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Interdependent Utility Functions in an Intergenerational Context

Tomoichi Shinotsuka

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Department of Economics
Otaru University of Commerce

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Department of Economics
Otaru University of Commerce
3-5-21, Midori, Otaru 047-8501 Japan
email: shino@res.otaru-uc.ac.jp

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Since Ray (1987) posed a question of representing nonpaternalistic functions (namely aggregators) in paternalistic form, Bergstrom (1999) identified sufficient conditions for a given list of linear aggregators to possess a unique list of utility functions (over consumption allocations) as the representations of the aggregators. Hori (2001) considered the representation problem for the case of a finite number of agents with possibly nonlinear aggregators. The model in this paper is a natural extension of Hori's (2001) to the case of countably many generations. As in Hori (2001), the aggregators in this paper may differ across generations and depend possibly on the utility levels of all other generations. We discuss two approaches to deal with infinite horizon. The first one explores monotonicity structures inherent in nonpaternalistic altruism. By means of lattice-theoretic arguments alone, we establish the existence of representations of nonpaternalistic functions in paternalistic form. A somewhat surprising feature of the lattice-theoretic approach is that the existence result is obtained without requiring that the degree of altruism is "small". The second approach uses the requirement of small degree of altruism in terms of "uniformly small Fréche derivative"(with respect to the utility level of other generations). We regard this approach as a natural extension of Hori's (2001) as we see later. Along the way, we discuss the case of linear aggregators. Our treatment in this paper is a little different from Bergstrom (1999) in that we view the infinite matrix as a representation of a continuous linear operator on l_∞. JEL Classification Numbers: D11, D64

1. Introduction

To analyze intertemporal resource allocation problems, the notions of intergenerational altruism have been playing a central role, especially in the recent literature. [2] Researchers in this field identified two models to make intergenerational altruism operational. To borrow terminologies from Ray (1987), the paternalistic model, on the one hand, incorporates intergenerational altruism into the utility function of each generation as a function of consumption allocations among all generations. On the other hand, the nonpaternalistic model captures intergenerational altruism by means of aggregators which relate the utility level of each generation to the utility levels of other generations as well as one's own consumption. The idea of nonpaternalistic altruism was formulated by Becker (1974) in the context of altruism among family members. In intergenerational contexts, the same idea was employed by Barro (1974), Kimball (1987), Ray (1987), Hori and Kanaya (1989), Hori (1992), and Hori (2000).

Ray (1987, 113-114) addressed the following question concerning the two approaches.

> The representation of nonpaternalistic functions in paternalistic form has also been the object of limited attention;.... But a systematic analysis of the relationship between these two frameworks is yet to be written, and appears to be quite a challenge, especially for models with an infinite horizon.

Several interesting results have been delivered ever since. Bergstrom (1999) identified sufficient conditions for a given list of linear aggregators to possess a unique list of utiliti functions (over consumption allocations) as the representations of the aggregators. Hori (2001) considered the representation problem for the case of a finite number of agents with possibly nonlinear aggregators. The model in this paper is a natural extension of Hori's (2001) to the case of countably many generations. As in Hori (2001), the aggregators in this paper may differ across generations and depend possibly on the utility levels of all other generations.

We discuss two approaches to deal with infinite horizon. The first one explores monotonicity structures inherent in nonpaternalistic altruism. By means of lattice-theoretic arguments alone, we establish the existence of representations of nonpaternalistic functions (namely aggregators) in paternalistic form. A somewhat surprising feature of the lattice-theoretic approach is that the existence result is obtained without requiring that the degree of altruism is "small". The requirement is well-known since Becker (1974) discussed the problem of "infinite regress". The second approach uses the requirement of small degree of altruism in terms of "uniformly small Fréche derivative"(with respect to the

[2] For prominet examples, the reader is referred to the references in Ray (1987) and Hori and Kanaya (1989).

utility level of other generations). We regard this approach as a natural extension of Hori's (2001) as we see later. Along the way, we discuss the case of linear aggregators. As Bergstrom (1999) showed, a certain infinite matrix with a dominant diagonal expresses the idea of small degree of altruism in this case and it offers a powerful tool to represent nonpaternalistic functions in paternalistic form (linearly and uniquely). Our treatment in this paper is a little different from Bergstrom (1999) in that we view the infinite matrix as a representation of a continuous linear operator on l_∞(the set of bounded utility allocations of all generations) into itself while Bergstrom viewed it as a certain limit of finite dimensional square matrices.

The rest of this paper is organized as follows. In the next section, we present the model. In section 3, we discuss the lattice-theoretic approach to the representation problem. In section 4, we discuss linear aggregators. In the last section, we prove the existence and uniqueness of representation.

2. The Model

For simplicity, we assume that there is one consumer for each generation. The integers t=1, 2, ...denote generations. For each t, $X_t = \mathbf{R}^l_+$ denotes the consumption set of generation t. [3] Let $X=\prod_{t=1}^{\infty} X_t$. For each t, let U_t be a nonempty subset of R^∞. A generic element $u_{-t} = (u_1,\ldots, u_{t-1}, u_{t+1}, \ldots) \in U_t$ signifies a profile of utilities other than generation t.

For each t, a real-valued function G_t on $X_t \times U_t$ is given. We call it the **aggregator for generation t**. Let $G=(G_1, G_2, \ldots)$ be the profile of the aggregators.

Representation Problem (RP): Given the profile G of aggregators, find a profile $u=(u_1, u_2, \ldots)$ of real-valued functions on X such that for each $x \in X$ and t, $u_t(x) = G_t(x, u_{-t}(x))$, where $u_{-t}(\cdot)$ denotes the profile with the t-th component $u_t(\cdot)$ deleted, $u_t(x)$ is strictly increasing in x_t and weakly increasing in $x_{-t} =(x_1,\ldots, x_{t-1}, x_{t+1}, \ldots)$. If RP has a solution $u = (u_1, u_2,\ldots)$, we call it a **representation of $G = (G_1, G_2,\ldots)$**.We call the t-th component $u_t(\cdot)$ of the representation u the **utility function of generation t** Two questions immediately arise.

Question 1: Does G have a representation?

Question 2: Is the representation unique?

[3] This specification of X_t is not essential. Technically,

3. The Lattice-Theoretic Approach to the Representation Problem

In this section, we assume the following on the aggregators.

Pointwise Boundedness (PB): For each t and $x_t \in X_t$, $\{G_t(x_t, u_{-t})$: $u_{-t} \in U_t\}$ is bounded.

Monotonicity(MON): For each t, $G_t(x_t, u_{-t})$ is strictly increasing in x_t and weakly increasing in x_{-t} and u_{-t}.

Now, we present the first main result.

Theorem 1: Under PB and MON, there exists a representation of a given profile of aggregators.

Proof. By **PB**, we can define the following real-valued functions. For each t and $x = (x_1, x_2, ...) \in X$, let $\alpha_t(x) = \inf\{G_t(x_t, u_{-t}): u_{-t} \in U_t\}$, $\beta_t(x) = \sup\{G_t(x_t, u_{-t}): u_{-t} \in U_t\}$. We consider the following function spaces. $\mathcal{U}_t = \{u_t(\cdot) \mid u_t(\cdot)$is weakly increasing and for each $x \in X$, $\alpha_t(x) \leq u(x) \leq \beta_t(x)\}$. The set $\mathcal{U}_t$ is non-empty since $\alpha_t(\cdot)$ and $\beta_t(\cdot)$ belong to it. For example, the function $u(\cdot)$ defined by $u(x) = \alpha_t(x)$ belongs to $\mathcal{U}_t$. Let $\mathcal{U} = \prod_{t=1}^{\infty} \mathcal{U}_t$. We equip $\mathcal{U}$ with the natural order $\geq$, i. e.$u \geq v$ if $u_t(x) \geq v_t(x)$ for every x and t. For $u = (u_1, u_2, ...)$, $v = (v_1, v_2, ...) \in \mathcal{U}$, let $u \wedge v = \inf\{u, v\}$ and $u \vee v = \sup\{u, v\}$. Then, for each $x \in X$, $(u \wedge v)(x) = (\min\{u_1(x), v_1(x)\}, \min\{u_2(x), v_2(x)\},...)$ and $(u \vee v)(x) = (\max\{u_1(x), v_1(x)\}, \max\{u_2(x), v_2(x)\},...)$. These operations, $\wedge$ and $\vee$, make $\mathcal{U}$ a complete lattice, i. e. for every non-empty subset $\mathcal{T}$ of $\mathcal{U}$, $\inf\mathcal{T}$ and $\sup\mathcal{T}$ exist and belong to $\mathcal{U}$.

For each $u(\cdot) = (u_1(\cdot), u_2(\cdot),...) \in \mathcal{U}$ and t, let $F(u(\cdot))_t(x) = G_t(x, u_{-t}(x))$. Clearly, $F(u(\cdot))_t(x)$ is strictly increasing in x_t and weakly increasing in x_{-t}. It is also trivial that $F(u(\cdot))_t(\cdot) \in \mathcal{U}_t$. Hence, the operator F maps $\mathcal{U}$ into itself. Clearly, F(u) is weakly increasing in u. Hence, by Tarski's fixed point theorem (1955), there exists $u(\cdot) = (u_1(\cdot), u_2(\cdot),...) \in \mathcal{U}$ such that for every x and t, $u_t(x) = G_t(x, u_{-t}(x))$. By **MON**, u_t satisfies the desired monotonicity properties. ■

Example 1

To see how crucial **PB** is, let us consider the following system of aggregators $G = (G_1, G_2, G_3, ...)$: $G_1(x_1, u_{-1}) = p \cdot x_1 + \alpha u_2$, $G_2(x_2, u_{-2}) = p \cdot x_2 + \beta u_1$, $G_t(x_t, u_{-t}) = x_t$ $(t = 3, 4, ...)$, where p is an l-dimensional vector with strictly positive components, and α and β are positive constants satisfying $\alpha\beta > 1$. Clearly G satisfies **MON** but violates **PB**. Suppose G possesses a system of utility functions $u = (u_1, u_2, u_3, ...)$. Then, $u_1(x) = p \cdot x_1 + \alpha u_2(x)$ and $u_2(x) = p \cdot x_2 + \beta u_1(x)$ for all x. Hence, $u_1(x) = \frac{p \cdot x_1 + \alpha p \cdot x_2}{1 - \alpha\beta}$. Since $1 - \alpha\beta < 0$, $u_1(x)$ cannot be strictly increasing in own consumption x_1 (or weakly increasing in x_2 for that matter). A contradiction obtains. Therefore, there is no system of utility functions for the aggregators.

4. The Contraction Approach to RP

We would like to be more ambitious in this section. Our goal here is to obtain a profile of **product continuous** utility functions as a **unique representation** of a given profile of aggregators.

To this end, we add a few more assumptions on the aggregators. For simplicity, we put a restriction on the domains of the aggregators: For each t, U_t is equal to l_∞. For $u \in l_\infty$, $||u||_\infty$ denotes the sup norm of u. $\mathbf{1}$ denotes the constant sequence (1, 1,...). Note that the domain of the aggregator, $X_t \times U_t$ is a subset of R^∞. We equip $X_t \times U_t$ with the **relative product topology**. From now on, we refer it as the **product topology**.

Continuity(CONT): For each t, the aggregator G_t is product continuous.

Uniform Boundedness(UB): For every $\alpha \in \mathbf{R}$, $\sup_{x\in X} \sup_t |G_t(x_t, \alpha\mathbf{1})| < \infty$.

Lipschitz Condition(LC): There exists $\delta \in (0, 1)$ such that for every t, x, u_{-t} and u_{-t}', $|G_t(x, u_{-t}) - G_t(x, u_{-t}')| \leq \delta ||u_{-t} - u_{-t}'||_\infty$.

CONT is standard. **UB** may be weakened at the cost of elaborating the choice of relevant function spaces (Boyd (1990)), which we do not pursue in this paper. **LC** expresses the idea that the utility level of each generation does not depend too much on those of others.

Theorem 2: Under CONT, UB, and LC, there uniquely exists a representation of a given profile of aggregators.

Proof. We set up different function spaces from those in the previous section. Let $\mathcal{U} = \{u = (u_1, u_2, ...) |$ For each t, u_t is a product continuous, real-valued function on X, and $\sup_{x\in X} \sup_t |u_t(x)| < \infty\}$. For $u = (u_1, u_2, ...) \in \mathcal{U}$, let $||u||_\infty = \sup_{x\in X} \sup_t |u_t(x)|$. By the standard argument, $\mathcal{U}$ is a Banach space under the norm $||u||_\infty$.

Let $\mathcal{U}^{inc} = \{u = (u_1, u_2, ...) \in \mathcal{U} |$For each t, u_t is weakly increasing.$\}$. Clearly, $\mathcal{U}^{inc}$ is a closed subset of U so that it is a complete metric space.

Now, we define a operator T on $\mathcal{U}^{inc}$. For $u = (u_1, u_2, ...) \in \mathcal{U}^{inc}$ and $x \in X$, let $T(u)(x) = (G_1(x_1, u_{-1}(x)), G_2(x_2, u_{-2}(x)), ...)$, where $u_{-t}(x) = (u_1(x), u_2(x), ..., u_{t-1}(x), u_{t+1}(x), ...)$ for every t. To see that T maps $\mathcal{U}^{inc}$ into itself, for every $x \in X$, $u = (u_1, u_2, ...) \in \mathcal{U}^{inc}$, and t, $G_t(x_t, -||u||\mathbf{1}) \leq G_t(x_t, u_{-t}(x)) \leq G_t(x_t, ||u||\mathbf{1})$ by **MON**.

Thus, for every t, $|G_t(x_t, u_{-t}(x))| \leq \max\{\sup_{x\in X} \sup_\tau |G_\tau(x_\tau, ||u||\mathbf{1})|, \sup_{x\in X} \sup_\tau |G_\tau(x_\tau, -||u||\mathbf{1})|\}$. Thus, by **UB**, $\sup_{x\in X} \sup_t |G_t(x_t, u_{-t}(x))| < \infty$. Clearly, for every t, $G_t(x_t, u_{-t}(x))$ is weakly increasing in x and product continuous in x. Hence, T maps $\mathcal{U}^{inc}$ into itself.

By **LC**, T is a contraction. Hence, by the contraction mapping theorem, there exists a unique $u^* = (u_1^*, u_2^*, ...) \in \mathcal{U}^{inc}$ such that $u^* = T(u^*)$, i.e. for every $x \in X$ and t, $u_t(x) = G_t(x_t, u_{-t}(x))$. By **MON**, $u_t(x)$ is strictly increasing in x_t.

5. Linear Representation Problem

We call a real-valued, increasing function ν_t on X_t a **felicity function of generation t.** $\nu = (\nu_1, \nu_2, ...)$ denotes a profile of felicity functions. Let $\nu = (\nu_1, \nu_2, ...)$ be a profile of felicity functions and let $\{a_{tj}\}_{t=1j=1}^{\infty\infty}$ be a double sequence such that for each t and j, $a_{tj} \geq 0$ and a_{tt}=0, and $\{a_{tj}\}_{j=1}^{\infty}$ is summable. We say that the aggregator $G_t(\cdot, \cdot)$ is **linear** if it is of the form $G_t(x_t, U_{-t}) = \nu_t(x_t)+\sum_{j=1}^{\infty} a_{tj} U_j$.

Linear Representation Problem (LRP): Given a profile of linear aggregators, find a profile of utility functions.

Two immediate questions arise.

Question 3: Does LRP possess a solution?

Question 4: Is a solution to LRP unique?

To give a positive answer to each question, we propose a condition which generalizes Hori's (2001). To this end, let B be the infinite matrix defined by

$$\begin{bmatrix} 1 & -a_{12} & -a_{13} & \cdot & \cdot \\ -a_{21} & 1 & -a_{23} & \cdot & \cdot \\ -a_{31} & -a_{32} & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & & \end{bmatrix}$$

Let b_{ij} be the (i, j)-element of the matrix B, i. e. b_{ij} = 1 if i = j, and b_{ij} = $-a_{ij}$ otherwise. Let n be a positive integer and let I_1 = {1, 2, ..., n},..., I_k = {n(k−1)+1, n(k−1)+2, ...,n(k−1)+n} (k = 2, 3, ...). Then, the set $\{I_k\}_{k=1}^{\infty}$ partitions the set N of all positive integers. For every i and j ∈N, let B_{ij} be the sub-matrix $[b_{lm}]_{l\in I_i, m\in I_j}$.

Dominant Diagonal Blocks (DDB): The matrix B has a dominant diagonal blocks, i.e. there exists n ∈

N and a norm $\| \bullet \|$ on R^n such that for all i, B_{ii} satisfies the Hawkins-Simon condition, and there exists a norm $||\cdot||$ on $\mathbf{R}^l$ such that $\sup_i \sup_{x\in \mathbf{R}^n:||x||=1} ||B_{ii}^{-1}x|| < \infty$ and $\sup_i \sup_{x\in \mathbf{R}^n:||x||=1} \Sigma_{j\neq i}^{\infty} ||B_{ii}^{-1} B_{ij} x|| < 1$.

DDB means that off-diagonal blocks are small in terms of some norm. However, its intuitive content is somewhat obscure. Therefore, it could be helpful to consider a special case of DDB with n = 1: In this case,all the diagonal blocks B_{i_i} degenerate into 1×1 matrix, 1.

Dominant Diagonal (DD): $\sup_t \sum_{j\neq t}^{\infty} a_{tj}$ <1.

The series $\Sigma_{j\neq t}^{\infty} a_{t_j}$ may be regarded as the degree of intergenerational altruism. Then, **DD** clearly expresses the idea that the degree of intergenerational altruism is small.

To see the relevance of **DDB**, let us look at the system of simultaneous equations:

$U_t = G_t(x_t, U_{-t}) = \nu_t(x_t)+\sum_{j=1}^{\infty} a_{tj} U_j$ (t=1,2,...).

We search for a bounded sequence U = (U_1, U_2,...) that solves the simultaneous equation. This immediately raises a question of invertibility of the continuous linear operator $T: l_\infty \rightarrow l_\infty$ represented by the infinite matrix B.

Let $I: l_\infty \to l_\infty$ be the identity operator. By DDB, $||T - I|| < 1$. Hence, T is invertible and $T^{-1} = \sum_{j=0}^{\infty}(I - T)^n$. See Lang(1969, Ch.5), for example. The last formula shows the inverse operator T is represented by a non-negative infinite matrix. See Araujo and Scheinkman (1979, Lemma 4.1) for detail. Thus, by DDB, LRP has the unique solution:

$$U = T^{-1}\nu(x)$$

Now, we discuss diagonal dominance introduced by Bergstrom (1999).

Bergstrom Dominant Diagonal (BDD): There exists a bounded sequence $d = (d_1, d_2, \ldots)$ such that for all t, $d_t > 0$, and $\inf_t (d_t - \sum_{j=1}^{\infty} a_{t_j} d_j) > 0$.

Suppose that the infinite matrix B satisfies BDD. Then, the continuous linear operator $T: l_\infty \to l_\infty$ represented by the infinite matrix B is invertible. The infinite matrix representing the inverse operator of T is of the following form: $DC^{-1}D^{-1}$, where $D = \mathrm{diag}(d_1, d_2, \ldots)$, $C = (c_{t_j})$, $c_{t_j} = (a_t d_j)/d_t$. Note that the existence of the inverse matrix of C follows from $||C - I|| < 1$, where I denotes the identity matrix and $||\bullet||$ the sup-norm.

6. Link between the Contraction Approach and DDB

In this section, we consider the logical implications of differentiable aggregators. To be more specific, we extend Hori's result (2001) by means of the contraction approach.

Smoothness(S):For each t and x_t , $G_t(x_t, u_{-t})$ is continuously Fréchet differentiable with respect to u_{-t}.

Let $D_{u_{-t}}G_t(x_t, u_{-t})$ be the derivative of $D_{u_{-t}}G_t(x_t, u_{-t})$ with respect to u_{-t}. Note that $D_{u_{-t}}G_t(x_t, u_{-t})$ is a sup norm continuous, linear functional on l_∞. By **MON**, it is nonnegative. By definition of the dual norm, $||D_{u_{-t}}G_t(x_t, u_{-t})||=\sup_{h\in l_\infty:||h||_\infty=1}|D_{u_{-t}}G_t(x_t, u_{-t})(h)|$. Since $D_{u_{-t}}G_t(x_t, u_{-t})$ is nonnegative, $||D_{u_{-t}}G_t(x_t, u_{-t})||$can be written as $D_{u_{-t}}G_t(x_t, u_{-t})(\mathbf{1})$. To see the link between the contraction approach in the previous section and the condition developed by Hori (2001), it is useful to consider the following condition.

Limited Utility Dependence (LUD): $\sup_{u\in\mathcal{U}^{inc}}\sup_t\sup_{x\in X}||D_{u_{-t}}G_t(x_t, u_{-t}(x))||<1$.

By the mean value theorem (see Lang (1969, Ch.5) for example), it is easy to see that **LUD** implies **LC**. **LUD** expresses the same intuition as **LC** by means of Fréchet derivatives.

In order to see the link between our results and Hori's(2001), we need to invoke the Yosida-Hewitt decomposition theorem: $D_{u_{-t}}G_t(x_t, u_{-t})$ can be expressed as

$D_{u_{-t}}G_t(x_t, u_{-t})(h)= \sum_{j\neq t}^{\infty} p_{tj}(x_t, u_{-t})h_j + \lambda_t(x_t, u_{-t})(h)$ for every $h\in l_\infty$, where $\{p_{tj}(x_t, u_{-t})\}_{j\neq t}^{\infty}$ is an absolutely summable, nonnegative sequence and $\lambda_t(x_t, u_{-t})$ is a purely finitely additive, nonnegative linear functional on l_∞.

Let $j_0 \neq t$, and let $e^{j_0} = \{e_j^{j_0}\}_{j\neq t}^{\infty}$ be the sequence defined by $e_{j_0}^{j_0} =1$ and $e_j^{j_0}=0$ for $j\neq t, j_0$. Then, $D_{u_{-t}}G_t(x_t, u_{-t})(e^{j_0})= p_{tj_0}(x_t, u_{-t})$. Since $D_{u_{-t}}G_t(x_t, u_{-t})(e^{j_0})$ is the partial derivative of $G_t(x_t, u_{-t})$ with respect to u_{j_0},denoted by $G_{tj_0}(x_t, u_{-t})$, $\{G_{tj}(x_t, u_{-t})\}_{j\neq t}^{\infty}$ is absolutely summable and nonnegative.

Let $a_{tj} = \sup_{u\in\mathcal{U}^{inc}}\sup_{x\in X}G_{tj}(x_t, u_{-t})(t \neq j)$. Clearly, $\sup_{u\in\mathcal{U}^{inc}}\sup_t\sup_{x\in X}\{\sum_{j\neq t}^{\infty} G_{tj}(x_t, u_{-t})\}\leq \sup_t \sum_{j\neq t}^{\infty}a_{tj}$. Now, let us consider the following two conditions.

Representability(REP): For each t, $x\in X$, and u_{-t}, the Fréchet derivative $D_{u_{-t}}G_t(x_t, u_{-t})$ can be represented as a summable sequence.

Uniformly Dominant Diagonal Blocks(UDDB): There exists a nonnegative infinite matrix $A =[a_{tj}]_{t=1j=1}^{\infty\infty}$ such that for each t, j, and (x_t, u_{-t}), $a_{tt} = 0$, $a_{tj} \geq \partial G_t(x_t, u_{-t})/\partial u_j$ and that the infinite matrix I-A satisfies DDB.

It follows from the above discussions that **UDDB**, along with **REP**, imply **LUD**. This explains why **UDDB**, the analogue of Hori's condition ((4.1) in Hori(2001)), is useful in obtaining the unique solution to the utility aggregation problem in an intergenerational set-up.

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Center for Business Creation, Otaru University of Commerce
3-5-21, Midori, Otaru, Hokkaido 047-8501, Japan　Tel +81-134-27-5290　Fax +81-134-27-5293
E-mail:cbc@office.otaru-uc.ac.jp

小樽商科大学ビジネス創造センター
〒047-8501 北海道小樽市緑３丁目５番２１号　Tel 0134-27-5290　Fax 0134-27-5293
E-mail:cbc@office.otaru-uc.ac.jp