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Implementation in Nash Equilibrium

Tomoichi Shinotsuka*
Koji Takamiya**

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*Department of Economics
Otaru University of Commerce

**Economics Department
Hokkaido University

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Tomoichi Shinotsuka

Department of Economics

Otaru University of Commerce

3-5-21, Midori, Otaru 047-8501 Japan

Koji Takamiya

Economics Department

Hokkaido University

Kita9 Nishi7 Sapporo 060-0808 Japan

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Correspondence: Tomoichi Shinotsuka

3-5-21, Midori, Otaru, Hokkaido 047-8501 Japan

e-mail: shino@res.otaru-uc.ac.jp

Abstract:

In a simple game, coalitions belonging to a given class are supposed to be "absolutely powerful" while others have no power. We attempt to make this distinction operational. Toward this end, we propose two axioms, Exclusion and Strong Non-Discrimination. Strong Non-Discrimination describes circumstances under which certain coalitions, the losing coalitions, can have no influence over social choice. Exclusion requires that there are situations in which certain coalitions, the winning coalitions, can exercise their power. We show that the weak core correspondence is the minimal correspondence satisfying Maskin Monotonicity and Strong Non-Discrimination. We also show that the weak core is the unique correspondence satisfying Nash implementability, Strong Non-Discrimination, and Exclusion.

0. Introduction

This paper studies Nash-implementation of social choice correspondences (SCC for short) on the class of simple games with ordinal preferences. In a simple game, coalitions belonging to a given class are supposed to be "absolutely powerful" while others have no decision power. In this paper, we attempt to make this distinction operational by proposing two axioms, Strong Non-Discrimination and Exclusion. Strong Non-Discrimination describes circumstances under which certain coalitions, the "losing" coalitions, have no influence over social choice: for any such coalition and any two alternatives, if all the individuals outside the coalition are indifferent between these alternatives, then the SCC should not discriminate between them, in the sense that one alternative is chosen, i.e. contained in the image of the SCC, if and only if the other alternative is chosen. Strong Non-Discrimination is a strengthening of a condition used in different contexts (Thomson (1987), Gevers (1986), and Nagahisa (1991, 1994)). Exclusion requires that there are situations in which certain coalitions, the "winning" coalitions, can exercise their power: for any such coalition, if all the members of the coalition have identical, non-trivial preferences (in the sense that at least two alternatives are not judged indifferent), and, furthermore, all the members of the complementary coalition have trivial preferences (for them all the alternatives are indifferent), then the coalition has the power of excluding at least one alternative. We call this axiom Exclusion.

Some remarks are in order on Strong Non-Discrimination and Exclusion. The former cannot be interpreted as saying that losing coalitions have no power. In fact, it tells us nothing if some member of the complementary coalition finds one alternative preferable to another. Similarly, Exclusion demands that a SCC should grant a winning

coalition a "right" to exclude some alternative only in the very special case when all of its members have common preferences and all the members of the complementary coalitions regard all alternatives indifferent. Therefore, Exclusion does not give unlimited power to the winning coalitions. However, we will see that Strong Non-Discrimination and Exclusion become considerably stronger when the important axiom of Maskin Monotonicity is imposed as well.

Recall that a SCC satisfies Maskin Monotonicity if it preserves the desirability of an allocation under transformations of preferences that raise the relative ranking of the allocation. Maskin Monotonicity and Strong Non-Discrimination imply that losing coalitions have no veto power (Lemma 3-10). Furthermore, these three axioms together imply that winning coalitions are "all-powerful" and losing coalitions are "completely powerless" (Corollary 3-6).

It is important to recall the distinction between the strong core and the weak core of simple games. An alternative is in the strong core of a simple game if there exist no winning coalition and another alternative that is at least as good for all members of the winning coalition and strictly preferred by some member of the winning coalition. An alternative is in the weak core if there exist no winning coalition and another alternative that is strictly preferred by all members of the winning coalition. The strong core satisfies Strong Non-discrimination and Exclusion but violates Maskin Monotonicity (see Remark 3-7). Hence, the distinction between the weak core and the strong core is critical.

We show that the weak core correspondence is the minimal correspondence satisfying Maskin Monotonicity and Strong Non-Discrimination. This is the most basic result in this paper but, strictly speaking, it is not a full characterization of the weak

core. Toward this end, we work with a domain on which the weak core is non-empty. Then, we show that the weak core is the unique correspondence satisfying Maskin Monotonicity, Strong Non-discrimination, and Exclusion. It is well-known that Maskin Monotonicity is necessary for Nash implementability. It turns out that the weak core is the unique correspondence satisfying Nash implementability, Strong Non-Discrimination, and Exclusion.

Implementability of the core has been investigated in different contexts.[1] Kara and Sönmez (1996) studied the strong core of two-sided matching problems. They proved that the core correspondence is Nash-implementable, and that if a SCC is Nash-implementable, individually rational and Pareto efficient, then it is a supercorrespondence of the core correspondence. Sönmez (1996) extended this result to generalized matching problems. Another strand of the literature deals with mechanisms implementing the core in economic environments. Focuses are not necessarily on Nash-implementation or even mechanisms. For example, Wilson (1978) proposed a two-stage (thus extensive form) game one of whose subgame perfect equilibrium outcomes is always in the weak core of the underlying exchange economy. Kalai *et al.* (1979) studied (strategic form) a different kind of implementation of the weak core of exchange economies in strong equilibrium. Recently, Serrano and Vohra (1997) constructed an extensive form mechanism that implements the weak core of private ownership economies in subgame prefect equilibrium. They also proposed a strategic form mechanism which implements the weak core of NTU games in Nash equilibrium.

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[1] For excellent surveys on implementation problems, see Maskin (1985) and Moore (1992), and Jackson (2000).

The plan of the paper is as follows: The next section provides preliminary definitions. Section 2 introduces the axioms for SCC's. Section 3 states the main results. And Section 4 gives concluding remarks.

1. Notations and Definitions

We fix the set X of (social) alternatives (or outcomes), and the finite set N of individuals. Denote by $\mathcal{R}$ the set of complete and transitive binary relations on X. Let $i \in N$, and $R^i \in \mathcal{R}$. We denote by R^i individual i's preference relation. For x, $y \in X$, xR^iy means that to individual i, x is at least as good as y. We denote the asymmetric part of R^i by P^i, and the symmetric part by I^i. We say that $R^i \in \mathcal{R}$ is trivial if individual i is indifferent among all alternatives, that is, for all x, $y \in X$, xI^iy. Denote by R_0^i individual i's trivial preference relation. We call $R = (R^i)_{i \in N} \in \mathcal{R}^N$ a preference profile . The trivial preference profile R_0 consists of the trivial preference relation of each individual, i.e. $R_0 = (R_0^i)_{i \in N}$.

1-1. Simple Games

Let W be a nonempty subset of 2^N. We call a subset of N a coalition. A coalition in W is a winning coalition and a coalition not in W is a losing coalition. Let R be a preference profile. For simplicity, we fix W from now on. Hence, we identify a simple game with a preference profile. Let x, $y \in X$ and let $S \subset N$. We say that x strongly dominates y via S if S is winning and for all $i \in S$, xP^iy. We say that x weakly dominates y via S if S is winning, x is at least as good as y for every individual in S, and x is preferred by some

individual in S. The <u>weak</u> <u>core</u> <u>of a</u> <u>simple</u> <u>game</u> R, denoted by C(R), is the set of all

alternatives that are not strongly dominated by any other alternative. The <u>strong core</u>

<u>of a simple game</u> R, denoted by SC(R), is the set of all alternatives that are not weakly

dominated by any other alternative.

1-2. Implementation

For each i∈N, let D^i be a nonempty subset of $\mathcal{R}$, interpreted as the set of <u>admissible</u>

<u>preferences</u> of individual i. For a coalition S, D^S denotes the Cartesian product $\prod_{i\in S}D^i$. A

<u>social</u> <u>choice</u> <u>correspondence</u> (henceforth, SCC) is a nonempty-valued correspondence φ:

$D^N \twoheadrightarrow X$. We call D^N the <u>domain</u> of φ. For $R^i\in\mathcal{R}$ and x∈X, let $L(R^i, x) = \{y\in X \mid xR^iy\}$.

For preference profiles R and R', call R' a <u>monotonic transformation of R at x</u> if for all

i∈N: $L(R^i, x)\subset L(R'^i, x)$. Denote by MT(R, x) the set of all monotonic transformations of

R at x. Let φ be a SCC. In order to freely perform monotonic transformations, the

domain D^N should satisfy a certain "richness condition". That is, we say that D^N is

<u>closed</u> <u>under</u> <u>monotonic</u> <u>transformations</u> (or simply, closed) if for all $R\in D^N$ and x∈X,

MT(R, x) $\subset D^N$. Under this assumption, the domain D^N contains the trivial profile R_0

because φ is nonempty-valued, and R_0 is a monotonic transformation of <u>any</u> R at <u>any</u> x.

A <u>game</u> <u>form</u> is a list (S, g) with $S=\prod_{i\in N}S^i$, where each S^i is a <u>strategy</u> <u>space</u> for

individual i, and g: S→X is an <u>outcome</u> <u>function</u>. A <u>game</u> is a list (S, g, R), where (S, g)

is a game form, and $R\in\mathcal{R}^N$. A strategy profile s∈S is a <u>Nash equilibrium of the game (S,</u>

<u>g, R)</u> if there is no i∈N such that for some $t^i\in S^i$: $g(s^{-i}, t^i)P^ig(s)$. Denote by NE((S, g, R))

the set of Nash equilibria of the game (S, g, R). Let φ be a SCC. Then the game form <u>(S,</u>

<u>g) Nash-implements φ</u> if for all $R\in D^N$: φ(R)=g(NE((S, g, R))). Say that φ is

<u>Nash-implementable</u> if there exists a game form which Nash-implements φ.

Necessary and sufficient conditions for Nash-implementation have been investigated in detail. A fundamental property is as follows. Let φ be a SCC.

<u>Maskin Monotonicity</u> (MMON):

For all R and $R' \in D^N$, if $x \in \varphi(R)$ and $R' \in MT(R, x)$, then $x \in \varphi(R')$.

Maskin (1977) proved that if a SCC is Nash-implementable, then it satisfies MMON. The converse of this claim does not hold true. Danilov (1992) and Yamato (1992) developed a sharper condition [2]. Let $i \in N$ and let $Y \subset X$. Let us denote Ess(i; φ; Y)={$y \in Y$ | There exists $R \in D^N$ such that $y \in \varphi(R)$ and $L(R^i, y) \subset Y$)}.

<u>Essential Monotonicity</u> (EMON):

For all R and $R' \in D^N$, if $x \in \varphi(R)$ and for all $i \in N$, Ess(i; φ; $L(R^i, x)$) $\subset L(R'^i, x)$), then $x \in \varphi(R')$.

Yamato (1992) proved that (i) if $|N| \geq 3$, any SCC satisfying EMON is Nash-implementable, and, (ii) under a certain mild condition imposed on admissible preferences, any Nash-implementable SCC satisfies EMON.

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[2] For other refined conditions of monotonicity, see e.g. Maskin (1985), Williams (1986), Repullo (1987), Saijo (1988), McKelvey (1989), Moore and Repullo (1990), Sjöström(1991) and Ziad (1998).

2. New Axioms for Social Choice Correspondences

In this section, we propose new axioms that describe how SCC's depend on the class of winning coalitions. Before we get to those, we start with a discussion of a standard axiom.

The first axiom says that if all individuals are indifferent between two alternatives, then the SCC should not treat the alternatives differently.

Non-Discrimination (ND):

For all $R \in D^N$, x and y∈X, if for all i∈N, xI^iy, then $x \in \varphi(R)$ if and only if $y \in \varphi(R)$.

The axiom appears in axiomatic characterizations of the Walrasian correspondence (see Thomson (1987), Gevers (1986) and Nagahisa (1991, 1994)). We will see that ND is also useful in the context of axiomatic studies of the weak core of simple games (see Lemma 3-4). We also consider a strengthening of ND to obtain further results.

The next axiom says that if all the individuals outside a losing coalition find two alternatives indifferent, then the SCC should not treat the alternatives differently.

Strong Non-Discrimination (SND)

For all $R \in D^N$, x, y∈X, and S⊂N, if for all i∈N−S: xI^iy and $S \notin W$, then $x \in \varphi(R)$ if and only if $y \in \varphi(R)$.

In other words, the SCC treats two alternatives symmetrically, independently of the preferences of the members of the losing coalition, as long as all the members of the

complementary coalition are indifferent between the alternatives. Hence, losing coalitions have no power then. However, SND is far from saying that losing coalitions are absolutely powerless. This is because SND is not applicable if two people outside a losing coalition have different preferences over the two alternatives.

The connection between ND and SND is straightforward. Since w does not contain the empty set, SND implies ND. The converse does not hold (for example, the Pareto correspondence violates SND but does satisfy ND).

The next axiom says that a winning coalition can exclude at least one alternative if all the member of the coalition have identical nontrivial preferences and all the members of the complementary coalition are indifferent among all the alternatives.

Exclusion (EX):

For all $S \in w$ and $R^S \in D^S$, if for all i and $j \in S$, $R^i = R^j$, and for all $i \in S$, $R^i \neq R_0^i$, then $\varphi(R_0^{-S}, R^S) \neq X$.

The axiom cannot be interpreted as saying that winning coalitions are all-powerful for the following reasons. First, excluding at least one alternative is not equivalent to picking the best alternative under the foregoing circumstances. Second, if some individual of the complementary coalition $N - S$ has non-trivial preferences, EX does not apply.

3. Main Results

In this section, we assume that every profile in the domain D^N of SCC's under consideration has a non-empty weak core and that D^N is closed under monotonic transformations. We start with the following important observation.

<u>Lemma 3-1</u>: The weak core correspondence satisfies MMON.

<u>Proof</u>: Let R, R' be two profiles and $x \in X$ be such that $R' \in MT(R, x)$ and $x \notin C(R')$. Thus under R', x is dominated by some other $y \in X$ via some $S \in W$. That is, for all $i \in S$, $yP'^i x$. Since $R' \in MT(R, x)$, this implies for all $i \in S$, $yP^i x$. Thus x is dominated by y under R. But this says that $x \notin C(R)$. Therefore, C satisfies MMON. ☺

<u>Theorem 3-2</u>: If a SCC φ satisfies MMON and SND, then φ is a supercorrespondence of the weak core C, that is, for any $R \in D^N$, $\varphi(R) \supset C(R)$.

<u>Proof</u>: Let $x \in X$ and $R \in D^N$ be such that $x \in C(R)$. Suppose that $x \notin \varphi(R)$. Let $y \in \varphi(R)$, $T^- = \{i \in N \mid xR^i y\}$, and $T^+ = \{i \in N \mid yP^i x\}$. Let R' be the profile obtained by raising y to the same indifference class as x and keeping the other indifference classes intact for each member of T^-:

For all $i \in T^-$, $xI'^i y$ and for all v, $w \in X - \{y\}$: $vR^i w$ if and only if $vR'^i w$ and $vI^i w$ if and only if $vI'^i w$, and for all $i \in T^+$, $R'^i = R^i$.

Note that $R' \in MT(R, x)$. Since D^N is closed, $R' \in D^N$. By MMON, $y \in \varphi(R')$. Further, since $L(R^i, x) = L(R'^i, x)$ for each $i \in N$, we have $R \in MT(R', x)$. Again, by MMON, $x \notin \varphi(R')$. Thus φ treats x and y differently under R'. Suppose that T^+ is losing. Then, by SND

either x, $y \in \varphi(R')$ or x, $y \notin \varphi(R')$, which is a contradiction. Hence, T^+ is winning. Thus under R', y dominates x via T^+. This says that $x \notin c(R')$. Note that $R' \in MT(R, x)$ since $L(R^i, x) = L(R'^i, x)$ for all $i \in N$. Lemma 3-1 tells us that c satisfies MMON. Thus $x \notin c(R)$. This is a contradiction. ☺

Remark 3-3: In Theorem 3-2 above, the set inclusion may be strict. Consider the SCC that chooses the whole set of alternatives X for every profile. This SCC satisfies both MMON and SND.

Lemma 3-4: If a SCC φ satisfies MMON, ND and EX, then φ satisfies the following property:

(★) For all $S \subset N$, $R^S \in D^S$, and $x \in X$, if $S \in W$ and for all $i \in S$, y and $z \in X - \{x\}$, xP^iy and yI^iz, then $\varphi(R_0^{-S}, R^S) = \{x\}$.

Proof: Let $S \in W$ and $x \in X$. Let R be a profile such that for all $i \in S$, y and $z \in X - \{x\}$, xP^iy and yI^iz, and for all $i \in N-S$, $R^i = R_0^i$.

Let R^*. D^N and $v \in \varphi(R^*)$. Since $R_0 \in MT(R^*, v)$ and D^N is closed, $R_0 \in D^N$. Then by MMON, $v \in \varphi(R_0)$. Since x is indifferent to v for any $i \in N$ at R_0, by ND $x \in \varphi(R_0)$. Note that $R \in MT(R_0, x)$. Then since D^N is closed, $R \in D^N$. Then, by MMON $x \in \varphi(R)$. On the other hand, by EX, $\varphi(R) \neq X$. That is, there is some alternative that does not belong to $\varphi(R)$. By construction, under R, any two alternatives other than x are indifferent for any member of N. Thus ND implies that any alternative other than x is excluded from $\varphi(R)$. ☺

<u>Theorem 3-5</u>: If a SCC φ satisfies MON, ND and EX, then φ is a subcorrespondence of the weak core, that is, for any $R \in D^N$, $\varphi(R) \subset c(R)$.

<u>Proof</u>: Let $x \in X$ and $R \in D^N$. Assume that $x \in \varphi(R)$, but $x \notin c(R)$. Then there exists $S \in W$ and $y \in X$ such that for all $i \in S$, $y P^i x$. Let R' be a profile such that for all $i \in S$, v and $w \in X - \{y\}$, $y P'^i v$ and $v I'^i w$, and for all $i \in N - S$, $R'^i = R_0^i$.

Note that $R' \in MT(R, x)$. Then by MMON, we have $x \in \varphi(R')$. By Lemma 3-4, $\varphi(R') = \{y\}$. This implies $x = y$, a contradiction. ☺

The following result is immediate from the foregoing theorems.

<u>Corollary 3-6</u>: c is the unique SCC that satisfies MMON, SND and EX.

<u>Remark 3-7</u>: The above Corollary 3-9 characterizes the weak core of simple games. This result is tight in the sense that the three properties MMON, SND and EX are logically independent. The example in Remark 3-3 establishes the independence of EX. For the independence of SND, fix x . X and let $\varphi(R) = \{x\}$ for all R. Then, φ obviously satisfies MMON and EX but violates SND. To show the independence of MMON, note that SC satisfies SND and EX but violates MMON. To illustrate a violation of MMON, let $X = \{x, y\}$, $N = \{1, 2\}$, $W = \{N\}$, $x P_1 y$, $y P_2 x$, $x I_1' y$, $y P_2' x$. Then, x . $\varphi(R)$, $R' \in MT(R, x)$, but x . $\varphi(R)$.

<u>Theorem 3-8</u>: Assume that $|N| \geq 3$. Then c is Nash-implementable.

<u>Proof</u>: In view of Yamato (1992), it suffices to show that c satisfies EMON. Suppose otherwise. Then,

(*) There exist R, $R' \in D^N$ such that $x \in c(R)$ and for all $i \in N$, Ess(i; c; $L(R^i, x)) \subset L(R'^i, x)$ but $x \notin c(R')$.

Since c satisfies MMON (Lemma 3-1), [$x \in c(R)$ and $x \notin c(R')$] in (*) above implies that $R' \notin MT(R, x)$. Hence, there exists $i \in N$ such that $L(R^i, x) - L(R'^i, x) \neq \phi$. Let $z \in L(R^i, x) - L(R'^i, x)$. By (*), $z \notin L(R'^i, x)$ implies $z \notin Ess(i; c; L(R^i, x))$. Therefore, for some $i \in N$,

(**)   For all $R'' \in D^N$, if $L(R''^i, z) \subset L(R^i, x)$, then $z \notin c(R'')$.

Now let R* be a profile such that for all $j \in N$, if $xI^{*j}z$ and for all v and $w \in X-\{z\}$, $vR^{*j}w$ if and only if vR^jw.

Thus for each $j \in N$, we have $L(R^j, x) \cup \{z\} = L(R^{*j}, x)$. This implies $L(R^j, x) \subset L(R^{*j}, x)$. Thus, $R^* \in MT(R, x)$. Since $x \in c(R)$ and c satisfies MMON, we have $x \in c(R^*)$. Since $L(R^i, x)$ contains z by assumption, $L(R^i, x) = L(R^{*i}, x)$. Since $xI^{*i}z$ by construction, $L(R^{*i}, z) = L(R^{*i}, x)$. Therefore $L(R^{*i}, z) = L(R^i, x)$. This equality and (**) together imply $z \notin c(R^*)$. On the other hand, $x \in c(R^*)$ and for all $j \in N$, $xI^{*j}z$. Hence, $z \in c(R^*)$. This is a contradiction.   ☺

Now we conclude with the following.

<u>Theorem 3-9</u>: Assume that $|N| \geq 3$. Then c is the unique SCC satisfying Nash implementability, SND and EX.

We discuss the logical relations between Maskin's '<u>no veto power</u>' condition and SND and EX. Let φ denote a SCC. Let S be a coalition. Say that S has <u>veto power</u> if

there are some $R \in D^N$ and $x \in X$ such that $x \notin \varphi(R)$ and for all $j \in N-S$, $L(R^j, x) = X$.

Maskin (1977) introduced the following definition.

No Veto Power (NVP): For all $i \in N$, $R \in D^N$, and $x \in X$, if for all $j \in N-\{i\}$, $L(R^j, x) = X$, then $x \in \varphi(R)$.

It directly follows from the definition that if a SCC satisfies EX, then every winning coalition has veto power.

Lemma 3-10: If a SCC φ satisfies MMON and SND, then no losing coalition has veto power.

Proof: Let φ satisfy MMON and SND. Let S be a losing coalition. Suppose that S has veto power. Then there are $R \in D^N$ and $x \in X$ such that for all $j \in N-S$, $L(R^j, x) = X$ and $x \notin \varphi(R)$. Since φ satisfies SND, $S \notin W$ implies $\varphi(R_0^{-S}, R^S) = X$. Since $L(R^j, x) = X = L(R_0^j, x)$, $R \in MT(x, (R_0^{-S}, R^S))$. Thus MMON implies $\varphi(R) = X$. This contradicts $x \notin \varphi(R)$. ☺

Maskin (1977) proved that if a SCC satisfies both MMON and NVP, and $|N| \geq 3$, then it is Nash implementable. Combining Lemma 4-15 with Maskin's result, we obtain the following.

Theorem 3-11: Assume $|N| \geq 3$. Let φ be a SCC. Assume that there does not exist $i \in N$ such that $\{i\} \in W$. Then if φ satisfies MMON and SND, then φ is Nash-implementable.

Remark 3-12: Theorems 3-2 and 3-11 immediately imply the following: Let there be at

least three individuals. Assume there does not exist a winning coalition consisting of only one individual. Then if a SCC satisfies MMON and SND, then it is Nash implementable and it is a supercorrespondence of the core.

4. Concluding Remarks

Though we require SCC's to be nonempty-valued, this requirement played little role in the foregoing arguments. Though the nonemptiness requirement has some conceptual appeal, there is a room for applying Occam's Razor. In fact, in axiomatic studies, some authors (see footnote 4, for example) do not insist on this requirement but explore how far they can go formally without it. In this section, we take this view.

In this section, we assume that the domain D^N contains at lest one simple game R whose weak core is non-empty.

Let φ be a correspondence from D^N to X, possibly empty valued for some $R \in D^N$. We introduce the following property:

Restricted Nonemptiness (RNEM)[3]: $\forall R \in D^N$: $C(R) \neq \emptyset \Rightarrow \varphi(R) \neq \emptyset$.

Theorem 4-1: The weak core is the unique correspondence satisfying MMON, SND, EX and RNEM.

Proof: By the foregoing arguments, the core C satisfies these four properties. We shall

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[3] Similar properties can be found, for example, in Norde *et al.* (1996), and Peleg *et al.* (1996). These papers study axiomatizations of solutions to strategic form games.

prove uniqueness. First, it is easy to verify that Theorem 3-2 still holds even if the nonempty-valuedness of φ is weakened to the assumption that φ satisfies RNEM. This establishes $\varphi \supset c$. Second, Theorem 3-5 is also true without nonempty-valuedness or RNEM. And in that case, nonempty-valuedness of the weak core c becomes superfluous. Recall that SND implies ND. Then all conditions for Theorem 3-5 are met. This proves $\varphi \subset c$. ☺

Remark 4-2: The properties appearing in Theorem 4-1 are logically independent. By adapting the examples in Remark 3-7 in a straightforward manner, we can show that each of MMON, SND and EX is independent of the three other properties. The following example shows RNEM is independent of the other three properties: Let φ be the solution such that $\varphi(R) = \varnothing$ for any $R \in D^N$. Then φ satisfies MMON, SND, and EX but violates RNEM.

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