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**Competition between Matching Markets**

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# COMPETITION BETWEEN MATCHING MARKETS

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The present paper investigates the possibility of domination of mediate matching over independent search. It is shown to be difficult although not impossible. The domination can happen in the case of binary utility, which is consistent with the data and history that mediated match has been dominated in simple jobs.

## 1. INTRODUCTION

Along with historically high rate of unemployment in Japan, resolution of mismatch between supply and demand of labour is increasingly demanded. Along with this trend, the number of private firms which mediate matching in the labour market is increasing. This paper examines whether there is a room for a mediated matching market to improve matching efficiency. For it to be possible, not only the mediated matching market exceeds the unmediated matching market in terms of matching efficiency, but also it must provide higher expected utility than unmediated search so that unemployed workers and firms with unfilled vacancies choose the mediated matching market. We show that the room for mediated matching to emerge is limited although not impossible. For it to be possible, the quality of jobs and workers must be sufficiently homogeneous and also the benefit to match must be high enough compared to the benefit of being single. In most of cases, unmediated matching dominates mediated matching.

Section 2 describes the assumptions used in the paper. Section 3 shows how an unmediated matching market is described by a stable marriage problem. Section 4 describes how match-makers whose objectives are to maximise the number of matches accomplish their objectives. Section 5 overviews large number results of these markets. Section 6 studies selection of markets through competition. Section 7 overviews data on the actual matching market in Japan. It shows that unmediated search dominates in the market of job-seekers with working experience, but the share of mediated search significantly increases in the matching market of high school graduates and that of simple jobs. Section 8 concludes.

## 2. MATCHING SESSION

We call job-seekers and firms with job vacancies *searching agents* collectively. They seek for each other to form a pair to undertake production. The quality of a match differs by pairs reflecting the heterogeneity of agents. It is bilaterally measured by utility of each agent as a total evaluation of characteristics a given pair brings about, such as productivity, wages and working conditions. Formally, utility of a job-seeker and a job vacancy is simultaneously drawn from a probability distribution on  $\mathbb{R} \times \mathbb{R}$  for all possible pairs between two groups. The distributions of any pairs are all independent and the identically distributed. The i.i.d. assumption implies that there is no ranking between agents *ex ante*. We assume that there is positive correlation of utility within a given pair. This is because monetary transfer between agents in the form of wages is available. If productivity of a worker when he is engaged in a particular job is sufficiently high but he is reluctant to work, then the firm may want to offer higher wages to compensate his reluctance. On the other hand, if a worker is eager to work but his productivity is low, then he may offer to accept lower wages. Therefore, the original utility profile is transformed through monetary transfer into another utility profile so that bilateral utility is levelled and the number of potential pairs which satisfy individual rationality of agents increases. The utility of staying unmatched is deterministically given. In general, it is a function of unemployment benefit, matching probability and conditional expected utility upon match in the next period, and discount rate. The lottery of utility is drawn when a matching session begins, but will not be disclosed until a given pair meets each other.

A matching session opens periodically. This is the only place where a searching agent can find a partner. In the matching session, there are various types of matching market and a searching agent can belong to only one of them. Search activities in each market requires devotion of agents and it prohibits them from participating in multiple markets. Matching markets are largely categorised into two types: one is a market which offers only a “place” to search and each agent must search for a partner by himself, and the other is a market in which a matchmaker acts as a proxy for agents. We call the former type of search *independent search* and the latter *mediated matching*. In the independent search, there may or may not be an organiser. An independent search market with an organiser may be able to gather more searching agents via advertisement, but autonomous search without an organiser is always left

as a choice in hands of a searching agent. Similarly, mediated matching is also an open market for matchmakers. A matchmaker does not let searching agents meet by themselves. He arranges which agents to meet. The choice of agents to allow meeting *is* the matching technology. Potentially, any matchmaker with any matching technology can join in a matching session. However, since the matching technology critically depends on shutdown of information shared by searching agents in the market, the implementability of matching technology depends on how well a matchmaker act as a proxy of a matching agent. We assume that a matchmaker can find out the preference order of an agent, for example asking “which is better” questions without disclosing the name of candidates. However, he cannot find out the *level* of an agent’s utility. Therefore, a matchmaker must rely on a matching algorithm which uses only a preference order. As a degenerate case, it can bring the same outcome as the independent search following the algorithm to find stable matchings shown in the next section. However, we focus on a case in which a matchmaker brings a different outcome. Another candidate of objectives of a matchmaker is maximisation of the number of matches. We will show that it is achievable only by using the preference order. We call a *maximum matching* the matching in which number of matches is maximised and a *maximum matching algorithm* the algorithm to maximise it. In short run, a matchmaker indeed has an incentive to maximise the number of matches, because he can use its past records for advertisement (and as will be shown, gathering more searching agents is critical for him). However, in long run, his viability depends on whether he can provide higher expected utility than other matching markets because, as we are now doing, superiority of his matching algorithm can be judged by an experiment. If his survivability is shown, then it means that growth of those matchmakers can raise matching efficiency of the economy.

A searching agent chooses a matching market when a matching session starts, based on expectation of utility each market brings. After a choice of a matching market, he is matched according to the rule (matching technology) the matchmaker uses if it is a mediated matching market. If he chooses independent search, the matching outcome will be one of stable equilibria as will be shown in the next section. After observing a final matching outcome, a searching agent has an option to reject it. It is obvious for independent search, but also a matchmaker cannot force a searching agent to accept someone unacceptable. The standard to choose a

matching market is expected utility it brings to a searching agent. It is affected by two factors: the matching probability and the conditional expected utility upon match. We assumed that a matchmaker maximises the number of matches. However, such an attempt may result in deterioration of the conditional expected utility upon match, and the overall expected utility may not rise. If it is true for the maximum matching algorithm, only independent search propagates in the economy. On the other hand, if there is such a technology, emergence of matching monopoly is possible. As will be seen later, the matching probability and, as a result, the expected utility are both an increasing function of the size of a matching market.

We assumed that utility within a pair is positively correlated. This assumption can be justified as follows. Crawford and Knoer (1981) showed a natural extension of a stable matching equilibrium when monetary transfer is available, called a *bargaining equilibrium*, in which monetary transfer is determined so that the total surplus over threats of two agents is equally divided into two. The threat equals the utility the agent receives in a reduced matching market in which the counterpart of a given pair becomes indifferent to being single. Namely, it is the utility he receives from the partner he will be matched, once a given pair becomes unavailable. Since the reduced matching market requires the threats of each agent to be determined again, we have to trace back recursively to a reduced matching market in which all agents are indifferent to being single. In each recursive market, multiple equilibrium is possible. Different beliefs on equilibrium in a recursive market bring different level of wages. The bargaining equilibrium is the stable matching equilibrium when utility is transferred between two sides of agents according to the bargaining procedure described above. However, the bargaining procedure is not peculiar to the stable matching process. If insulation of information leads to an unstable matching as an equilibrium, utility is transferred in exactly the same way as bargaining equilibrium. Suppose that there are two sets of agents  $M$  and  $W$ . Denote by  $u_i(j)$  the payoff of agent  $i \in M$  (or  $\in F$ ) when he/she is matched with  $j \in W$  (or  $\in M$ ) under the condition that monetary transfer is not available. Since the matrix of  $u$  completely characterises the matching market, we use the function  $u : M \times W \cup W \times M \rightarrow \mathbb{R}$  to denote the market itself. We denote the payoff of agent  $i$  to remain single by  $c_i$ . Then, the payoff of agent  $i$  when matched with  $j$

in market  $u$ , denoted by  $\tilde{u}(j; u)$  is given by the rule

$$\tilde{u}_i(j; u) = \max \left\{ c_i, \min \left[ u_i(j) + u_j(i) - c_j, \frac{u_i(j) + u_j(i) + t_i(j; u) - t_j(i; u)}{2} \right] \right\}$$

where  $t_i(j; u)$  and  $t_j(i; u)$  are the threats of players  $i$  and  $j$  in market  $u$ . If matched, the total utility of the two over the sum of threats is symmetrically divided and added on each threat. The rule guarantees individual rationality and that the payoff does not exceed the amount the given pair can collectively achieve however the threat of agent  $i$  relative to agent  $j$  is large. The threats are determined by constructing a new market  $u'$  so that

$$\begin{aligned} u'_i(j) &= c_i \\ u'_j(i) &= c_j \\ u'_k(l) &= u_k(l) \quad (k, l \text{ otherwise}) \end{aligned}$$

Let  $(f', \tilde{u}')$  be a bargaining equilibrium of market  $u'$ . Then

$$\begin{aligned} t_i(j; u) &= \tilde{u}'_i \\ t_j(i; u) &= \tilde{u}'_j \end{aligned}$$

To determine  $\tilde{u}'_i$  and  $\tilde{u}'_j$ , threats in market  $u'$  must be fixed. It is done by repeating the above operation recursively until the market reduces to the one in which all elements except one pair are replaced by the utility of being single.

**Proposition 1.** *If the elements of  $u$  are independently distributed, for given  $i \in M$  and  $j \in W$ ,  $\tilde{u}_i(j)$  and  $\tilde{u}_j(i)$  are positively correlated.*

*Proof.* Let's denote by  $u^{(\sigma)}$  a reduced matching game constructed from  $u$  in which elements listed in  $\sigma$  and its transpose are substituted by the payoff of being single, where

$\sigma = \{(i, j) \in M \times W \mid u_i(j) = c_i \text{ and } u_j(i) = c_j\}$ . Also, denote the payoff of  $i$  when matched with  $j$  in market  $u^{(\sigma)}$  by  $\tilde{u}_i^{(\sigma)}(j)$ . If  $\sigma = \{(1, 2)\}$ , then it is set that  $u_1(2) = c_1$  and  $u_2(1) = c_2$  in the market  $u$ . Also note that  $u^{(\emptyset)} = u$ . Any path of recursive reduction of the game is represented by an increasing sequence of  $\sigma$ . If a reduced game represented by  $u^{(\sigma')}$  is constructed from a reduced game  $u^{(\sigma)}$ , then  $\sigma' \supset \sigma$ . The recursive reduction stops when  $\sigma$  lists all the

elements in the lower triangle of  $u$ . Take any reduced game  $u^{(\sigma)}$  and consider a match between  $m \in M$  and  $n \in W$ . Then,

$$(1) \quad \tilde{u}_m^{(\sigma)}(n) = \begin{cases} \frac{U_{mn} + t_m(n) - t_n(m)}{2} & \text{if } -(U_{mn} - 2c_m) \leq t_m(n) - t_n(m) \leq U_{mn} - 2c_n \\ c_m & \text{otherwise} \end{cases}$$

where  $U_{mn} := u_m(n) + u_n(m)$  and  $t_m(n)$  is the equilibrium payoff of agent  $m$  in the game  $u^{(\sigma')}$  where  $\sigma' = \sigma + (m, n)$ . Note that if  $t_m(n) - t_n(m) > U_{mn} - 2c_n$ ,  $n$  declines to match and  $m$  is forced to stay single. We denote by function  $f^{(\sigma')} : M \cup W \rightarrow M \cup W$  the allocation in the bargaining equilibrium of the game  $u^{(\sigma')}$ . It has a property that for  $\forall m \in M$ ,  $f^{(\sigma')}(m) \in W + \{m\}$ ,  $\forall w \in W$ ,  $f^{(\sigma')}(w) \in M + \{w\}$  and  $f^{(\sigma')} \circ (f^{(\sigma')})^{-1}$  is an identity mapping. If the allocation in game  $u^{(\sigma')}$  is such that  $f^{(\sigma')}(m) = s$  ( $s \neq m$ ) and  $f^{(\sigma')}(n) = r$  ( $r \neq n$ ), then  $t_m(n) = \tilde{u}_m^{(\sigma')}(s)$  and  $t_n(m) = \tilde{u}_n^{(\sigma')}(r)$ . If  $s = m$ ,  $t_m(n) = c_m$ . If  $r = n$ ,  $t_n(m) = c_n$ . Since  $(m, n) \in \sigma'$  and  $\forall \sigma'' \supset \sigma'$ ,  $(m, n) \in \sigma''$ ,  $t_m(n)$  and  $t_n(m)$  are *not* a function of  $u_m(n)$  and  $u_n(m)$ . Since the elements of  $u$  are mutually independent,  $u_m(n)$ ,  $u_n(m)$  and  $t_m(n) - t_n(m)$  are mutually independent.

Coming back to the first layer of recursive operation and applying equation (1) recursively, the above independence result implies

$$\tilde{u}_i(j) = \begin{cases} \frac{U_{ij}}{2} + \frac{T_{is} - T_{jr}}{4} & \text{if } -(U_{ij} - 2c_i) \leq t_i(j) - t_j(i) \leq U_{ij} - 2c_j \\ c_i & \text{otherwise} \end{cases}$$

where

$$T_{xy} := \begin{cases} U_{xy} + t_x(y) - t_y(x) & \text{if } -(U_{xy} - 2c_x) \leq t_x(y) - t_y(x) \leq U_{xy} - 2c_y \\ c_x & \text{otherwise} \end{cases}$$

Similar for  $\tilde{u}_j(i)$ . Therefore, for any cases in the above equation, conditional covariance of  $\tilde{u}_i(j)$  and  $\tilde{u}_j(i)$  can be written

$$\text{Cov}(\tilde{u}_i(j), \tilde{u}_j(i)) = \frac{\text{Var}(U_{ij})}{4} + \frac{\text{Var}(T_{is}) + \text{Var}(T_{jr})}{16} \geq 0$$

where we assume  $\text{Var}(T_{is}) = 0$  if  $T_{is}$  degenerates to  $c_i$  for the purpose of concise exposition. The last inequality holds with strict inequality if either  $\tilde{u}_i(j)$  or  $\tilde{u}_j(i)$  does not degenerate to  $c_i$  or  $c_j$ . Therefore, unconditional covariance is  $\text{Cov}(\tilde{u}_i(j), \tilde{u}_j(i)) > 0$  for all  $i \in M$  and  $j \in W$ .  $\square$

In the experiments undertaken below, we directly generate  $\tilde{u}$  randomly so that it has positive correlation within any pairs and calculate matching outcomes as if utility is not transferable. This is equivalent to generating  $\tilde{u}$  in which the bargaining over wages is already embedded, and doing matching experiment for the first layer of the recursive matching process of the Crawford and Knoer's bargaining equilibrium. Note that if there are  $n$  agents for each side of the market, direct calculation of a bargaining equilibrium must solve  $n^2!$  reduced markets (if  $n = 50$ , it is more than  $1.6 \times 10^{7411}(!)$ ) for a particular belief on multiple equilibria, which requires gigantic calculation power.

### 3. INDEPENDENT SEARCH

Major characteristics of independent search is that it has no organiser in the market. We assume perfect information so that once searching agents in this market start searching, profile of each agent is perfectly revealed. Based on the disclosed information, an agent forms a preference over all candidates in the market in the form of utility. He also has a preference over staying single. He has a right to leave the market without being matched. Naturally, all agents seeks for a best partner, but the interests of each agent mutually conflict. The best candidate he pursues may have one of his competitors as her best. Once that person accepts her, she becomes infeasible for him. Namely, his choice is limited to the *feasible* candidates who will not decline once he proposes to them. Being single is always a feasible choice. Gale and Shapley (1962) showed that there exists at least one equilibrium in which every agent proposes to and actually be accepted by the best feasible candidate (including the status of being single). Other candidates who are ranked above the partner in equilibrium in his preference list are all *infeasible* for him in the sense that they are matched with more preferable partner in equilibrium. Therefore, the equilibrium has a property that they have no incentive to change from the equilibrium partner. In this sense, such an equilibrium is called a *stable matching*. In general, stable matching has multiple equilibria and they are partially ranked by either group of matching agents. In



our case, two groups are a group of workers and that of firms. There exists a stable matching in which all workers agree as the most preferable one among all stable matchings, which we call a *worker-optimal stable matching*. However, this is the worst outcome among all the stable solutions for firms. Similarly, there exists a *firm-optimal stable matching*. Again, it is the worst stable matching for workers. Between these extreme stable matchings,<sup>1</sup> all other stable outcomes are ordered in lattice structure.

The optimal stable equilibria can be found by an algorithm used in the constructive proof of the existence of stable equilibria by Gale and Shapley (1962). To find a worker-optimal stable matching, we make workers advance to firms and let firms wait for a propose from a worker. The algorithm is divided into two operations *proposal* and *refusal*. At first, all firms are engaged with a dummy worker who they unanimously dislike most. In the operation *proposal*, a worker proposes to a firm who is at the top of his preference among the firms which haven't refused him yet and calls a operation *refusal* by the firm proposed. However, the dummy worker never proposes since it is known that he is never accepted by any firms. The dummy worker is used in the algorithm to judge if the first layer of the recursive structure has finished or not. In the operation *refusal*, the firm proposed must choose either to engage with the person newly proposed or to refuse him and keep the current "fiance". Subsequently, the refused worker out of the two enters in the operation *proposal*. As far as a firm replaces the fiance with the newcomer, the process enters the deeper layer of the recursive structure of the algorithm.<sup>2</sup> Obviously, by swapping the role of workers and firms in the algorithm, we can obtain the firm-optimal stable equilibrium. To find all other stable equilibria, we introduce a *breakmarriage* operation. It breaks exogenously a pair already matched in equilibrium as if the worker in the pair is refused by the firm. Then, he has to search for another firm from a set of firms which is ranked below the current partner.

In a stable matching problem, there remains ambiguity concerning the revelation of preference of agents. In any stable matchings, there must exist at least one agent who has an incentive not to reveal his preference truthfully. Therefore, there is difficulty in specifying the matching pattern in equilibrium. Fortunately, the *number* of matches is not affected by truth revelation.

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<sup>1</sup>They may degenerate to the same unique stable matching.

<sup>2</sup>This recursive algorithm is an improved version of the Gale-Shapley algorithm proposed by McVitie and Wilson (1971). The basic reasoning stays the same as the Gale-Shapley algorithm.

Roth (1984) showed that the outcome brought by misrepresentation of preference is also a stable matching and the set of agents which stay unmatched remains the same. The difference in expected utility conditional upon match, which is brought by different preference revelation, may affect the expected utility of choosing independent search. However, no searching agent have information about other searching agents prior to entrance to the independent search market. A searching agent does not know what kind of stable matchings are possible, how many stable matchings can exist, what kind of preference revelation strategy he should take, so on. Therefore, we simply assume that searching agents anticipate that all stable matchings emerge with the same probability.

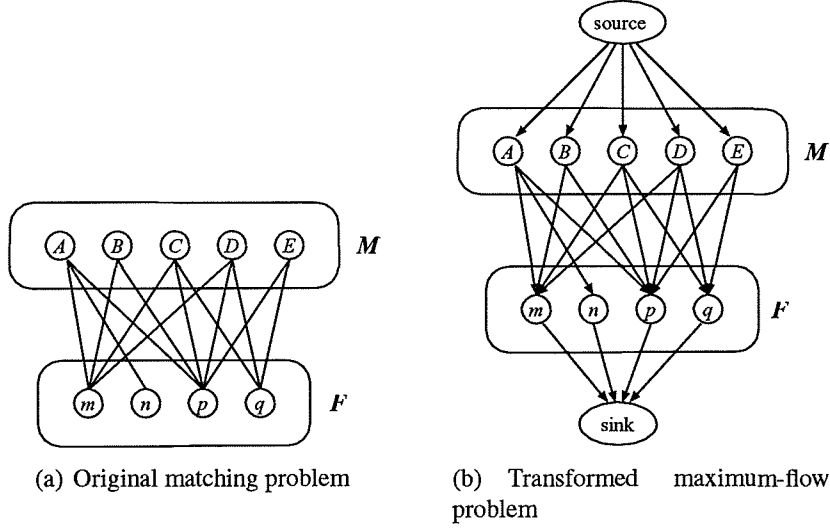
#### 4. MAXIMUM MATCHING MARKET

We are seeking different matching outcome for a matchmaker from a stable matching. It implies that a maximum matching is an unstable matching. Therefore, shutdown of information traffics between searching agents is inevitable. If there is someone who envies another matched pair and the target agrees to elope, the whole matching may break down.

In the maximum matching market, a matchmaker only needs to know a pairwise preference of each searching agent between staying single and a candidate. That is, whether a candidate is acceptable or not. Then, he draws a graph, denoting each agent by a dot, or a vertex, and connects two agents in different groups who are mutually acceptable by an edge. The problem for the matchmaker is to find a maximum matching on this graph. Since a pair connected with an edge represents a pair mutually acceptable, such a matching is enforceable. Formally, we denote a set of vertexes by  $V$ . In a general graph, edges can be drawn between any two vertexes and denoted by a pair of the starting vertex and the end vertex such as  $v_1v_2$  where  $v_1, v_2 \in V$ . Therefore, the set of edges  $E$  is  $E \subset V \times V$ . A graph  $G$  is defined as  $G = (V, E)$ .  $V$  and  $E$  are sometimes written as  $V(G)$  and  $E(G)$  to emphasise that they are a set of vertexes or edges of graph  $G$ . If  $V' \subset V$  and  $E' \subset E$ , then  $G' := (V', E')$  is called a subgraph of  $G$ . We can use the following theorem by Berge to find a maximum matching, which we show without a proof.

**Theorem 1. (Berge)** *A matching  $C$  of a graph  $G$  is the maximum matching if and only if there exists no  $C$ -increasing path in  $G$ .*

FIGURE 1. Transformation of the problem



A *path* is a subgraph  $H \subset G$  in which  $V(H) = \{v_1, v_2, v_3, \dots, v_n\}$  and  $E(H) = \{v_1v_2, v_2v_3, v_3v_4, \dots, v_{n-1}v_n\}$ .  $v_1$  and  $v_n$  are called *end-points*. If the path is such that either  $v_i v_{i+1} \in C$  for  $i \equiv 0 \pmod{2}$  and  $v_i v_{i+1} \notin C$  for  $i \equiv 1 \pmod{2}$ , or  $v_i v_{i+1} \notin C$  for  $i \equiv 0 \pmod{2}$  and  $v_i v_{i+1} \in C$  for  $i \equiv 1 \pmod{2}$ , then  $H$  is called *C-alternate path*. If the end-points are not saturated by  $C$ , i.e.  $v_1, v_n \notin C$ , the *C-alternate path* is called *C-increasing path*. Indeed, if there exists a *C-increasing path*, one can increase the number of matches by one by swapping the edges which belongs to  $C$  and those not.

We use this fact to find a maximum matching. We search for a *C-increasing path* for all possible edges. If we find one, then we can increase the number of matches by one by swapping  $C$  and  $\bar{C}$ . Then, we continue to search for another *C-increasing path* for new  $C$ . If we find out that there is no more *C-increasing path*, then  $C$  is one of maximum matching. However, this algorithm is a little complicated for implementation. Therefore, we transform a maximum matching problem into a maximum flow problem by adding the source upstream of all vertexes in  $M$  (or  $F$ ) and the sink downstream of all vertexes in  $F$  (or  $M$ ) (Figure 1; See Sedgewick (1988)). It gives an efficient algorithm to find the maximum-flow. The algorithm used here is equivalent to finding out a new *C-increasing path* recursively until there cannot find any more such a path. We regard Figure 1(b) as a network of water pipes. Each pipe has the same capacity of water current. Also each vertex has the same capacity to pass water through as a pipe. At the upstream end of each pipe, there is a stop cock. We show an example of the

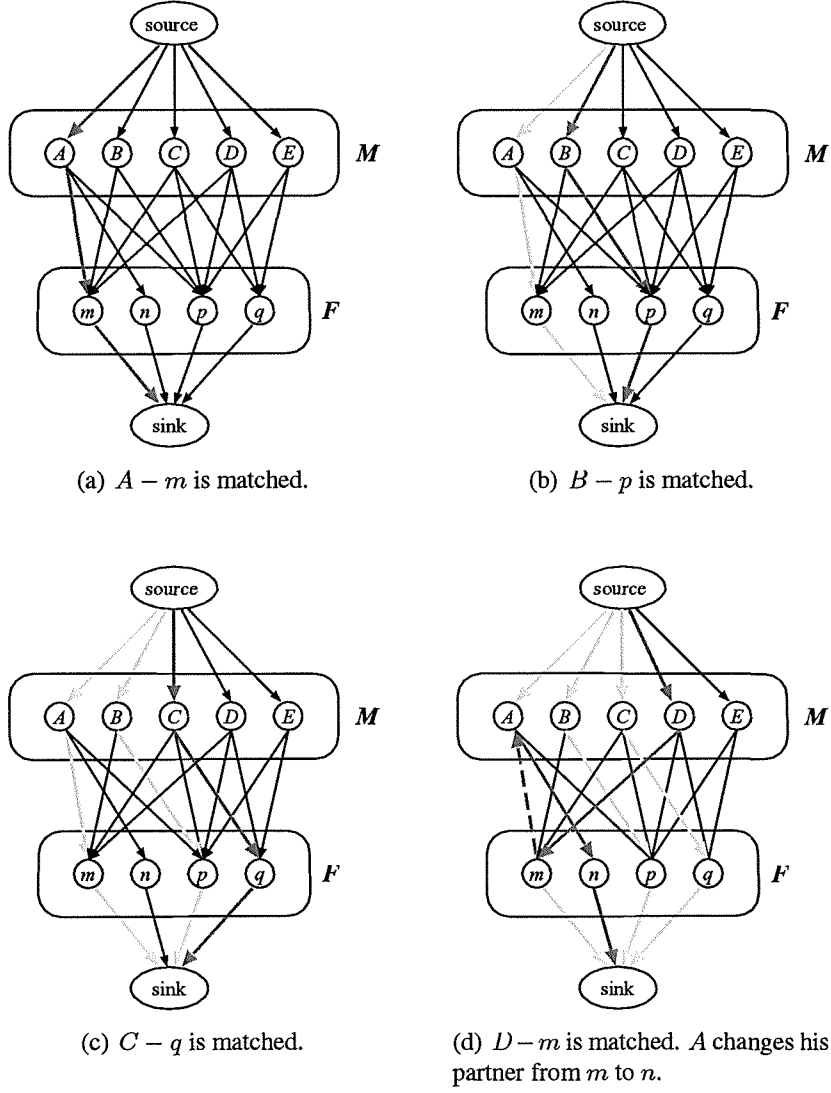
algorithm in Figure 2. There are the same number of stopcocks as the number of vertexes in  $M$ . We open those stopcocks in order as far as it leads water to the sink (if not, even if you open the cock, water does not flow). We first open the cock at the source to  $A$  (Figure 2(a)). From  $A$ , there are two pipes available, to  $m$  and  $p$ . We arbitrarily choose to open the cock to  $m$ . By opening the cock from  $m$  to the sink, we made one unit of water current flow from the source to the sink. Next, we open the cock at the source to  $B$ , knowing that the path (source) –  $B$  –  $p$  – (sink) adds another unit of water current (Figure 2(b)). Beginning from the cock to  $C$ , we can find a path (source) –  $C$  –  $q$  – (sink) (Figure 2(c)). In Figure 2(d), we open a cock to  $D$ . There are three pipes from  $D$ , to  $m$ ,  $p$  and  $q$ , and all of them are already occupied by other water currents. However, we can still open the cock to  $D$ . We open the cock  $D \rightarrow m$ . It does not make additional flow yet since vertex  $m$  and the pipe  $m \rightarrow$  (sink) is already at capacity. We additional open cocks  $A \rightarrow n$ . Then, it makes (source) –  $D$  –  $m$  – (sink) begin to flow, since water which flows  $A$  escapes to  $n$  instead of  $m$ . Therefore, we found four units of water current flow from the source to the sink. Note that this algorithm finds out only one of all possible matching coverings.

**Proposition 2.** *There exists at least one matching which contains all vertexes of degree one which achieves the maximum matching.*

*Proof.* Any paths which includes the edge that is incident to a vertex of degree one must have the vertex as an end point. If the edge is covered by a matching  $C$ , there is no  $C$ -increasing path which includes this vertex. By Theorem 1, such a recursive operation to find a  $C$ -increasing path ultimately leads to a maximum matching. Therefore, this vertex remains matched until the maximum matching is reached.  $\square$

This proposition implies that we can start the matching algorithm by beginning from the “saving lonely people first” principle. By starting to match all agents who have only one candidate, we can construct a maximum matching. It shows the ‘egalitarian’ property of the maximum matching algorithm.

FIGURE 2. Algorithm to find maximum-flow



## 5. LARGE-NUMBER RESULTS

For the two algorithm, stable matching and maximum matching, large number results are known. A result for a stable matching is obtained by Dagsvik (2000) for a case that utility of agents are drawn from a special case of an extreme value distribution.

**Theorem 2. (Dagsvik)** *Let  $\varepsilon_i$  ( $i = 1, 2, 3, 4$ ) be almost surely positive i.i.d. random variable with cumulative distribution function*

$$\exp(-1/x) \quad (x > 0).$$

Let the utility of an agent in  $M$  when he meets an agent in  $W$  be  $a\varepsilon_1$  where  $a > 0$ , and let his utility when he stays single be  $\varepsilon_2$ . Similarly, the utility of an agent in  $W$  when she meets an agent in  $M$  is  $b\varepsilon_3$  where  $b > 0$ , and her utility when she stays single is  $\varepsilon_4$ .

Then, the asymptotic number of realised match is given by

$$X = \frac{1}{2} \left[ \frac{1}{ab} + |M| + |W| - \sqrt{\left( \frac{1}{ab} + |M| + |W| \right)^2 - 4|M||W|} \right].$$

*Proof.* See Theorem 2 and Corollary 1 in Dagsvik (2000). □

If we keep  $|W| = k|M|$  ( $k > 0$ ) and let  $|M|, |W| \rightarrow \infty$ , then  $X/\min\{M, W\} \rightarrow 1$ . So, asymptotically, it matching all agents in the smaller group.

A theorem by Bollobás and Thomason (1985) tells us that the number of matches converges to smaller number of two groups as the size of groups becomes large in the maximum matching market. We define a probability space on a set of graphs of given order  $n$ , and we call it a random graph. Assume that each edge is drawn with fixed probability  $p(n)$ ,<sup>3</sup> and we denote the set of such graphs by  $\mathcal{G}(n; p)$ .

**Theorem 3. (Bollobás and Thomason)** *Let*

$$(2) \quad p = \frac{\log n + 2 \log \log n + \omega(n)}{2n}$$

where  $\omega(n) \rightarrow \infty$ . Then almost every  $\mathcal{G}(n; p)$  has a matching covering all but at most one of the vertexes of non-zero degree.

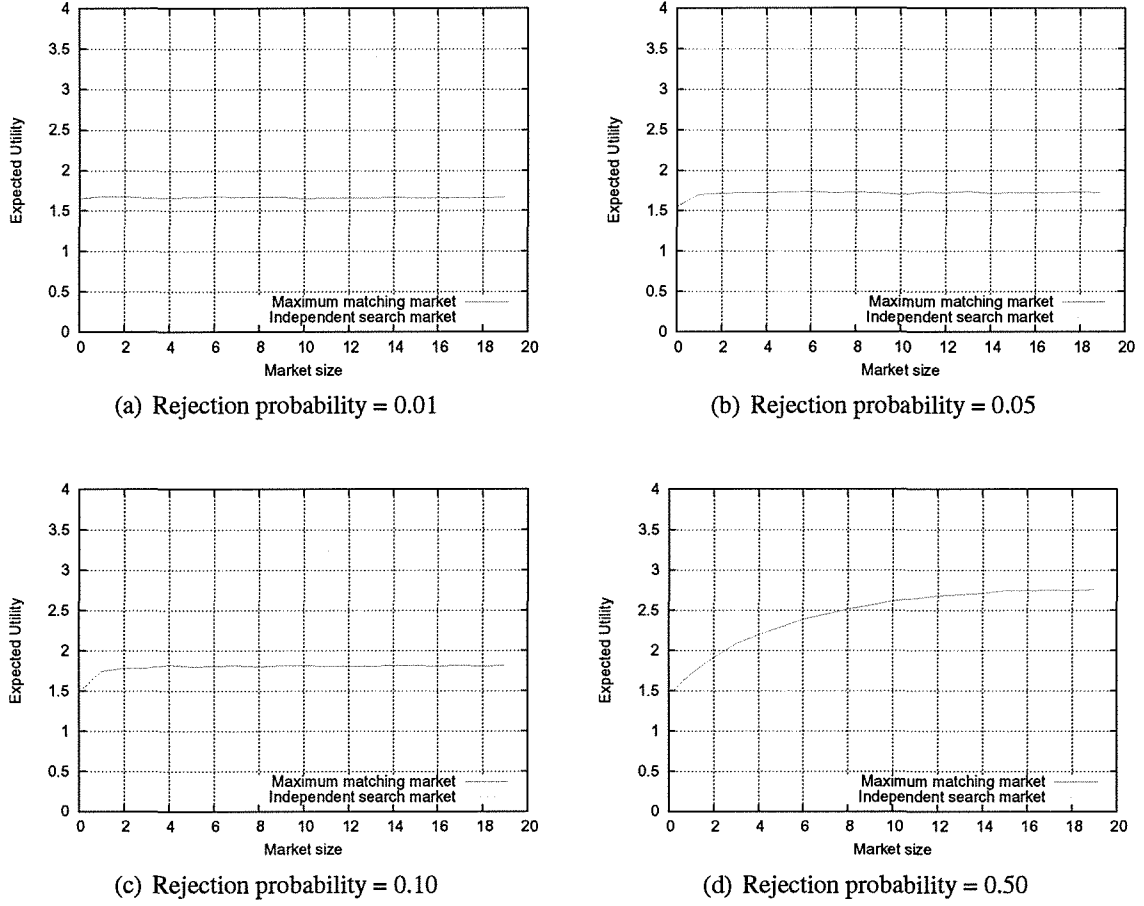
*Proof.* See Bollobás and Thomason (1985). □

In the theorem,  $p$  can be decreasing in  $n$  as far as it satisfies (2). From the threshold property of search models, we should assume that  $p(n) = p$  for all  $n$ . Thus, the condition  $\omega(n) \rightarrow \infty$  holds. It means that a maximum matching algorithm matches all agents in the smaller group as the size of market becomes large.

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<sup>3</sup>We allow  $p$  to be a function of the parameter  $n$ .

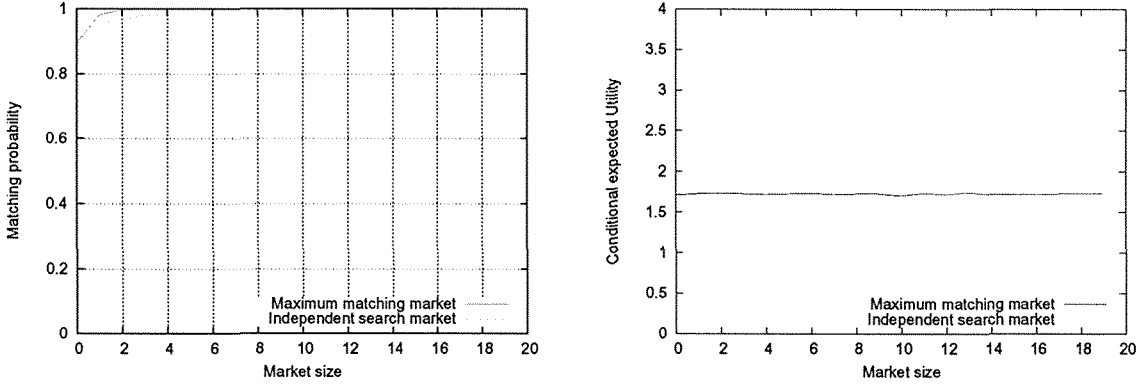
FIGURE 3. Expected utility of independent search and maximum matching



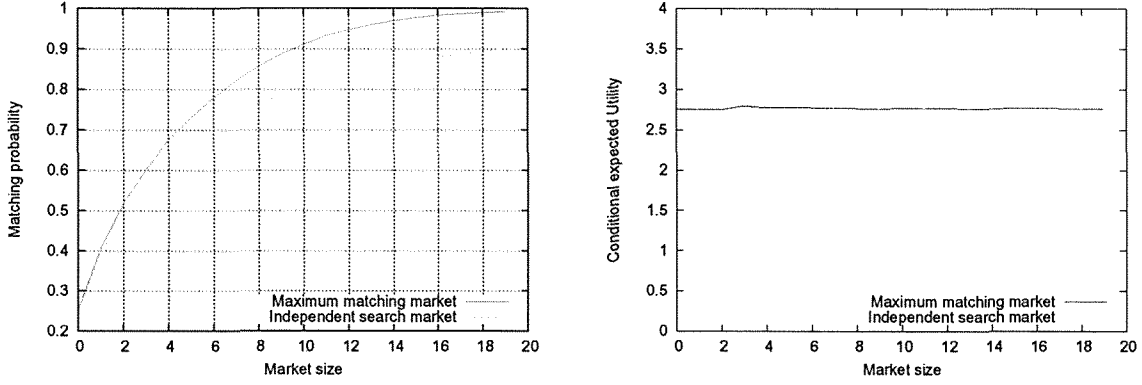
## 6. COMPETITION BETWEEN MATCHING MARKETS

The results by Dagsvik (2000) and Bollobás and Thomason (1985) imply that both independent search and maximum matching match all agents in the smaller group as the market size goes infinity. In this section, we obtain the small number results. By applying the algorithms argued in Section 3 and 4 to given number of agents who have particular utility profile, we obtain the number of matches and utility of each agent that independent search and maximum matching bring. Repeating this procedure for randomly generated utility profiles, we can estimate the expected number of matches and expected utility of each matching process for given number of agents. Figure 3 shows the result when utility is drawn from an *i.i.d.* standard log-normal distribution and the reservation utility of each agent is 0.0976517, 0.193041, 0.277606 and 1.00000 so that probability to reject is 1%, 5%, 10% and 50%, respectively. The cases in which worker-firm ratio is one are shown in the graph. Basic properties do not change for other

FIGURE 4. Decomposition of the expected utility



(a) Rejection probability = 0.05



(b) Rejection probability = 0.50

threshold values or other worker-firm ratios. As explained in Section 2, normally utility within a pair is positively correlated. The matching probability and expected utility shown here should be understood as a lower bound for each number of agents. When market size is one, the number of matches and expected utility of independent search is the same as maximum matching, since conflict of interests between agents does not matter in this obvious case. As the market size grows, the expected utility of independent search increases unboundedly whereas that of maximum matching becomes quickly bounded. Figure 4 shows matching probability and conditional expected utility on match. By definition, the maximum matching market provides the highest possible matching probability. The graph shows, as the large number results predict, the matching probability converges to one as the market size goes infinity for both markets. The small number result shows that the matching probability of independent search is fairly good compared to maximum matching. Also, its conditional expected utility on match increases unboundedly. This unboundedness comes from the assumption that the support of the density



function of utility is unbounded. Since an agent always matches the best feasible candidate in independent search, the number of feasible candidates tends to increase as the market size becomes larger. If utility is drawn from a probability distribution  $F(u)$ , the distribution of the best feasible candidate is  $F(u)^n$  where  $n$  is number of feasible candidates and a random variable. Since matching probability converges to one, the conditional probability upon match is also  $F(u)^n$  asymptotically. Under the current assumption that the support of utility is unbounded, the conditional expected utility upon match unboundedly increases as the market size increases. On the other hand, the conditional expected utility on match of the maximum matching market is constant regardless of the market size. Under the maximum matching algorithm, only information on whether there is an edge between agents or not is used and the preference order of candidates above the threshold is ignored. Therefore, it is natural that we should expect that, under any choices of matching covering, the utility level of the partner is randomly chosen.

Matching probability of the maximum matching market is always higher than the independent search market. On the other hand, conditional expected utility upon match is always higher in the independent search market. In general, we cannot tell in which market the *unconditional* expected utility is higher. Let us denote by  $p_m, p_i$  matching probability in the maximum matching market and the independent search market, respectively. Also, we denote conditional expected utility upon match by  $u_m$  and  $u_s$  in each market, and the threshold utility by  $c$ . Then, we have the following result.

**Proposition 3.** *The expected utility of a maximum matching market is greater than or equal to an independent search market if and only if*

$$(3) \quad p_m/p_s \geq (u_s - c)/(u_m - c)$$

*holds.*

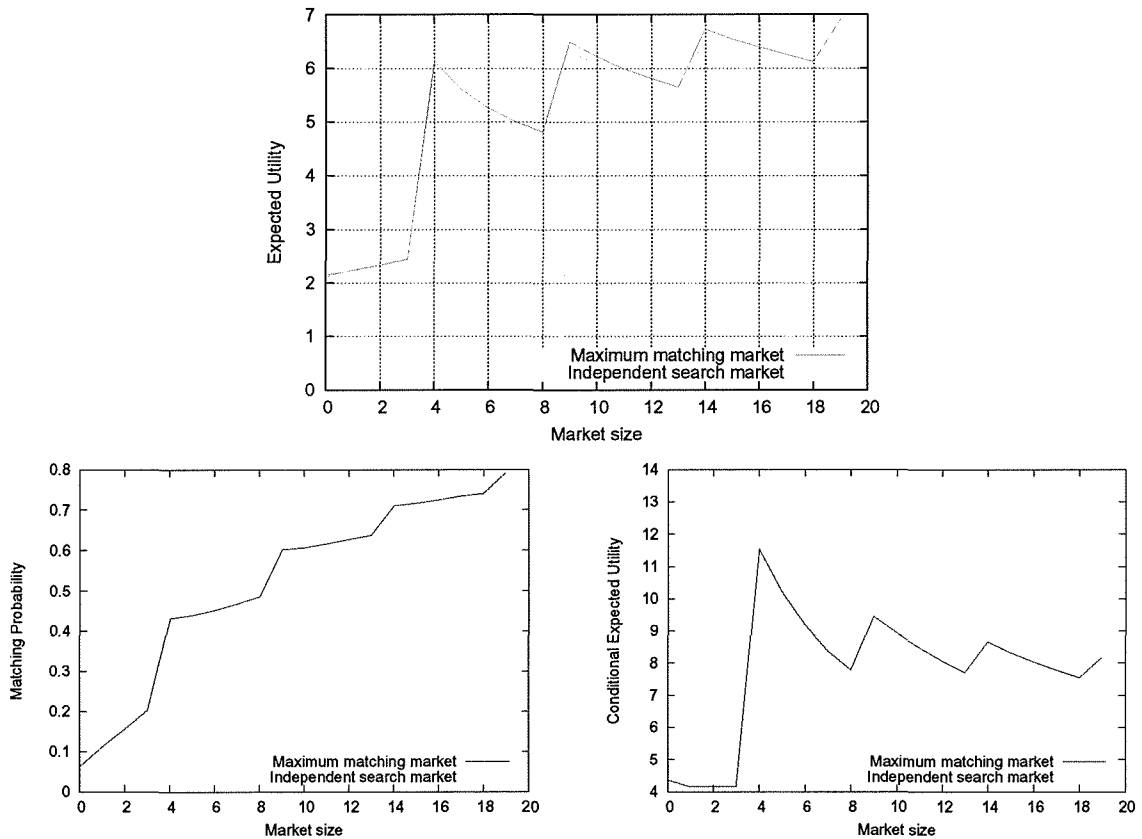
*Proof.* Immediate from  $p_m u_m + (1 - p_m) c \geq p_s u_s + (1 - p_s) c$  and  $u_i \geq c_i$  for  $\forall i \in \{m, s\}$ . □

It says that the increasing rate of matching probability brought by the shift from independent search to maximum matching must be larger than or equal to the increasing rate of the conditional expected rent upon match (in terms of utility) brought by the shift in the opposite

direction. In general, there are cases that this condition holds. However, it is unlikely for a maximum matching market to have larger expected utility than an independent search market for each market size when utility of agents is drawn from a class of  $s$  or log-normal distributions. Grid search for various parameters of these distributions never finds out such a case. Note that support of the density of a beta distribution is  $[0, 1]$ , so that conditional expected utility upon match becomes bounded also for the independent matching markets. For a market size in which maximum matching dominates independent search to exist, increase of  $u_s$  brought by expansion of the market size and, therefore, increase of feasible candidates must be limited for some range of the market size. A simple example in which maximum matching dominates independent search is the binary utility case. An agent bears utility of, say, one when he is matched and zero when unmatched. Then, the right-hand side of equation (3) is always one regardless the size of two markets. Since  $p_m > p_s$  for all market sizes, inequality of equation (3) holds. Another example is the case when utility is drawn from  $LN(3, 0.01)$  with probability 0.2 and from  $LN(0, 1)$  with probability 0.8 and the threshold of utility is 2. It can be interpreted as there are two states, “good” and “bad”. The good state comes with probability 0.2 and the mean of candidates is high. The bad state has lower mean of zero and it comes with probability 0.8. Note that the variance of probability distribution of “good” status is small. So, when an agent is in a “good” state, there is a discrete “jump” in utility and utility conditional upon the “good” state has small variance. Also note that, in this example, the corresponding graph is “sparse”, which means that probability to find a partner is low.

This result suggests that the class of  $s$  is too “smooth” for maximum matching to dominate independent search. Note that we compared expected utility of two markets for a given market size. It implicitly assumes that organisers and matchmakers have the same ability to gather searching agents. If there is a matchmaker who can attract more searching agents than any organisers of the independent search market, then maximum matching can dominate independent search. If it is the case, the matchmaker can raise enrolment fee which is equivalent to the rent of enrolling this market.

FIGURE 5. A matchmakers dominates independent search



## 7. MATCHMAKERS IN THE REAL WORLD

The fact that private matching institutions did not play a major role in the labour market, at least until recently, is partially reflection of world-wide agreement that profit-seeking institutions are not socially suitable for matching between workers and firms. Convention number 96 of ILO adopted in 1949 (which is revision of the original convention adopted in 1933) says, “fee-charging employment agencies<sup>4</sup> conducted with a view to profit ..... shall be abolished within a limited period of time determined by the competent authority” (article 3, 1.), “such agencies shall not be abolished until a public employment service is established” (article 3, 2.) and “fee-charging employment agencies not conducted with a view to profit ..... (a) shall be required to have an authorisation from the competent authority and shall be subject to the supervision of the said authority; (b) shall not make any charge in excess of the scale of charges submitted to and approved by the competent authority or fixed by the said authority, with strict

<sup>4</sup>Fee-charging employment agencies are defined as matching institutions between workers and firms which levies any kind of fees, whether or not they conduct with a view to profit, and they don't include newspapers or other publications unless they are not wholly or mainly for the purpose of acting as intermediaries (article 1).

regard to the expenses incurred; .....” (article 6). It was ratified by 38 countries and clearly intended to prohibit profit-making from matching activities. Although Japan denounced this convention, its policy was along with it.

However, growing consent about the importance of flexibility in the labour market had driven the revision of the convention to admit private employment agencies as an important part of the matching institutions in the labour market (ILO convention number 181; 1997). Thinking in the other way around, it can be said that such a regulation as ILO decision in 1949 was indeed *necessary* because otherwise profit-seeking matchmakers dominated independent search in some part of the labour market and exploited workers. History shows that there were rooms for matchmakers to exist and raise profit from mediation. Although it had been illegal, it was generally observed until recently that there were matchmakers who monopolise in gathering simple workforce in slums such as San-ya (Tokyo), Kamagasaki (Osaka), Kotobuki-cho (Yokohama) and Hakata-chikko (Fukuoka). Firms —typically, construction companies— sought young and powerful workforce and workers wanted a job where labour intensity is less harsh. A matchmaker who gathers good workforce was paid a good rake-off by firms. Since announced labour condition and wages are not always trustful (there were even cases that a matchmaker “arrests” a muscled worker he aimed at to the workplace), there was a good reason to restrict such private mediation. In this particular case, the matchmaker stand on firm’s side and attempt to misguide workers by giving wrong information.

History obviously shows the survivability of matchmakers, at least in some fields of jobs. Are they playing a certain role in the modern matching process as well? Because of the long-continued consensus seen in ILO’s convention 96 that profit should not be pursued in the labour matching, private matchmakers has not been major players in the labour market. However, there existed non-profit matchmakers. To define the range of matchmakers, we must note that a matchmaker is defined as an institute which strictly narrows the scope of matching candidates on behalf of matching agents. An institute which deputise for matching agents to collect information but does not narrow information of candidates is not regarded as a matchmaker in our context, since it brings the same outcome as independent search. Table 1 shows the media of matching that firms in Japan with more than 30 employees used to employ workers in year 2000. In our definition, media such as “newspaper ads”, “job ad magazines and cites” and

TABLE 1. Media of situations-vacant advertisement

## 1. Workers with working experience

Number of employees	30 – 99	100 – 299	300 – 999	1,000 – 4,999	5,000 or above	Total
PES	58.9%	63.2%	57.0%	51.1%	41.2%	59.6%
Meeting session held by PES	1.2%	2.8%	6.0%	6.6%	8.1%	2.1%
Meeting session held by own	0.2%	2.0%	4.6%	8.1%	19.0%	1.2%
Private mediators	2.4%	6.2%	11.4%	16.0%	28.5%	4.3%
Job ad magazines & cites	14.3%	22.0%	31.5%	42.1%	53.5%	18.1%
Newspaper ads	26.1%	39.8%	45.4%	44.6%	51.1%	31.3%
Firm's web page	2.1%	6.3%	15.7%	26.7%	43.3%	4.7%
Personal connection	23.0%	24.5%	17.9%	17.2%	15.1%	22.9%
Other	23.0%	20.0%	21.8%	26.6%	25.7%	22.3%

## 2. University graduates

Number of employees	30 – 99	100 – 299	300 – 999	1,000 – 4,999	5,000 or above	Total
PES	23.5%	17.1%	14.5%	9.0%	8.1%	18.6%
Meeting session held by PES	15.0%	12.7%	10.1%	7.8%	3.6%	12.8%
Meeting session held by own	19.0%	32.8%	50.7%	64.7%	73.9%	32.9%
Private mediators	0.0%	2.8%	4.6%	6.4%	3.9%	2.2%
Job ad magazines & cites	18.6%	24.0%	48.3%	70.8%	87.6%	29.8%
Newspaper ads	6.4%	7.7%	8.8%	11.7%	15.6%	7.7%
Firm's web page	10.8%	19.9%	40.6%	66.9%	87.9%	23.7%
Recommendation by school instructors	27.7%	46.8%	42.4%	47.1%	56.4%	38.4%
Recruiter	1.6%	2.7%	4.2%	10.3%	20.2%	3.2%
Personal connection	19.2%	17.3%	15.8%	14.8%	14.3%	17.6%
Other	13.8%	5.8%	7.5%	6.4%	4.9%	9.4%

## 3. High school graduates

Number of employees	30 – 99	100 – 299	300 – 999	1,000 – 4,999	5,000 or above	Total
PES	44.5%	42.2%	35.6%	28.5%	24.2%	42.2%
Meeting session held by PES	9.7%	8.4%	9.4%	10.5%	4.0%	9.3%
Meeting session held by own	4.1%	7.7%	17.5%	27.3%	25.5%	7.4%
Private mediators	0.0%	0.0%	0.5%	4.4%	3.4%	0.2%
Job ad magazines & cites	4.0%	7.6%	14.9%	28.2%	31.5%	7.2%
Newspaper ads	8.0%	5.7%	3.8%	6.3%	8.1%	6.9%
Firm's web page	3.0%	6.8%	14.0%	29.9%	32.2%	6.3%
Recommendation by school instructors	50.0%	57.3%	67.0%	68.7%	65.1%	54.5%
Recruiter	0.4%	1.1%	0.7%	2.3%	2.7%	0.7%
Personal connection	5.3%	4.9%	7.2%	3.6%	6.0%	5.3%
Other	1.8%	2.8%	4.0%	2.1%	2.7%	2.3%

**Note:** PES = The Public Employment Security Office. Sum of each choice may exceed 100 percent, since multiple answer is allowed.

**Source:** Ministry of Health, Labour and Welfare, *Survey on Employment Management (Koyô Kanri Chôsa)* (2001).

“firm’s web page” listed in Survey are excluded from matchmakers. The entries “recommendation by school instructors” and a part of “private mediators” are interpreted as matchmakers. “Private mediators” regarded as matchmakers depend on their strategy to match. If they adopt a passive strategy, for example, simply passing all information of candidates, they should be excluded from matchmakers. However, because of lack of precise statistics, we classify them as matchmakers. School teachers especially in junior high schools and high schools have strong incentive to make all the students find a job. We assume that they maximise the number of matches.<sup>5</sup> The public employment security office helps matching between job-seekers and jobs but since it does not intentionally narrow the scope of candidates, it should be also excluded from matchmakers when it acts independently. However, one of missions of PES is to cooperate with schools to provide maximum information of jobs. In this case, the school takes the initiative of matching arrangement. Therefore, we categorise PES as independent search for workers with working experience and as a matchmaker for students.

“Personal connection” needs some interpretation. It includes mostly head-hunting *without* search activities in the case of workers with working experience. The person is known to the firm through, say, everyday business contact. In the case of students, it is personal connection of their parents. When a firm decides to hire a worker through personal connection, it judges him as acceptable (i.e. above the threshold) and also employment of him is better than entering costly search process. The same holds for the worker side. Therefore, to conform this case to our definition, although matchmaking through personal connection actually does not rely on any mediating institute, it should be regarded as a trivial case that an imaginary matchmaker introduces only one candidate with no cost (obviously the matchmaker does not pursue maximum matching). The choice of a candidate should be regarded as stochastic, which is resolved by the time of recruiting. Since the sampling of a candidate in personal connection is made before an agent chooses a matching market, certain proportion of matching agents has a better candidate than the one that any other matching institute are expected to be able to provide. Therefore, regarding the emergence of personal connection as stochastic, it is expected that a fixed proportion of matching agents always chooses matching through personal connection.

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<sup>5</sup>The university the author belongs makes matching for internship every year. After try-and-error, we found that the matching procedure ended up with matching those jobs with less applicants first and those with many applicants last. We followed the procedure shown in Proposition 2.

The proportion of agents which chooses personal connection is affected by the performance of the other matching institutions. If the performance of the best matching institution improves, then the share of personal connection decreases. If it deteriorates, vice versa. We regard personal connection as a third category, a trivial matchmaker which has earlier timing of revelation of a candidate, which is independent from maximum matchmakers and independent search.

Table 1 shows that top 3 media used to hire workers with working experience is the public employment security office (59.6%), advertisements on newspapers (31.3%) and personal connection (22.9%). For university graduates, it is recommendation by school instructors (38.4%), meeting session held by the firm (32.9%) and job advertisement magazines and cites (29.8%). For high school graduates, recommendation by school instructors (54.5%), the public employment security office (42.2%) and meeting session held by the public employment security office (9.3%). Obviously, independent search dominates in the employment of workers with working experience and matching via matchmakers dominates in the employment of high school graduates. The share of independent search is 117.0% for workers with working experience, 110.1% for university graduates and 37.8% for high school graduates, allowing for double counts. Here, we summed up the entries which are categorised as matchmakers regardless of multiple answers. Similarly, the share of matchmakers is: 4.3%, 59.2% and 96.9%, respectively. Because of double counts, comparison between matchmakers and independent search is meaningless. However, if the share of double counts can be safely regarded as constant, we can say that the share of matchmakers increases in the order of workers working experience, university graduates and high school graduates. The share of independent search increases in the exactly opposite order. It suggests that jobs of high school graduates are more suitable for matching through a matchmaker than that of university graduates and workers with working experience. It may be able to attribute it to the relatively simple nature of jobs of high school graduate workers. Suppose that those jobs have a property that, once fixed criteria is satisfied for the achievement of a given job, additional achievement is not required. For example, a cashier of a supermarket needs to be able to operate a cash register at an appropriate speed, but no additional creativity is required. Also, suppose that utility of a worker does not change largely whatever job he engages in. If his purpose of work is only to earn income and if wage

TABLE 2. Number of temporary help services in Japan

Fiscal year	Category general		Category special		Total	
1986	615	(—)	3,266	(—)	3,881	(—)
1987	1,153	(87.5%)	5,788	(77.2%)	6,941	(78.8%)
1988	1,320	(14.5%)	5,764	(−0.4%)	7,084	(2.1%)
1989	1,619	(22.7%)	6,270	(8.8%)	7,889	(11.4%)
1990	1,785	(10.3%)	7,080	(12.9%)	8,865	(12.4%)
1991	2,081	(16.6%)	7,304	(3.2%)	9,385	(5.9%)
1992	2,032	(−2.4%)	7,231	(−1.0%)	9,263	(−1.3%)
1993	2,005	(−1.3%)	6,610	(−8.6%)	8,615	(−7.0%)
1994	2,070	(3.2%)	6,688	(1.2%)	8,758	(1.7%)
1995	2,105	(1.7%)	6,914	(3.4%)	9,019	(3.0%)
1996	2,354	(11.8%)	7,165	(3.6%)	9,519	(5.5%)
1997	2,632	(11.8%)	6,627	(−7.5%)	9,259	(−2.7%)
1998	3,026	(15.0%)	6,985	(5.4%)	10,011	(8.1%)
1999	3,352	(10.8%)	6,326	(−9.4%)	9,678	(−3.3%)

**Note:** The number in the parentheses shows the rate of growth over the previous fiscal year.

**Source:** Ministry of Health, Labour and Welfare.

rate is similar for all related jobs, this property is satisfied. Then, it is natural that a maximum matchmaker dominates independent search, since utility is regarded as binary.

It is notable that private mediators are increasing in the field of temporary help service. Along with the revision of ILO Convention No. 181, the policy for temporary help services (THS) in Japan has drastically changed in 1999. Until June 1999, THS business was basically put under regulation so that its fields of activity is limited. However, deregulation in 1999 removed the limitation on the scope of THS activities in principle.<sup>6</sup> Table 2 shows the number of temporary help service in Japan. In the table, firms in “category special” are the type which employs workers as its own full-time employees and dispatch them to customers as workforce. Firms in “category general” are the type which pools “registered” workers without hiring and send them temporarily to the customers. As the table shows, the latter grew significantly in late 1990’s despite the depressive economic situation.

<sup>6</sup>Since 1985, THS activities were limited to the following 16 fields: software development, machine designing, broadcast equipment operation, broadcast program direction, business equipment operation, translation and stenography, secretary, filing, research, accounting, trading document writing, demonstration, courier, building cleaning, building equipment operation and maintenance, and reception and parking supervision. In 1996, the following 10 fields were added: R&D, business management planning, book edition, advertisement designing, interior coordinating, announcement, OA instruction, telemarketing, sales engineer, and stage setup. However, after the deregulation in 1999, THS is basically admitted in any fields except the following fields: stevedores, construction, security service, medical activities, negotiation on labour conditions, licensed professions such as lawyer, chartered accountant, or licensed tax accountant. However, temporary help to the manufacturing sector is regulated for the moment as a part of transition process when this manuscript is written.



TABLE 3. Temporary help labour force

Fiscal year	Registered staff	Workforce in unit of full-time worker
1986	144,709	87,367
1987	268,300	143,324
1988	312,903	166,233
1989	427,958	195,377
1990	510,267	233,765
1991	624,200	283,143
1992	653,598	262,059
1993	574,686	235,980
1994	575,879	238,300
1995	612,056	254,957
1996	724,248	298,530
1997	855,330	340,059
1998	895,274	306,914
1999	1,067,949	394,502

**Note:** The number of registered workers is used in the column of “number of staffs”. In the column of “labour force in unit of full-time worker”, part-time workers are weighted by their actual work hours compared to full-time workers.

**Source:** Ministry of Health, Labour and Welfare.

Sometimes, there is no clear distinction between THS and committed recruiting, especially in the case of so-called *shinsotsu-haken* (staffs who joined THS right after the graduation of the school). In this case, most of THS staffs get a position at the firm they dispatched as a permanent worker after the service period has expired. Thus, THS cannot be distinguished from committed recruiting. A THS firm searches from their pool a staff who meets the demand of a customer and the customer normally has an option to replace the staff. It is a safer way to recruit for both a customer and a THS staff, compared to entering directly in the employment relationship. If the quality of a match turned out to be unsatisfactory after they entered in an employment relationship, both parties have to enter in the costly process of search. In the case of THS mediation, replacement cost is much smaller. It is similar to the internship of university students in that it reduces the risk of mismatch. THS can be used in this way because the jobs of new employees are simple in general and also a firm can collect more information about his potentiality while making him work on easy jobs.

It is critical for a THS company to find out correctly the characteristics of each staff to meet the demand of customers. It is easier in the professional fields such as IT engineers, since skills are sometimes easily confirmed by established tests or qualifications. However, finding out general skills such as human interacting is difficult in general. Autor (2003) pointed out

TABLE 4. Major jobs of temporary help staffs

	Number of registration	Share
Business equipment operation	175,304	40.1%
Accounting	52,790	12.1%
Machine designing	41,228	9.4%
Software development	35,109	8.0%
Filing	29,222	6.7%
Trading document writing	26,058	6.0%
Reception and parking supervision	18,858	4.3%
Telemarketing	10,546	2.4%
Others	47,828	10.9%

Source: Ministry of Labour, Health and Welfare.

that THS firms might be providing general skill training to screen the ability of the workers, regardless the risk of escape of workers after the training. Also, it is reported that an attempt to set up and refine questionnaires to find out characteristics of workers by THS companies was successful in significantly reducing the trouble of mismatch. Despite these attempts, the fields of THS is limited in jobs which require special skills or simple jobs as Table 4 shows. It is consistent with our result that matchmakers dominate in the jobs which are illustrated by binary utility.

## 8. CONCLUSION

The present paper attempted to mediate between matching analyses developed in the context of the game theory and that of the graph theory. It showed that in most of cases, independent search dominates mediated matching, but under certain conditions, the opposite can arise. It is consistent with the history that mediated match dominated in simple jobs.

Section 2 provided a feasible way of obtaining stable matching and maximum matching.. Section 3 showed how an independent search is described by a stable marriage problem. Section 4 showed an algorithm of which matchmakers can make use to attain maximum matching. The problem is transformed into a maximum flow problem and the objective is turned into to maximise flow from a source connected to either group to a sink connected to the other group. Section 5 summarised the results concerning to the case when the size of a market goes to infinity. Matchmakers can attain complete matching and it is known that independent search can also do so for a special case. Section 6 studied which matching market survives through competition. For a wide class of distribution function which determines the quality of

a match, independent search dominates mediated markets. Section 7 gave an overview of the labour matching market in Japan. It showed that independent search dominates for workers with working experience, but the share of mediated matching is larger for high school graduates. It is consistent with our result that mediated matching can dominate in the case of binary utility.

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