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Budget Distribution Problem

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Summary

In this paper, we consider the budget distribution problems defined by Herrero et al. (1999). We provide axiomatic characterizations of the rights-weighted solution, introduced by Bergantiños and Mendez-Naya (1999), and pseudo rights-weighted solution both of which are generalizations of the right-egalitarian solution due to Herrero et al.. The functional equation approach produces sharper results with simpler proofs.

Key words: Budget distribution problem; rights-egalitarian solution, rights-weighted solution, pseudo rights-weighted solution, additivity,

JEL Classification: C71, C74

1. Introduction

Herrero, Maschler, and Villar (1999, HMV) formulated a general notion of budget distribution problems. A budget distribution problem deals with a situation that there is a budget to be distributed among a group of agents, each of whom has monetary claims. A distinguishing feature of the problem is this. The budget can be positive or negative, a claim can be negative, and the budget may be larger or smaller than the sum of the claims. Hence, the whole

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class of budget distribution problems is large enough to include the class of bankruptcy problems (O´Neill(1982) and Aumann and Maschler (1985)) and that of surplus sharing (Moulin (1987) and Chun (1988)). HMV provided axiomatic characterizations of the rights-egalitarian solution on the class of all budget distribution problems. As the terminology suggests, the rights-egalitarian solution satisfies symmetry, the requirement that two agents with the equal claims should receive equal amount. Hence, the solution may be appropriate in contexts where no apriori discrimination allowed . On the other hand, there are situations in which discrimination among agents may be necessary (Moulin (2000, p.645)). Hence, axiomatic studies of budget distribution problems without requiring symmetry are of some importance, and this is what we do in this paper.

Additivity axioms play central roles in this paper. Full additivity says that agents receive the same amount whether the solution is applied to two budget distribution problems separately, or whether the solution is applied to the sum of the problems. Full additivity was introduced by Bergantiños and Mendez-Naya (1999, BM). BM proposed the ´´rights-weighted´´solution by introducing agents´exogeneous shares of net surplus. The rights-weighted solution is the unique solution satisfying full additivity and either one of two bounds axioms. We also show that replacing the bounds axiom with continuity gives a further generalization of the rights-weighted solution. In the context of surplus sharing, Moulin (1987) and Chun (1988) explored implications of additivity with respect to budget given an arbitrary claim vector. We propose a weakening of this axiom (when applied to surplus sharing), partial additivity, that requires additivity with respect to buget only when the claims vector is zero. While full additivity is powerful, partial additivity is also useful when combined with other axioms.

For axiomatic characterizations, we employ the functional equation approach. The merits of this approach are the following. First of all, the approach produces a simpler proof than BM's. Second, we can replace a certain bounds axiom used by BM by a weaker one and yet axiomatize the rights-weighted solution. Third, the functional equation approach allows us to handle interesting smaller domains such as the class of surplus sharing problems and that of bankruptcy problems.

Relevant mathematical facts are gathered in Appendix.

2. Notation and definitions

Let N be a finite set of agents. The set N is fixed throughout this note. For simplicity, let N = {1,...,n}. we call a pair (B, $\mathbf{c}$) a **budget distribution problem** if (B, $\mathbf{c}$) $\in \mathbb{R} \times \mathbb{R}^N$. We call B and $\mathbf{c}$ the **budget** and **claims vector**, respectively. Henceforth, we use boldface to denote vectors in $\mathbb{R}^N$. A budget describes a given amount of money to be allocated among the agents. Let $\mathcal{A}$ be the set of all budget distribution problems. If a budget distribution problem (B, $\mathbf{c}$) satisfies (B, $\mathbf{c}$) $\in \mathbb{R}_+ \times \mathbb{R}_+^N$ and B $\leq \sum_{i \in N} c_i$, we call it a **bankruptcy problem**. Let $\mathcal{B}$ be the set of all bankruptcy problems. If a budget distribution problem (B, $\mathbf{c}$) satisfies (B, $\mathbf{c}$)$\in \mathbb{R}_+ \times \mathbb{R}_+^N$ and B $\geq \sum_{i \in N} c_i$, we call it a **surplus-sharing problem**. Let $\mathcal{S}$ be the set of all surplus sharing problems. Let $\mathcal{D} \subset \mathcal{A}$. A solution on $\mathcal{D}$ is a map F : $\mathcal{D} \rightarrow \mathbb{R}^N$ such that for all (B , $\mathbf{c}$) $\in \mathcal{D}$, budget balancedness holds, i.e. B $=\sum_{i \in N}$ F_i(B , $\mathbf{c}$) , where F(B, $\mathbf{c}$) = (F_1(B , $\mathbf{c}$),..., F_n(B , $\mathbf{c}$)). The **rights-egalitarian solution** selects for all (B , $\mathbf{c}$) $\in \mathcal{D}$ and for all j $\in$N, (B -$\sum_{i \in N} c_i$) /n + c_j . This solution was introduced by HMV.

A solution F on $\mathcal{D}$ is called a **rights-weighted solution** if there exists $\boldsymbol{\lambda} = (\lambda_i)_{i \in N} \in \mathbb{R}_+^N$ with $\sum_{i \in N} \lambda_i = 1$ such that for all (B, $\mathbf{c}$) $\in \mathcal{D}$, F(B, $\mathbf{c}$) = (B -$\sum_{i \in N} c_i)\boldsymbol{\lambda} + \mathbf{c}$. This solution was introduced by BM. The number λ_i is agent i ´s s*h*are of net surplus. If we do not insist on nonnegativity of weights, we obtain the following solution. A solution F on $\mathcal{D}$ is called a **pseudo rights-weighted solution** if there exists $\boldsymbol{\lambda} = (\lambda_i)_{i \in N} \in \mathbb{R}^N$ with $\sum_{i \in N} \lambda_i = 1$ such that for all (B, $\mathbf{c}$) $\in \mathcal{D}$, F(B, $\mathbf{c}$) = (B -$\sum_{i \in N} c_i)\boldsymbol{\lambda} + \mathbf{c}$. Clearly, a rights-weighted solution is a pseudo rights-weighted solution but the converse does not always hold.

In this paper, we discuss the following axioms.

Full Additivity (FAD): For all (B, $\mathbf{c}$) , (B´, $\mathbf{c}$´) $\in \mathcal{D}$, if (B + B´, $\mathbf{c}$ + $\mathbf{c}$´) $\in \mathcal{D}$, then

F(B + B´, $\mathbf{c}$ + $\mathbf{c}$´) = F(B , $\mathbf{c}$)+ F(B´, $\mathbf{c}$´).

Partial Additivity (PAD): For all B, B´$\in \mathbb{R}$, if (B, $\mathbf{0}$) $\in \mathcal{D}$, (B´, $\mathbf{0}$) $\in \mathcal{D}$, and (B + B´, $\mathbf{0}$) $\in \mathcal{D}$, then

F(B + B´, $\mathbf{0}$) = F(B , $\mathbf{0}$)+ F(B´, $\mathbf{0}$).

Responsibility (RES): For all (B, $\mathbf{c}$)$\in \mathcal{D}$, and all i $\in$N,

F(B, $\mathbf{c}$) = (0,$\bullet\cdot\cdot$,0 , c_i, 0,$\cdot\cdot\cdot$,0)+ F(B - c_i, ($\mathbf{c}_{-i}$, 0)).

Compatibility (COM): For all (B, $\mathbf{c}$) $\in \mathcal{D}$,
B $= \sum_{i \in N} c_i$ implies F(B, $\mathbf{c}$) = $\mathbf{c}$.

Claim lower bound (CLD): For all (B, $\mathbf{c}$) $\in \mathcal{D}$,
B $\geq \sum_{i \in N} c_i$ implies F(B, $\mathbf{c}$) $\geq \mathbf{c}$.

Claim upper bound (CUB): For all (B, $\mathbf{c}$) $\in \mathcal{D}$,
B $\leq \sum_{i \in N} c_i$ implies F(B, $\mathbf{c}$) $\leq \mathbf{c}$.

Maximum and minimum aspiration (MMA): Both CLD and CUB hold.

Continuity (CONT): For all $\mathbf{c} \in \mathbb{R}^N$ with (B$'$, $\mathbf{c}$) $\in \mathcal{D}$ for some B$'$,
B $\mapsto$F(B, $\mathbf{c}$) is continuous at some $B_0 \in \mathbb{R}$.

Symmetry (SYM): For all (B, $\mathbf{c}$) $\in \mathcal{D}$, and all i, j $\in$ N,
$c_i = c_j$ implies F_i(B, $\mathbf{c}$) = F_j(B, $\mathbf{c}$).

Net Surplus (NS): For all (B, $\mathbf{c}$), (B$'$, $\mathbf{c}'$) $\in \mathcal{D}$,
B -$\sum_{i \in N} c_i$ = B$'$-$\sum_{i \in N} c_i'$ implies F(B, $\mathbf{c}$) - $\mathbf{c}$ = F(B$'$, $\mathbf{c}'$) - $\mathbf{c}'$.

If a solution satisfies FAD, then agents are indifferent between solving two problems separately and solving the sum of the problems. BM introduced this axiom in discussing budget distribution problems. Clearly, a similar interpretation is applicable to PAD. Moulin (1987) and Chun (1988) introduced additivity with respect to budget given an arbitrary claim vector in discussing surplus sharing problems. The axioms RES says that each agent is indifferent between solving a problem directly, and receiving her claims and then solving the problem with her claims deleted. The axiom COM requires that all agents should receive their claims if the sum of the claims is equal to the budget. HMV introduced these two axioms. It is easy to see that FAD and COM together imply RES. In an earlier draft, these authors also discussed MMA. But, BM used this axiom for the axiomatizations of the rights-weighted solution and the rights-egaliaritan solution. Clearly, MMA implies CLD and CUB. Either CLD or CUB implies COM. The converse does not hold. For this point, see example 1 in the next section. NS says that the net payment F(B, $\mathbf{c}$) - $\mathbf{c}$ depends only on net surplus B -$\sum_{i \in N} c_i$.

3. Results

BM proved the following result.

Proposition 1(BM): A solution on $\mathcal{A}$ satisfies FAD and MMA if and only if it is a rights-weighted solution.

We prove a sharper result. Our proof is based on the functional equation approach and simpler than that of BM´s.

Proposition 2: A solution F on $\mathcal{A}$ satisfies FAD and at least one of CLB and CUB if and only if it is a rights-weighted solution.

Proof: It is easy to prove that a rights-weighted solution on $\mathcal{A}$ satisfies FAD and both CLB and CUB. Conversely, let F be a solution on $\mathcal{A}$ satisfying FAD and at least one of CLB and CUB. First, by FAD, F(B, $\mathbf{c}$) = F(B - $\sum_{i\in N} c_i$, $\mathbf{0}$) + F($\sum_{i\in N} c_i$, $\mathbf{c}$). Let ϕ_i(x) = F_i (x, $\mathbf{0}$), where i $\in$N. Clearly, the function ϕ_i inherits FAD from F. If CLB holds, ϕ_i(x) $\geq$0 for all x $\geq$0. If CUB holds, ϕ_i(x)$\leq$0 for all x $\leq$ 0. Therefore, by Fact 5 in Appendix , there exists λ_i such that ϕ_i(x) = λ_ix for all x $\in \mathbb{R}$. If CLB holds, λ_i =ϕ_i(1)$\geq$ 0. If CUB holds, λ_i =-ϕ_i(-1)$\geq$ 0. Thus, $\lambda_i \geq 0$.

Let $\boldsymbol{\lambda} = (\lambda_1,...,\lambda_n)$ and let $\beta(\mathbf{c}) = (\beta_1(\mathbf{c}),...,\beta_n(\mathbf{c}))$ = F($\sum_{i\in N} c_i$, $\mathbf{c}$) - $\mathbf{c}$. Then F(B, $\mathbf{c}$) = (B -$\sum_{i\in N} c_i)\boldsymbol{\lambda}$ + $\mathbf{c}$ +$\beta(\mathbf{c})$ for all (B, $\mathbf{c}$) $\in \mathcal{A}$. Note that if CLB holds, $\lambda_i \geq 0$, $\beta(\mathbf{c}) \geq$0, and $\sum_{i\in N} \lambda_i = \sum_{i\in N} F_i(1,\mathbf{0}) = 1$.

If CUB holds, $\lambda_i \geq 0$, $\beta(\mathbf{c}) \leq$0, and$\sum_{i\in N} \lambda_i = \sum_{i\in N} -\phi_i(-1) =$
$\sum_{i\in N} -F_i(-1,\mathbf{0}) = 1$.
Finally, $\sum_{i\in N} \beta_i(\mathbf{c}) = \sum_{i\in N} F_i(\sum_{i\in N} c_i,\mathbf{c}) - \sum_{i\in N} c_i. =$
$\sum_{i\in N} c_i - \sum_{i\in N} c_i. = 0$.
Hence $\beta(\mathbf{c}) = 0$ if at least one of CLB and CUB holds. ■

What happens if we replace either one of CLB and CUB by COM in Proposition 2? To answer this question, we start with the following example.

Example 1: Let f:$\mathbb{R} \to \mathbb{R}$ be an additive function whose graph {(x, y) | x $\in \mathbb{R}$, y = f(x)}, is dense in $\mathbb{R}^2$ (see Appendix). Define the function F on $\mathcal{A}$ by F_j(B, $\mathbf{c}$) = f(B -$\sum_{i\in N} c_i$) + c_j, for every j = 1,..., n - 1, F_n(B, $\mathbf{c}$) = -(n - 1)f(B - $\sum_{i\in N} c_i$) + c_n +B - $\sum_{i\in N} c_i$. It is easy to check that F satisfies FAD. Since f (0) = 0, F satisfies COM. We show that F violates both CLB and CUB. Since the graph of f is dense in $\mathbb{R}^2$, there exist x > 0 and y < 0 such that f(x) < 0 and f(y) > 0. Let (B, $\mathbf{c}$) and (B´, $\mathbf{c}$´) be such that B - $\sum_{i\in N} c_i$ = x and B´- $\sum_{i\in N} c_i$ ´= y. Then, B - $\sum_{i\in N} c_i > 0$ but F_j(B, $\mathbf{c}$) = f(B - $\sum_{i\in N} c_i$) + $c_j < c_j$. Also B´- $\sum_{i\in N} c_i$ ´< 0 but F_j(B, $\mathbf{c}$) = f(B´- $\sum_{i\in N} c_i$ ´)+ $c_j > c_j$. Thus, there exists a non-linear solution that satisfies FAD and COM.

There are distinct features of the solution in Example 1. First, the solution is discontinuous everywhere. Second, it violates SYM. These observations naturally lead us to the following questions. What happens if we require CONT in addition to FAD and COM? What happens if we require SYM in addition to FAD and COM? We answer these questions in the following two propositions.

Proposition 3: A solution on $\mathcal{A}$ satisfies FAD , COM, and CONT if and only if it is a pseudo rights-weighted solution.

Proof: Define the functions $\phi_i(\cdot)$, $\beta(\cdot)$ and the vector $\boldsymbol{\lambda}$ as in the proof of Proposition 2. By CONT, the function $\phi_i(\cdot)$ is bounded on some non-degenerate interval. Thus, by Fact 5 in Appendix $\phi_i(\cdot)$ is linear on $\mathbb{R}$. However, we can no longer conclude that $\boldsymbol{\lambda}$ is nonnegative. But, thanks to COM, we still have $\sum_{i\in N} \beta_i(\mathbf{c}) = \sum_{i\in N} F_i(\sum_{i\in N} c_i,\mathbf{c}) - \sum_{i\in N} c_i. = \sum_{i\in N} c_i - \sum_{i\in N} c_i. = 0.$■

Proposition 4: A solution on $\mathcal{A}$ satisfies FAD, COM, and SYM if and only if F is a rights-egalitarian solution.

Proof: It is clear that a rights-egalitarian solution satisfies FAD, COM, and SYM. Conversely, F be a solution on $\mathcal{A}$ satisfying FAD, COM, and SYM. First, by FAD, F(B, $\mathbf{c}$) = F(B -$\sum_{i\in N} c_i$, $\mathbf{0}$) + F($\sum_{i\in N} c_i$, $\mathbf{c}$). By SYM , F_j(B -$\sum_{i\in N} c_i$, $\mathbf{0}$) =(B -$\sum_{i\in N} c_i$)/n for all j $\in N$.

By COM, F($\sum_{i\in N} c_i$, $\mathbf{c}$) = $\mathbf{c}$. ■

Remark : Proposition 4 looks very similar to Proposition 2 in HMV. In Proposition 4, FAD takes the place of composition in Proposition 2 in HMV.

Corollary to Proposition 3(BM): A solution on $\mathcal{A}$ satisfies FAD, MMA, and SYM if and only if F is the rights-egalitarian solution.

Proof: Since MMA implies COM, the conclusion immediately follows from Proposition 4. ■

The functional equation approach easily handles restricted domains such as $\mathcal{S}$ and $\mathcal{B}$.

Proposition 4: A solution on $\mathcal{S}$ satisfies FAD and CLB(resp. CONT) if and only if it is a (resp. pseudo) rights-weighted solution.

Proof: Very similar to that of Proposition 1. To take nonnegativity constraints into account, we need Fact 6 in Appendix. ■

Proposition 5: A solution on $\mathcal{B}$ satisfies FAD and CUB(resp. CONT) if and only if it is a (resp. pseudo) rights-weighted solution.

Proof: Very similar to that of Proposition 1. Use Facts 5 and 6 in Appendix. ■

Now we replace FAD by PAD in Propositions 2.

Proposition 6: A solution on $\mathcal{A}$ satisfies PAD, RES, and at least one of CUB and CLB if and only if it is a rights-weighted solution.

Proof: By RES, F(B, $\mathbf{c}$) = $\mathbf{c}$ + F(B -$\sum_{i\in N} c_i$, $\mathbf{0}$). For each i $\in$N, let ϕ_i(x) = F_i(x, $\mathbf{0}$). The rest of the proof is the same as that of Proposition 2. ■

Proposition 7: A solution on $\mathcal{A}$ satisfies PAD, RES, and CONT if and only if it is a pseudo rights-weighted solution.

Proof: By RES, F(B, $\mathbf{c}$) = $\mathbf{c}$ + F(B -$\sum_{i\in N} c_i$, $\mathbf{0}$).

Proposition 8: A solution on $\mathcal{S}$ satisfies PAD, RES and CLB if and only if it is a rights-weighted solution.

Proof: Very similar to that of Proposition 6. To take nonnegativity constraints into account, we need Fact 6 in Appendix. ■

Proposition 9: A solution on $\mathcal{B}$ satisfies PAD, RES and CUB if and only if it is a rights-weighted solution.

Proof: Very similar to that of Proposition 6. Use Facts 5 and 6 in Appendix. ■

In the foregoing argument, we pointed out that composition is a close substitute for FAD. Is there any other? The answer is yes, as shown by the next two propositions.

Proposition 10: If a solution on $\mathcal{A}$ satisfies FAD and COM, then it satisfies NS.

Proof: Let (B, **c**), (B´, **c**´) $\in \mathcal{A}$ be such that B -$\sum_{i \in N} c_i$ =B´-$\sum_{i \in N} c_i$´. By FAD and COM, F(B, **c**) = F(B, **c**) + F(B - B´, **c** - **c**´)= F(B, **c**)+ F($\sum_{i \in N} c_i - \sum_{i \in N} c_i$´, **c** - **c**´)= F(B, **c**) + (**c** -**c**´). Thus, F(B, **c**) - **c** = F(B, **c**´) - **c**´. ■

Proposition 11: A solution on $\mathcal{A}$ satisfies NS and SYM if and only if it is the rights-egalitarian solution.

Proof: Let (B, **c**) $\in \mathcal{A}$. By NS, F(B, **c**) - **c** = F(B, **c**´) - **c**´. By SYM, F_j(B -$\sum_{i \in N} c_i$, **0**) =(B -$\sum_{i \in N} c_i$)/n for all j $\in N$. This completes the proof. ■

Appendix. Facts on Cauchy´s Equation

In this Appendix, we collect useful facts on Cauchy´s equation. For details, see Chapter 2 in Aczél and Dhombres (henceforth AD).

Fact 1 (Hamel): There exists a subset H of $\mathbb{R}$ such that every real number x can be represented as x = $r_1 h_1 + ... + r_n h_n$, where $h_i \in$ H, $r_i \in \mathbb{Q}$ (i = 1,...,n).Also, the representation is unique up to 0 coefficients.

A subset of $\mathbb{R}$ satisfying the condition in Fact 1 is called a **Hamel basis** and the formula in Fact 1 is called the **Hamel expansion of x**. The existence of a Hamel basis follows from the axiom of choice.

Fact 2 (Theorem 2.2.10 in AD): Let H be a Hamel basis. For each h $\in$ H, choose an arbitrary value of f(h). For each real number x, define f(x) via the Hamel expansion of x: f(x) = $r_1 f(h_1) + ... + r_n f(h_n)$.Then, the funcion f: $\mathbb{R} \to \mathbb{R}$ is additive.

Fact 3 (Corollary 2 . 2 . 13 in AD): Any subset of $\mathbb{R}$ of positive Lebesgue measure contains a Hamel basis. The Cantor set (which has zero Lebesgue measure) also contains a Hamel basis.

Fact 4 (Theorem 2.1.8 in AD): If an additive function f : $\mathbb{R} \to \mathbb{R}$ is not linear on $\mathbb{R}$, its graph is dense in $\mathbb{R}^2$.

Fact 5 (Theorem 2. 1. 8 in AD): An additive function f: $\mathbb{R} \to \mathbb{R}$ is linear on $\mathbb{R}$ if and only if it is bounded from above (or below) on a set of positive Lebesgue measure.

The following fact is extremely important when we consider restricted domains such as the class of surplus sharing problems and that of bankruptcy problems.

Fact 6 (Theorem 2.1.1 in AD): An additive function f: $\mathbb{R}_+ \to \mathbb{R}$ can be uniquely extended to all of $\mathbb{R}$, with additivity preserved.

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