W^{0}$. Hence, the possibility of collusion yields the additional social cost, using (1),

$$EW^{B}-EW^{NB}=(1-\alpha)(W^{0}-W^{0}(\overline{e}))+\alpha\rho\tau R(\overline{e})/(1+\nu)>0.$$

This result is the same as the collusion deterrence mechanism in Laffont and Tirole (1991). The reason why the use of the non-benevolent agency is socially costly compared with the benevolent agency is that costs are incurred both direct-

⁴⁾ This implies that another trade-off problem exists as the agency's rent cannot be extracted without reducing the inefficient type's cost-reducing incentive.

ly $(\alpha \rho \tau R(\overline{e})/(1+v))$ and indirectly $((1-\alpha)(W^{\circ}-W^{\circ}(\overline{e})))$ through giving up the rent to the agency. Hence, there is no additional cost by using the nonbenevolent agency if Congress can fully extract the agency's rent without reducing the inefficient type's incentive. To this end, the next subsection proposes a mechanism which fully extracts the agency's rent while the use of the non-benevolent agency or the possibility of collusion yields no additional social cost.

4.3. The Three-Stage Announcement Game

In this subsection, we propose a collusion deterrence mechanism which implements the same allocation as the benevolent agency case. Suppose that Congress sets up the following three-stage announcement game.

- Stage 1: Congress asks the firm for an announcement. The firm chooses either $a=\underline{\theta}$ or $\overline{\theta}$. If the firm chooses $a=\underline{\theta}$, then the game ends. If it chooses $a=\overline{\theta}$, then the game proceeds to stage 2.
- Stage 2: Congress asks the agency for the report. The agency chooses either $r = \theta$ or ϕ if $\sigma = \phi$, while $r = \phi$ if $\sigma = \phi$. If the agency chooses $r = \theta$ then the game ends. If it chooses $r = \phi$, then the game proceeds to stage 3.
- Stage 3: Congress asks the firm for an announcement again. The firm chooses either $a = \underline{\theta}$ or $\overline{\theta}$ again.

The extensive-form game in Figure 2 shows the three-stage announcement game when the firm is efficient. At stage 1, the efficient type chooses either $a=\underline{\theta}$ or $\overline{\theta}$. When $a=\overline{\theta}$, the game proceeds to stage 2 and so on.

Further, suppose that Congress presents the following outcome function as an enforcement rule. Letting a_1 , $a_2 \in \{\underline{\theta}, \overline{\theta}\}$ denote announcements of the firm at stage 1 and 3, respectively,

$$g(a_1, r, a_2) = \begin{cases} (\underline{t}^*, \underline{C}^*, \underline{q}^*, w^*) & \text{if } a_1 = \underline{\theta}, \\ (-\infty, \underline{C}^*, \underline{q}^*, \widehat{w}) & \text{if } a_1 = \overline{\theta} \text{ and } r = \underline{\theta}, \\ (\overline{t}^*, \overline{C}^*, \overline{q}^*, w^*) & \text{if } a_1 = \overline{\theta} \text{ and } r = \overline{\theta}, \\ (\underline{t}_0, \underline{C}_0, \underline{q}_0, w^*) & \text{if } a_1 = \overline{\theta}, r = \phi \text{ and } a_2 = \underline{\theta}, \\ (\overline{t}_0, \overline{C}_0, \overline{q}_0, w^*) & \text{if } a_1 = \overline{\theta}, r = \phi \text{ and } a_2 = \overline{\theta}, \end{cases}$$
(A1)

where \hat{w} is any finite value such that $\hat{w} > w^* + R(\bar{e}_0)/(1+v)$.

Now, we want to find a subgame perfect equilibrium in the three-stage announcement game with the outcome function (A1). It is easily seen that the optimal strategy for the inefficient type is $a_1 = a_2 = \overline{\theta}$ under the outcome function (A1). Therefore, we restrict our attention to a subgame perfect equilibrium when the firm is efficient.



Fig. 2. The Three-stage Announcement Game When the Firm is Efficient

The payoff structure associated with (A1) is as follows. For the efficient type,

$$\underline{U}(a_1, r, a_2) = \begin{cases} \underline{t}^* - k(\underline{e}^*) = 0 & \text{if } a_1 = \underline{\theta}, \\ -\infty & \text{if } a_1 = \overline{\theta} & \text{and } r = \underline{\theta}, \\ \underline{t}_0 - k(\underline{e}_0) = R(\overline{e}_0) & \text{if } a_1 = \overline{\theta}, r = \phi & \text{and } a_2 = \underline{\theta}, \\ \overline{t}_0 - k(\overline{e}_0 - \Delta \theta) = R(\overline{e}_0) & \text{if } a_1 = \overline{\theta}, r = \phi & \text{and } a_2 = \overline{\theta}. \end{cases}$$
(17)

For the agency,

$$V(a_1, r, a_2) = \begin{cases} \widehat{w} - w^* > R(\overline{e}_0) / (1+v) \text{ if } a_1 = \overline{\theta} \text{ and } r = \underline{\theta}, \\ 0 \text{ otherwise.} \end{cases}$$
(18)

The efficient type's payoff and the agency's are written in that order at the terminal nodes in Figure 2.

Lemma 1: Suppose the firm is efficient. A subgame perfect equilibrium in the three-stage announcement game with the outcome function (A1) is as follows.

$$\begin{cases} a_1 = a_2 = \underline{\theta} \text{ if } \sigma = \underline{\theta} \\ a_1 = \overline{\theta}, \ a_2 = \underline{\theta} \text{ if } \sigma = \phi \end{cases} \text{ for the efficient type,}$$

and

$$r = \underline{\theta}$$
 if $\sigma = \underline{\theta}$ for the agency.

Proof : We will first find the subgame-perfect equilibrium strategy by backward induction. Second, it will be shown that this equilibrium is collusion-proof.

First, we suppose that there is no collusion. Since the allocation $(\underline{t}_0, \underline{C}_0, \underline{q}_0)$ is incentive compatible, the efficient type tells the truth at stage 3: $a_2 = \underline{\theta}$. If $\sigma = \underline{\theta}$, then at stage 2, the agency gets some positive rent $\widehat{w} - w^* > 0$ by choosing $r = \underline{\theta}$ while zero rent by choosing $r = \phi$. Hence, the agency chooses $r = \underline{\theta}$ at stage 2. At stage 1, knowing that $r = \underline{\theta}$, the efficient type never chooses $a_1 = \overline{\theta}$ since it obtains $t = -\infty$. If $\sigma = \phi$, then the agency, at stage 2, trivially chooses $r = \phi$. Knowing this, the efficient type, at stage 1, prefers $a_1 = \overline{\theta}$ which yields the positive rent of $R(\overline{e}_0)$ to $a_1 = \underline{\theta}$ which yields zero rent.

Next, suppose that collusion is possible. When $\sigma = \underline{\theta}$, the efficient type can improve its payoff by $R(\overline{e}_0)$ if it makes the agency choose $r = \phi$ instead of $r = \underline{\theta}$. Therefore, if the efficient type gives a side-transfer to the $r = \phi$ agency, then the agency gets $\widetilde{V}_{\max} = R(\overline{e}_0)/(1+v)$ at most by choosing $r = \phi$. However, since $\widehat{w} - w^* > R(\overline{e}_0)/(1+v)$, the agency chooses $r = \underline{\theta}$. Hence, the equilibrium is not affected by the possibility of collusion. Q. E. D.

We obtain the following proposition from lemma 1.

Proposition 1: When the agency is non-benevolent, the three-stage announcement game with the outcome function (A1) is collusion-proof and attains the same social welfare level as the benevolent agency case. That is, the possibility of collusion does not lower the social welfare level.

Proof: To show the social welfare level, we must verify the allocations implemented in the subgame perfect equilibrium. When $\sigma = \theta$, the allocation for each type is the first-best: $(\underline{t}^*, \underline{C}^*, \underline{q}^*, w^*)$ and $(\overline{t}^*, \overline{C}^*, \overline{q}^*, w^*)$. Therefore, the expected social welfare of W^* is obtained with probability α . And when $\sigma = \phi$, the allocation for each type is the one under asymmetric information: $(\underline{t}_0, \underline{C}_0, \underline{q}_0, w^*)$ and $(\overline{t}_0, \overline{C}_0, \overline{q}_0, w^*)$. Therefore, the expected social welfare of W^0 is obtained with probability $1-\alpha$. Hence, the expected social welfare before auditing is $\alpha W^* + (1-\alpha) W^0$ which is the same as the benevolent agency case in (11). Q. E. D.

The major difference from the two-stage game is that there is the announcement stage for the firm before the reporting stage for the agency. Congress can detect the firm's lie at stage 1 by the agency's report at stage 2, and impose the punishment if it lies. Therefore, the firm tells the truth at stage 1 to avoid the



Fig. 3. Alternative Timing of Audit

punishment. Thus, both party's rents are fully extracted when the agency learns the true productivity parameter.

However, when the agency learns nothing, the efficient type can tell a lie *at stage 1* because it knows that its true type will not be revealed by the agency's report. Therefore, the game proceeds to stage 3 and Congress has to give up the rent to the efficient type. This result depends on the assumption that the firm has perfect information about the agency's signal. That is, the firm has already learned what signal the agency has received when it announces at stage 1.

Alternatively, suppose that the agency receives the signal after stage 1. That is, we assume that the audit is carried out after the first announcement stage as shown in Figure 3. Thereby, at stage 1, the firm cannot learn what signal the agency receives. In other words, the firm has imperfect information at stage 1. Assume that the firm also assigns α to the probability of $\sigma = \theta$. In Figure 2, the two nodes in stage 1 constitute an information set. Therefore, at stage 1 the firm does not know which node is reached.

Suppose that Congress presents the following outcome function as an enforcement rule.

$$g(a_1, r, a_2) = \begin{cases} (\underline{t}^*, \underline{C}^*, \underline{q}^*, w^*) & \text{if } a_1 = \underline{\theta}, \\ (-\infty, \underline{C}^*, \underline{q}^*, \widehat{w}) & \text{if } a_1 = \overline{\theta} \text{ and } r = \underline{\theta}, \\ (\overline{t}^*, \overline{C}^*, \overline{q}^*, w^*) & \text{if } a_1 = \overline{\theta}, r = \overline{\theta}, \\ (\underline{t}^*, \underline{C}^*, \underline{q}^*, w^*) & \text{if } a_1 = \overline{\theta}, r = \phi \text{ and } a_2 = \underline{\theta}, \\ (\overline{t}^*, \overline{C}^*, \overline{q}^*, w^*) & \text{if } a_1 = \overline{\theta}, r = \phi \text{ and } a_2 = \overline{\theta}, \end{cases}$$
(A2)

where \hat{w} is any finite value such that $\hat{w} > w^* + R(\bar{e}^*)/(1+v)$.

Lemma 2: In the three-stage announcement game with the outcome function (A2), the efficient type chooses $a_1 = \underline{\theta}$ for any $\sigma \in \{\underline{\theta}, \phi\}$, that is, it tells the truth

at stage 1.

Proof: The proof is similar to lemma 1. Given $a_1 = \overline{\theta}$ and $r = \phi$, the efficient type chooses $a_2 = \overline{\theta}$ and obtains $\overline{t}^* - k(\overline{e}^* - \Delta \theta) = R(\overline{e}^*)$ since the allocation $(\underline{t}^*, \underline{C}^*, \underline{q}^*)$ is not incentive compatible. Therefore, it is willing to pay the agency up to $R(\overline{e}^*)/(1+v)$ in return for reporting ϕ . However, since $\overline{w} - w^* > R(\overline{e}^*)/(1+v)$, the agency never chooses $r = \phi$ when $\sigma = \underline{\theta}$. That is, the agency chooses $r = \underline{\theta}$ if $\sigma = \underline{\theta}$ and ϕ if $\sigma = \phi$. Therefore, when it chooses $a_1 = \overline{\theta}$, the efficient type obtains $R(\overline{e}^*)$ if $\sigma = \phi$, but it obtains $-\infty$ if $\sigma = \underline{\theta}$. Thus, the efficient type obtains the expected payoff of $-\infty$ by choosing $a_1 = \overline{\theta}$ unless α is zero. Hence, the efficient type never chooses $a_1 = \overline{\theta}$ for any signal. Q. E. D.

We can easily see that the inefficient type's optimal strategy is $a_1 = a_2 = \overline{\theta}$. From lemma 2, the allocation is the first-best: $(\underline{t}^*, \underline{C}^*, \underline{q}^*, w^*)$ and $(\overline{t}^*, \overline{C}^*, \overline{q}^*, w^*)$ whether $\sigma = \underline{\theta}$ or $\sigma = \phi$. Therefore, Congress obtains the full information social welfare level: W^* . Thus, we obtain the following proposition.

Proposition 2: Suppose that the firm cannot learn the agency's signal before the first announcement. The three-stage announcement game with the outcome function (A2) is collusion-proof and attains the first-best social welfare level even if the agency is non-benevolent.

Thus, in the three-stage announcement game, the timing of the audit is crucial because it affects the level of welfare obtained.

5. Conclusions

We showed that by making the firm announce before auditing, this collusiondeterrence mechanism, even when collusion is possible, does not incur any additional social costs. In the three-stage game, the agency plays the role of detecting the firm's lie. Thus, Congress can punish the firm and deter collusion while also extracting both parties' rent when the agency succeeds in discovering the firm's true type. This implies that there is no trade-off between extraction of the agency's rent and the firm's cost-reducing incentive.

Further, we showed that the first-best social welfare is attained when the firm has imperfect information about the signal received by the agency. In order to conceal the agency's signal from the firm, it is necessary to make the agency audit *after* the first announcement of the firm. If, due to the timing of audit, the firm has imperfect information, the first-best allocation is implemented. Thus, the timing of audits is a non-negligible factor when designing collusion deterrence mechanisms in hierarchical regulatory contracts.

Doctoral Student, Faculty of Economics, Hokkaido University

References

- 1. Baron, D. P. and R. B. Myerson. (1982), "Regulating a Monopolist with Unknown Costs" *Econometrica*, Vol. 50, pp. 911-930.
- Demski, J. S. and D. E. M. Sappington. (1987), "Hierarchical Regulatory Control" RAND Journal of Economics, Vol. 18, pp. 369-383.
- 3. Kofman, F. and J. Lawarree. (1993), "Collusion in Hierarchical Agency" *Econometrica*, Vol. 61, pp. 629-656.
- 4. Kofman, F. and J. Lawarree. (1996)," A Prisoner's Dilemma Model of Collusion Deterrence" Journal of Public Economics, Vol. 59, pp. 117-136.
- 5. Laffont, J.-J. and J. Tirole. (1986), "Using Cost Observation to Regulate Firms" Journal of Political Economy, Vol. 94, pp. 614-641.
- 6. Laffont, J.-J. and J. Tirole. (1991), "The Politics of Government Decision-Making: A Theory of Regulatory Capture" Quarterly Journal of Economics, Vol. 106, pp. 1089-1127.
- 7. Tirole, J. (1986), "Hierarchies and Bureaucracies: On the Role of Collusion in Organizations" Journal of Law, Economics, and Organization, Vol. 2, pp. 181-214.