

## Looking at the Cournot-Nash Equilibrium by using the MATHEMATICA Graphics†

--- You can choose any values of parameters in the Cournot model ---

Masaru Uzawa \*  
Otaru University of Commerce

July 13, 1998  
January 1, 2000  
February 20, 2000

### Summary

We can show the graphical representation of Cournot Duopoly Model by using the fully integrated technical computing system **MATHEMATICA graphics**. Each firm's profit depends upon two firms' outputs.

At first, we formalize the Cournot model in MATHEMATICA language and then show the 3-dimensional profit surfaces. To emphasize the profit surface, we can show the modified profit surface in which it is zero profit if it was negative one.

Next, we have Cournot's reaction function and show two firms' reaction curves, respectively. We immediately get the Cournot equilibrium at which two firms' reaction curves cross each other. We know the Cournot equilibrium is not Pareto efficient because we can see the lense-shaped area in which each firm gets larger profit than one in the Cournot equilibrium. The profit surfaces are truncated by the profit level of Cournot equilibrium.

This paper is written as MATHEMATICA notebook format. You can choose any values of parameters of Cournot model and get the associated Cournot equilibrium. You may also get the HTML format file from this MATHEMATICA notebook format and you can make impressive presentation by using Microsoft's PowerPoint97.

---

† This research is partially supported by the Japan Society for the Promotion of Science Research Grant (#10630001: "Computer Assisted Learning (CAL) in Economics Programs and Experimental Economics" (from April 1998 to March 2000)). I would like to thank Professors Frank S.T. Hsiao (University of Colorado at Boulder), Michael Lovell (Wesleyan University), Manimay Sengupta (Otaru University of Commerce), and Nobuo Wakabayashi (Otaru University of Commerce) for their helpful comments and suggestions. Of course, I am sole responsible for remaining errors and ambiguities.

\* Otaru University of Commerce, 3-5-21 Midori, Otaru, Hokkaido 047-8501, Japan

Tel & FAX: +81-134-27-5310

E-mail: uzawa@res.otaru-uc.ac.jp

URL: <http://www.res.otaru-uc.ac.jp/~uzawa/welcome.html>

URL: <http://www.res.otaru-uc.ac.jp/~uzawa/cal-economics/cal-ee.html>

## 1 . Getting started

### --- Choice of values of parameters in Cournot model ---

We consider the Cournot dupoly in which two firms produce the homogeneous good. We call the ourput of firm 1 as  $q_1$  and the output of firm 2 as  $q_2$ .

The demand curve for those firms is as follows:

$$p = a - b (q_1 + q_2)$$

where  $p$  is the price of one unit of good.

Also we assume that the cost function for both firms are as follows:

$$c_1 = m q_1, c_2 = m q_2$$

You can choose the values of parameters for  $a, b, m$  and get the associated Cournot equilibrium

The default values of parameters are as follows:

```
In[1] := a = 25; b = 1; m = 1; ppt = 50;
```

where the value of  $ppt$  shows the resolution in the screen. The more the value of  $ppt$ , the more dense graphical representation.

**After you can choose any values of parameters for  $a, b, m$ , and  $ppt$ , you do follow the procedure:  
Please choose the following menu step by step from the menubar located in upper second line:**

Kernel  
Evaluation  
Evaluate Notebook

*[You may wait 2 or 3 minites depending upon your using machine.]*

Please wait until the expression in the third line changing from

Running ... [file name you are running]

to

[file name you are running].

You can see the processes getting the Cournot equilibrium by scrolling the screen when you have the message of Cournot9807engnb in the third line.

Of course, you can follow the default case by scrolling the screen at ease.

## 2 . Procedures to get the Cournot Equilibrium

The demand curve depends on the default values of parameters or values you may choose. In this case, we have the following demand curve:

$$\text{In}[2] := p = a - b (q1 + q2)$$

$$\text{Out}[2] = 25 - q1 - q2$$

Also we suppose that the cost functions for both firms are as follows:

$$\text{In}[3] := c1 = m q1$$

$$\text{Out}[3] = q1$$

$$\text{In}[4] := c2 = m q2$$

$$\text{Out}[4] = q2$$

Now let us define the profit to be the difference of sales and cost.

The profit of firm 1 is denoted by  $pa1$  and the profit of firm 2 is  $pa2$ , respectively.

We can get the following relations:

$$\text{In}[5] := pa1 = p q1 - c1$$

$$\text{Out}[5] = -q1 + q1 (25 - q1 - q2)$$

$$\text{In}[6] := pa2 = p q2 - c2$$

$$\text{Out}[6] = -q2 + (25 - q1 - q2) q2$$

We have another simplified expressions of two firms' profits.

$$\text{In}[7] := pa1simplified = \text{Simplify}[pa1]$$

$$\text{Out}[7] = -q1 (-24 + q1 + q2)$$

$$\text{In}[8] := pa2simplified = \text{Simplify}[pa2]$$

$$\text{Out}[8] = -q2 (-24 + q1 + q2)$$

Now we want to show you the firm 1's profit surface. To do so, some operations are in order.

$q1Max$  and  $q2Max$  refer to the maximum output of firm 1 and firm 2, respectively.

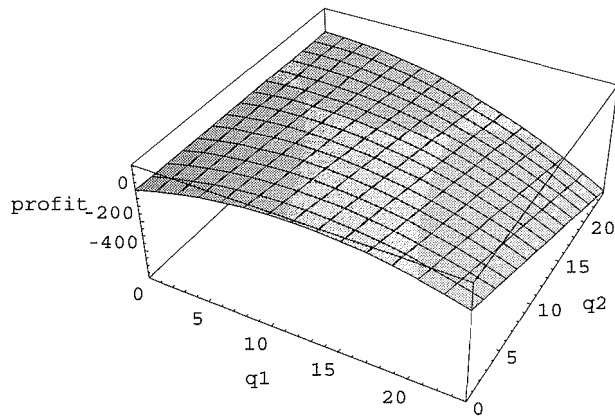
$$\text{In}[9] := q1Max = (a - m) / b$$

$$\text{Out}[9] = 24$$

$$\text{In}[10] := q2Max = (a - m) / b$$

$$\text{Out}[10] = 24$$

```
In[11]:= profit1 = Plot3D[pai1, {q1, 0, q1Max}, {q2, 0, q2Max},
    AxesLabel -> {"q1", "q2", "profit "}]
```

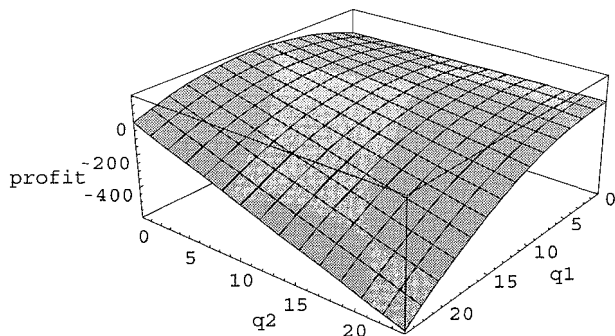


```
Out[11]= - SurfaceGraphics -
```

**Figure 1. The shape of firm 1's profit surface**

We can look the firm 1's profit surface from another point of view.

```
In[12]:= profit1another = Plot3D[pai1, {q1, 0, q1Max}, {q2, 0, q2Max},
    AxesLabel -> {"q1", "q2", "profit "},
    ViewPoint -> {2.5, 2.0, 1.5}]
```



```
Out[12]= - SurfaceGraphics -
```

**Figure 2. The shape of firm 1's profit surface viewed from another point of view**

By comparing two graphs, you can understand how the surface of firm 1's profits are.

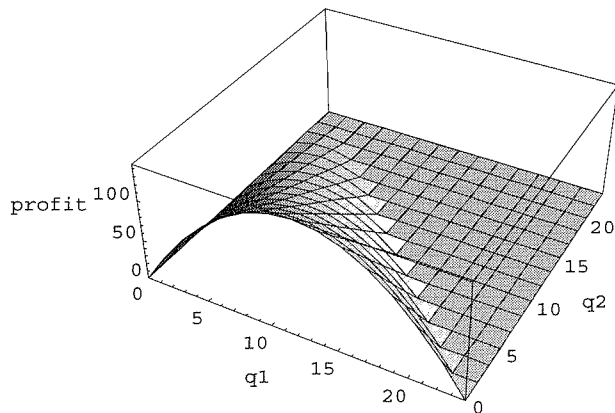
We would like to check the subtle shape of the profit surface of firm 1.

Because we are only interested in positive profit of firm 1, we can dispose the part of negative profits of firm 1.

In other words, we set the negative profit level of firm 1 as if it were zero profit.

We call this graph as the firm 1's modified profit surface.

```
In[13]:= profit1modified = Plot3D[pai1, {q1, 0, q1Max}, {q2, 0, q2Max},
    AxesLabel -> {"q1", "q2", "profit "},
    PlotRange -> {0, (q1Max/2)^2}]
```

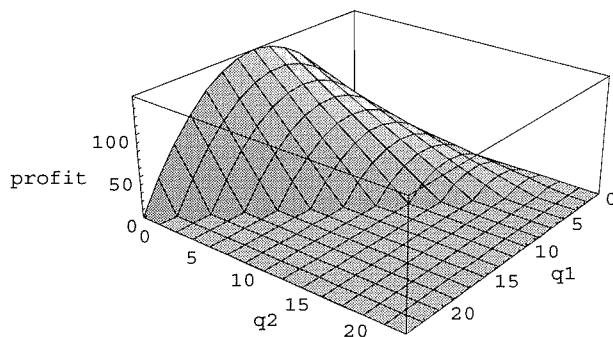


```
Out[13]= - SurfaceGraphics -
```

**Figure 3.** The shape of firm 1's modified profit surface

We can look the firm 1's modified profit surface from another point of view.

```
In[14]:= profit1modifiedanother = Plot3D[pai1, {q1, 0, q1Max}, {q2, 0, q2Max},
    AxesLabel -> {"q1", "q2", "profit "},
    PlotRange -> {0, (q1Max/2)^2}, ViewPoint -> {2.5, 2.0, 1.5}]
```

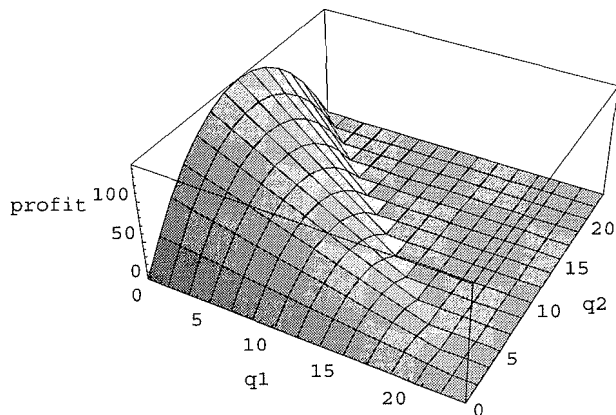


```
Out[14]= - SurfaceGraphics -
```

**Figure 4.** The shape of firm 1's modified profit surface viewed from another point of view

Also we have the firm 2's modified profit surface.

```
In[15]:= profit2modified=Plot3D[pai2, {q1, 0, q1Max}, {q2, 0, q2Max},
    AxesLabel -> {"q1", "q2", "profit "},
    PlotRange -> {0, (q2Max/2)^2}]
```

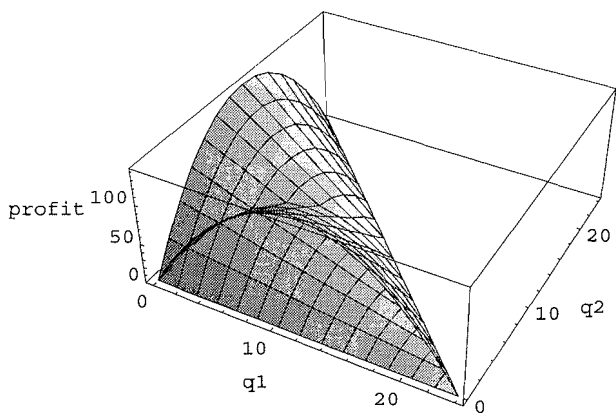


```
Out[15]= - SurfaceGraphics -
```

**Figure 5. The shape of firm 2's modified profit surface**

By displaying both figures 3 and 5 in together, we obtain the following graph:

```
In[16]:= profit1profit2=Show[profit1modified, profit2modified,
    AxesLabel -> {"q1", "q2", "profit "}]
```

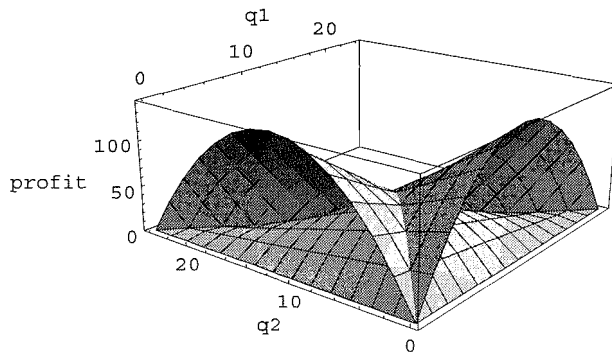


```
Out[16]= - Graphics3D -
```

**Figure 6. Joint representation of two firms' modified profit surfaces**

By viewing the figure 6 from the left hand corner, you will find what relation exists between two firm's profit surfaces.

```
In[17]:= jointprofit1profit2another = Show[profit1profit2,
      ViewPoint -> {-2.3, -1.8, 1.0}]
```



```
Out[17]= - Graphics3D -
```

**Figure 7. Joint representation of two firms' modified profit surfaces viewed from another point of view**

We are now considering the new concept.

Taking other firm's output constant or assuming Cournot expectation, you can get the output that maximizes own firm's profit.

Namely, we can get **the Cournot's reaction function or curve**.

The first order conditions for maximization(foc) are as follows. And reaction functions of two firms are derived from foc.

```
In[18]:= foc = {D[pai1, q1] == 0, D[pai2, q2] == 0}
```

```
Out[18]= {24 - 2 q1 - q2 == 0, 24 - q1 - 2 q2 == 0}
```

```
In[19]:= reactionFunction1 = D[pai1, q1]
```

```
Out[19]= 24 - 2 q1 - q2
```

```
In[20]:= reactionFunction2 = D[pai2, q2]
```

```
Out[20]= 24 - q1 - 2 q2
```

Now we have two firms' reaction functions, respectively.

```
In[21]:= reaction1 = Solve[foc[[1]], q1][[1]]
```

```
Out[21]= {q1 -> (24 - q2)/2}
```

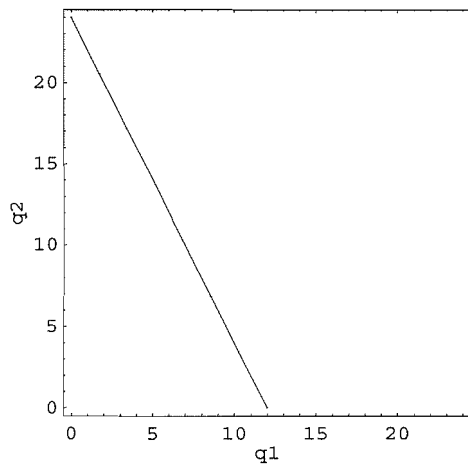
```
In[22]:= reaction2 = Solve[foc[[2]], q2][[1]]
```

```
Out[22]= {q2 -> (24 - q1)/2}
```

From reaction function, you can easily draw the reaction curve.

At first, we draw the reaction curve of firm 1.

```
In[23]:= grReaction1 =
  ContourPlot[reactionFunction1, {q1, 0, q1Max}, {q2, 0, q2Max},
    Contours -> {0}, ContourShading -> False, FrameLabel -> {q1, q2}]
```



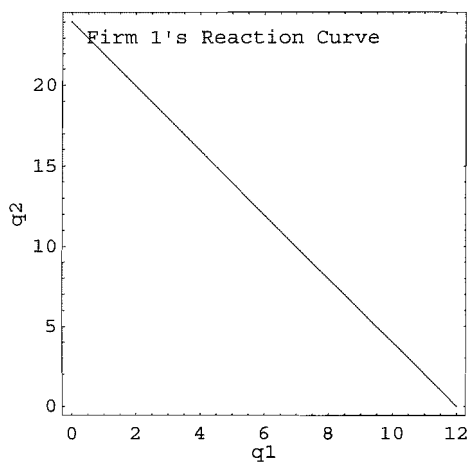
```
Out[23]= - ContourGraphics -
```

**Figure 8. Firm 1's reaction curve**

We can make legend as "Firm 1's Reaction Curve" in this graph.

I think it is very convenient to do so because many lines appear in later stages.

```
In[24]:= textReaction1 =
  Graphics[Text["Firm 1's Reaction Curve", {0 + 5, q2Max - 1}]];
grReactionCurve1 = Show[grReaction1, textReaction1]
```



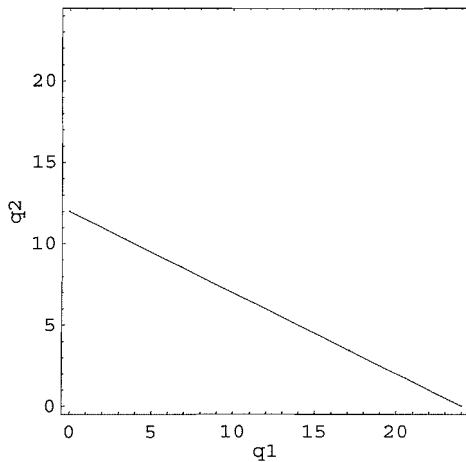
```
Out[24]= - Graphics -
```

**Figure 9. Firm 1's reaction curve with legend**

Next, you can draw the reaction curve of firm 2.



```
In[25]:= grReaction2 =
  ContourPlot[reactionFunction2, {q1, 0, q1Max}, {q2, 0, q2Max},
    Contours -> {0}, ContourShading -> False, FrameLabel -> {q1, q2}]
```

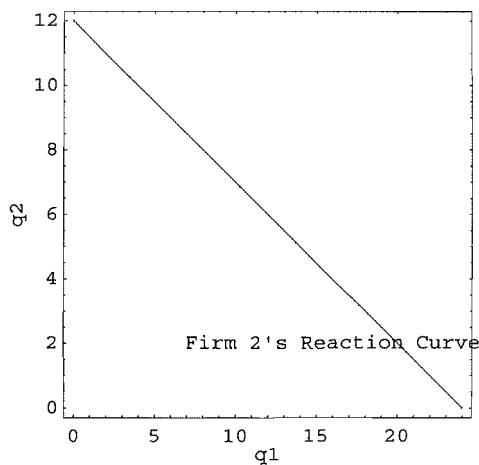


```
Out[25]= - ContourGraphics -
```

**Figure 10. Firm 2's reaction curve**

Also we can make legend as "Firm 2's Reaction Curve" in this graph.

```
In[26]:= textReaction2 =
  Graphics[Text["Firm 2's Reaction Curve", {q1Max - 8, 0 + 2}]];
grReactionCurve2 = Show[grReaction2, textReaction2]
```



```
Out[26]= Null (- Graphics -)
```

**Figure 11. Firm 2's reaction curve with legend**

### 3. Searching for the Cournot=Nash Equilibrium

We can get the Cournot=Nash equilibrium on the cross point of two firms' reaction curves.[See Shy (1998) and Uzawa (1998) for the formal definition of Cournot=Nash equilibrium.]

```
In[27]:= quantityCournot = Solve[foc, {q1, q2}][[1]]
```

```
Out[27]= {q1 → 8, q2 → 8}
```

In general expression, we have the equilibrium outputs of both firms in Cournot equilibrium by using *MATHEMATICA*. [See Uzawa(1998)].

Here, we denote the equilibrium outputs of both firms in Cournot equilibrium as  $q1_{\text{Cournot}}$  and  $q2_{\text{Cournot}}$ , respectively.

```
In[28]:= q1Cournot = (a - m) / (3 b)
```

```
Out[28]= 8
```

```
In[29]:= q2Cournot = (a - m) / (3 b)
```

```
Out[29]= 8
```

The price ( $price_{\text{Cournot}}$ ) is determined from demand curve.

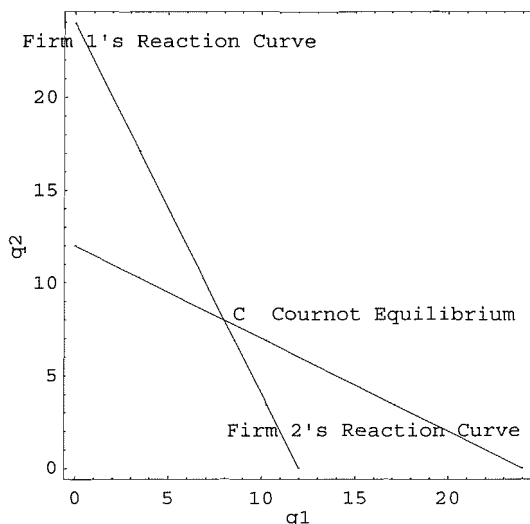
```
In[30]:= priceCournot = a - b (q1Cournot + q2Cournot)
```

```
Out[30]= 9
```

You can see the Cournot equilibrium at which both firms' reaction curve cross in the graphical expression.

```
In[31]:= textCournot = Graphics[Text["C Cournot Equilibrium",  
{q1Cournot + 8, q2Cournot + 0.3}]];
```

```
In[32]:= cournotNashEquilibrium = Show[grReaction1, grReaction2,
      textReaction1, textReaction2, textCournot]
```



```
Out[32]= - Graphics -
```

Figure 12. Two firms' reaction curves and the Cournot equilibrium

#### 4. Two firms' profit surfaces truncated by the profit level through the Cournot Equilibrium profits

In general, we have the firm 1's profit at the Cournot equilibrium as follows.[See also Uzawa(1998)].

```
In[33]:= pailCournot = ((a - m) / 3 b) ^ 2
```

```
Out[33]= 64
```

We want to draw the profit surface of firm 1 truncated by the profit level through the Cournot equilibrium profit. To do so, some operations are in order.

numRange refers to the profit level we want to truncate the profit surface.

numPoints refers to the numbers of points in the graph.

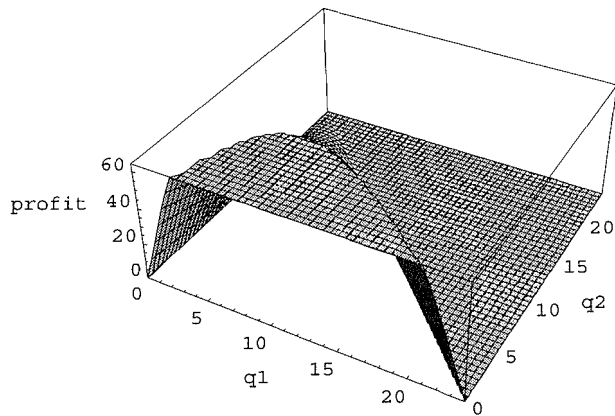
```
In[34]:= numRange = ((a - m) / (3 b)) ^ 2
```

```
Out[34]= 64
```

```
In[35]:= numPoints = 2 (a - m) / b
```

```
Out[35]= 48
```

```
In[36]:= profit1atCournot = Plot3D[pai1, {q1, 0, q1Max}, {q2, 0, q2Max},
  AxesLabel -> {"q1", "q2", "profit "}, PlotRange -> {0, numRange},
  PlotPoints -> numPoints]
```



Out[36]= - SurfaceGraphics -

**Figure 13. The shape of firm 1's profit surface truncated by the Cournot equilibrium profit level**

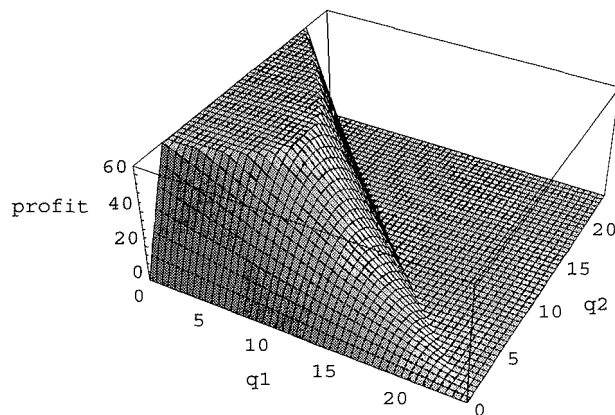
Also, we have the firm 2's profit in the Cournot equilibrium.

```
In[37]:= pai2Cournot = ((a - m) / 3 b) ^ 2
```

Out[37]= 64

We want to draw the profit surface of firm 2 truncated by the profit level through the Cournot equilibrium profit.

```
In[38]:= profit2atCournot = Plot3D[pai2, {q1, 0, q1Max}, {q2, 0, q2Max},
  AxesLabel -> {"q1", "q2", "profit "}, PlotRange -> {0, numRange},
  PlotPoints -> numPoints]
```



```
Out[38]= - SurfaceGraphics -
```

**Figure 14.** The shape of firm 2's profit surface truncated by the Cournot equilibrium profit

As you see the oligopoly textbook, you can see the lens-shaped area by combining these profit maps into one phase.

At the profit level of

```
In[39]:= pailCournot
```

```
Out[39]= 64
```

you can see two curves through profit level of the Cournot equilibrium profits.

These curves cross twice each other. Of these two points, the next one is the Cournot=Nash equilibrium output combination.

```
In[40]:= {q1Cournot, q2Cournot}
```

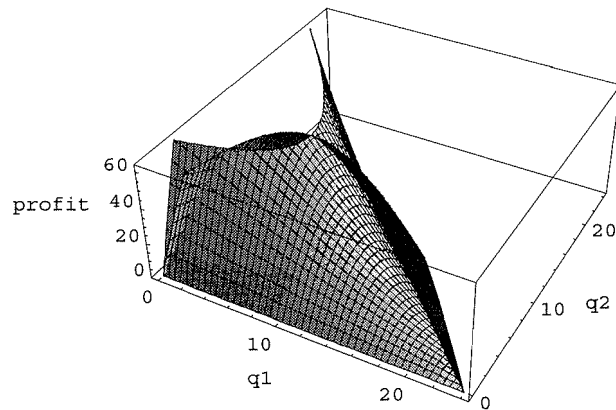
```
Out[40]= {8, 8}
```

The profits combination of the Cournot equilibrium is as follows:

```
In[41]:= {pailCournot, pai2Cournot}
```

```
Out[41]= {64, 64}
```

```
In[42]:= cournotProfits = Show[profit1atCournot, profit2atCournot]
```

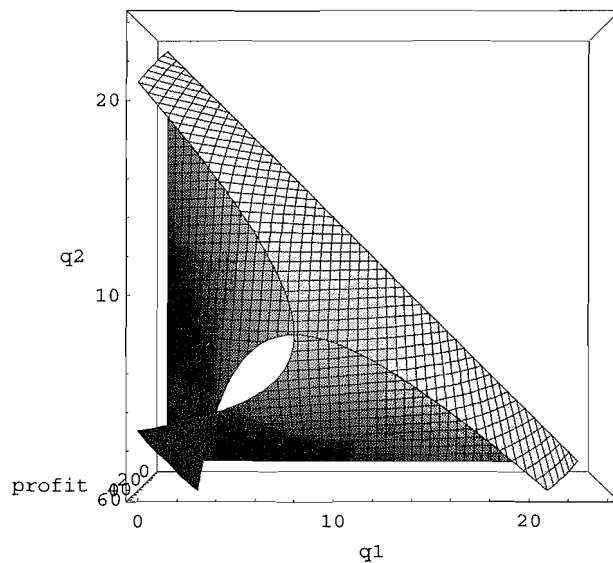


```
Out[42]= - Graphics3D -
```

**Figure 15. Joint representation of two firms' profit surfaces truncated by the Cournot equilibrium profit**

By viewing the figure 15 from the top-left hand corner, you will find what relation exists between two firm's profit surfaces.

```
In[43]:= cournotProfitsAnother = Show[cournotProfits, ViewPoint -> {0, 0, 3.0}]
```

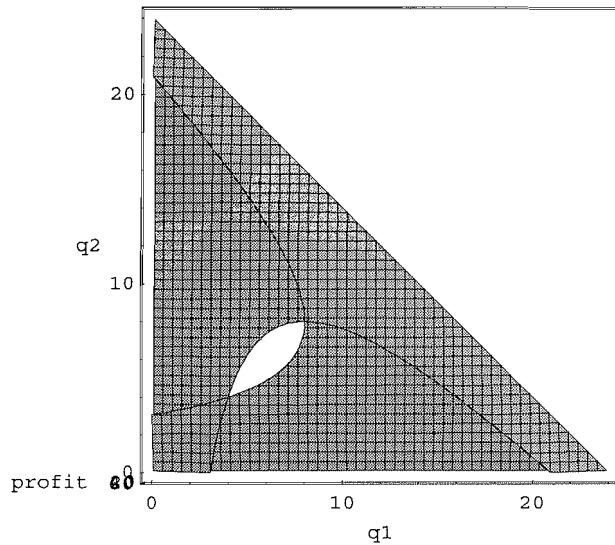


```
Out[43]= - Graphics3D -
```

**Figure 16. Joint representation of two firms' profit surfaces truncated by the Cournot equilibrium profit viewed from above**

If you see the figures 15 or 16 from the above far away, you can find the two curves emerged in your sight as depicted in the textbook.

```
In[44]:= cournotProfitsAnother = Show[cournotProfits, ViewPoint -> {0, 0, 50.0}]
```



```
Out[44]= - Graphics3D -
```

**Figure 17. Lens-shaped area curved by two firms' profit surfaces truncated by the Cournot equilibrium profit level**

Because two isoprofit curves cross each other at two points, it is evident that there exist many points in the lens-shaped area which show larger profits combination of both firms than one at the Cournot equilibrium.

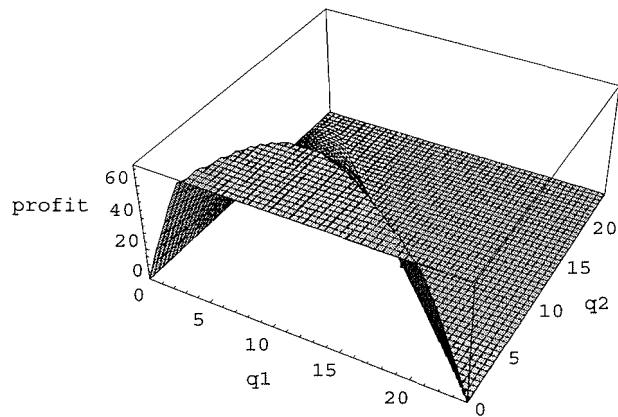
**In other words, the Cournot equilibrium is not Pareto efficient profit combination.**

Now we want to show you the case in which two firms cooperate and get the joint profit maximization. Each firm gets the half profit of the joint profit maximization, for simplicity.

```
In[45]:= halfMonopoly = ((a - m) / (2 b)) ^ 2 / 2
```

```
Out[45]= 72
```

```
In[46]:= profit1atHalfMonopoly = Plot3D[pai1, {q1, 0, q1Max}, {q2, 0, q2Max},
  AxesLabel -> {"q1", "q2", "profit "},
  PlotRange -> {0, halfMonopoly},
  PlotPoints -> ppt]
```



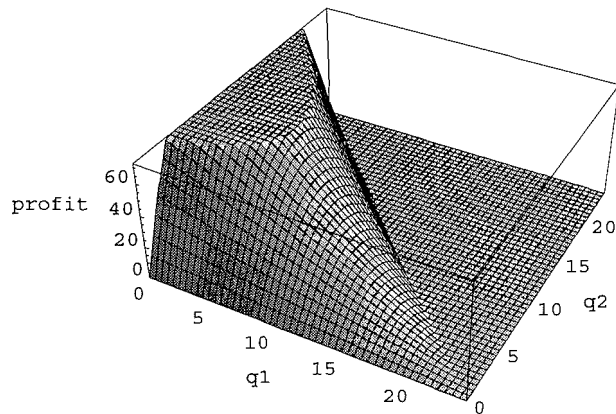
```
Out[46]= - SurfaceGraphics -
```

**Figure 18.** The shape of firm 1's profit surface truncated by shared cooperative profit level

We also have the same type of firm 2's profit surface truncated by shared cooperative profit level.



```
In[47]:= profit2atHalfMonopoly= Plot3D[pai2, {q1, 0, q1Max}, {q2, 0, q2Max},
  AxesLabel -> {"q1", "q2", "profit "},
  PlotRange -> {0, halfMonopoly},
  PlotPoints -> ppt]
```



Out[47]= - SurfaceGraphics -

**Figure 19. The shape of firm 2's profit surface truncated by shared cooperative profit level**

Two isoprofit curves through the half profit of the joint profit maximization just touch each other.

You can see that **this profit combination is Pareto efficient profit combination** .

What's more, this profit level is larger than the one at the Cournot equilibrium.

For your reference, we present here the profit combination in the Cournot equilibrium, again.

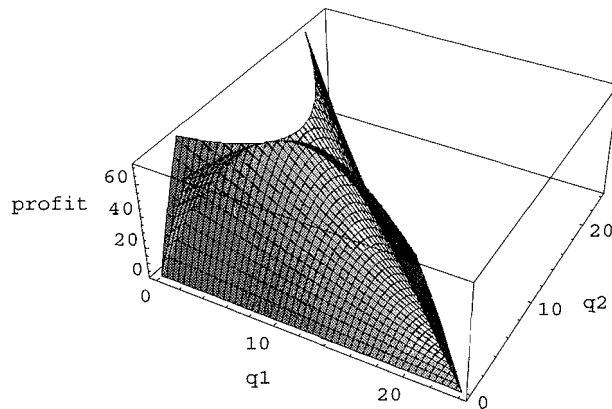
```
In[48]:= {pai1Cournot, pai2Cournot}
```

Out[48]= {64, 64}

```
In[49]:= {halfMonopoly, halfMonopoly}
```

Out[49]= {72, 72}

```
In[50]:= cournotProfitsHalfMonopoly =
          Show[profit1atHalfMonopoly, profit2atHalfMonopoly]
```



```
Out[50]= - Graphics3D -
```

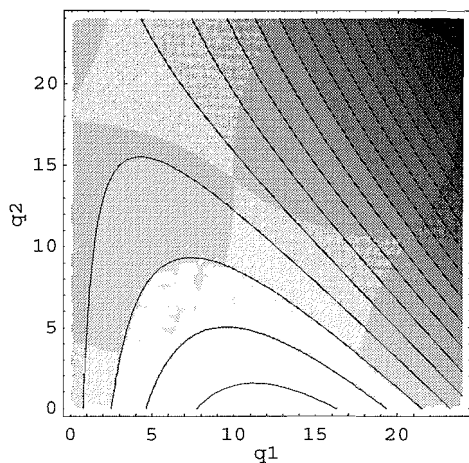
**Figure 20. Joint representation of two firms' profit surfaces truncated by shared cooperative profit level**

## 5. Iso-profit curves for both firms

Next, we can show you the iso-profit curve in the another graph. The iso-profit curve of firm 1 refers to the output combinations of both firms which provide the same profit level to firm 1. The same is true for the iso-profit curve of firm 2.

Please note that the profit level is larger in the light dark area than in the heavy dark one. Namely, the lower the firm 1's iso-profit curve, the larger profit level for firm 1.

```
In[51]:= cournot1Contour = ContourPlot[pai1, {q1, 0, q1Max}, {q2, 0, q1Max},
      Contours -> 20, PlotPoints -> 2 * ppt, FrameLabel -> {"q1", "q2"}]
```



```
Out[51]= - ContourGraphics -
```

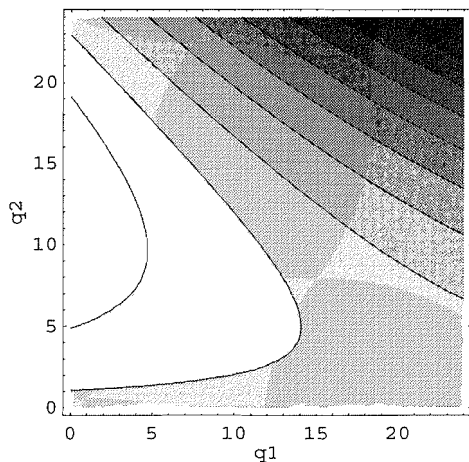
**Figure 21. Firm 1's iso-profit contour set**

In the same way, we can show you the iso-profit curve of firm 2.

Please note that the profit level is larger in the light dark area than in the heavy dark one.

Namely, the lefter the firm 2's iso-profit curve, the larger profit level for firm 2.

```
In[52]:= cournot2Contour = ContourPlot[pai2, {q1, 0, q1Max}, {q2, 0, q2Max},
      PlotPoints -> 100, FrameLabel -> {"q1", "q2"}]
```



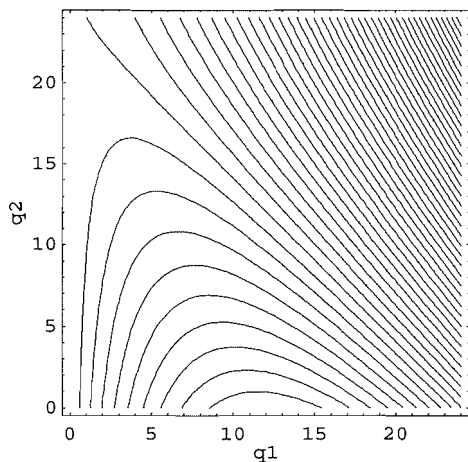
```
Out[52]= - ContourGraphics -
```

**Figure 22. Firm 2's iso-profit contour set**

Now we look at the iso-profit curves of both firms from different point of view.

The heavy lines refer to the different level of profits in iso-profit curves.

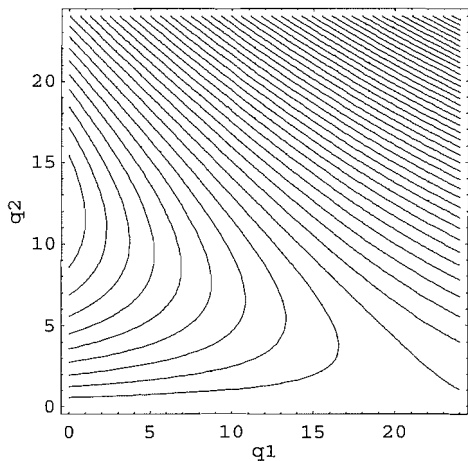
```
In[53] := gr1Cournot =
          Show[cournot1Contour, Contours -> 50, ContourShading -> False]
```



```
Out[53]= - ContourGraphics -
```

**Figure 23. Firm 1's iso-profit contour set without shading**

```
In[54] := gr2Cournot =
          Show[cournot2Contour, Contours -> 50, ContourShading -> False]
```

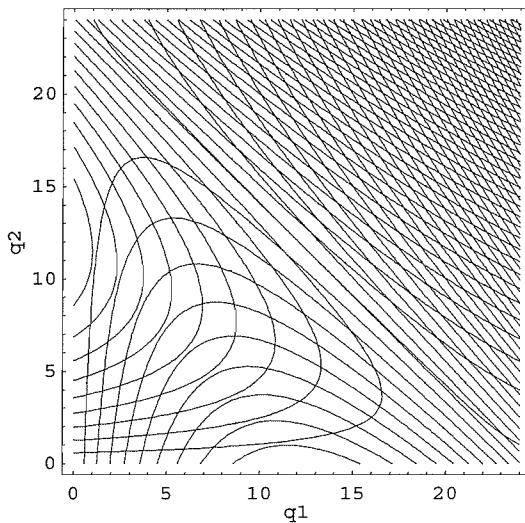


```
Out[54]= - ContourGraphics -
```

**Figure 24. Firm 2's iso-profit contour set without shading**

We make consolidating these two iso-profits maps into one in which you can find the tangential point of iso-profit curves to each other. These tangential points consist of the set of Pareto efficient profit allocations.

```
In[55] := isoprofits = Show[gr1Cournot, gr2Cournot]
```

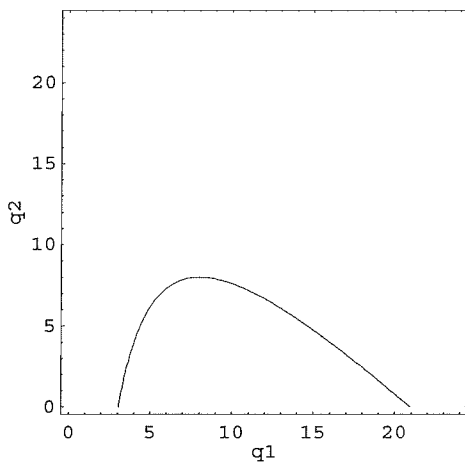


```
Out[55]= - Graphics -
```

**Figure 25. Joint representation of two firms' profit contour Sets**

Let us draw the isoprofit curve of firm 1 through which goes the Cournot equilibrium.

```
In[56] := iso1ProfitCournot = ContourPlot[pai1, {q1, 0, q1Max}, {q2, 0, q2Max},
      Contours -> {pai1Cournot}, ContourShading -> False,
      PlotPoints -> 2 * ppt, FrameLabel -> {q1, q2}]
```

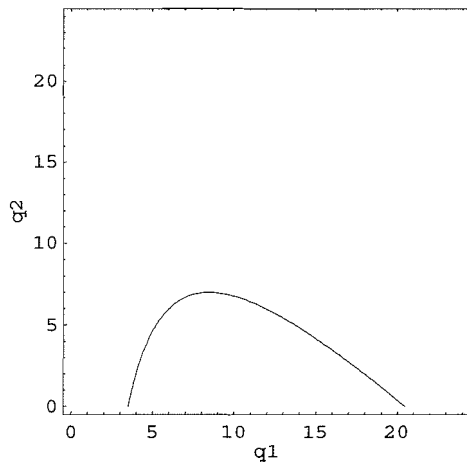


```
Out[56]= - ContourGraphics -
```

**Figure 26. Firm 1's iso-profit curve through the Cournot equilibrium**

Also we can draw the iso-profit curve at which firm 1 gets the half of joint profit maximization.

```
In[57]:= isolProfitHalfMonopoly = ContourPlot[pai1,
      {q1, 0, q1Max}, {q2, 0, q2Max}, Contours -> {halfMonopoly},
      ContourShading -> False, PlotPoints -> 100, FrameLabel -> {q1, q2}]
```

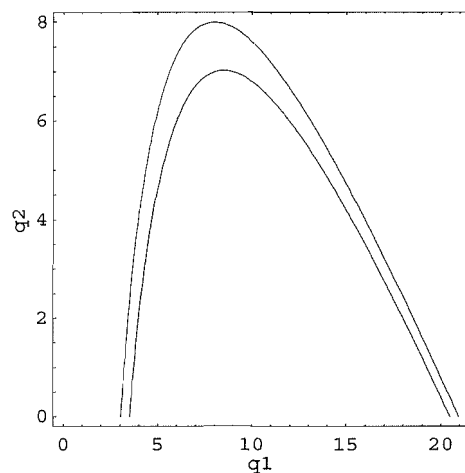


```
Out[57]= - ContourGraphics -
```

**Figure 27. Firm 1's isoprofit curve through shared cooperative solution**

For your reference, we preset these two iso-profit curves in one graph. The lower iso-profit curve refers to the iso-profit curve at which firm 1 gets larger profit than in the Cournot equilibrium.

```
In[58]:= isolCournotHalfMonopoly =
      Show[isolProfitCournot, isolProfitHalfMonopoly]
```

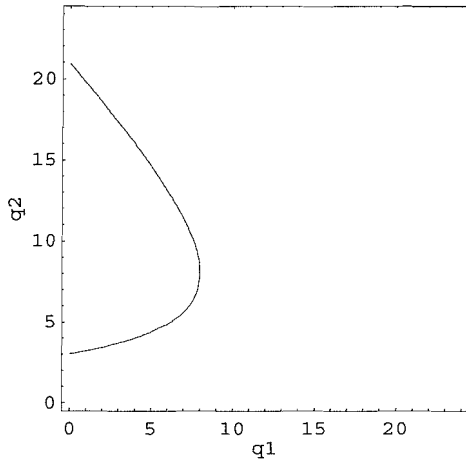


```
Out[58]= - Graphics -
```

**Figure 28. Joint representation of firm 1's two iso-profit curves**

Let us draw the isoprofit curve of firm 2 through which goes the Cournot equilibrium.

```
In[59]:= iso2ProfitCournot = ContourPlot[pai2, {q1, 0, q1Max}, {q2, 0, q2Max},
      Contours -> {pai2Cournot}, ContourShading -> False,
      PlotPoints -> 100, FrameLabel -> {q1, q2}]
```

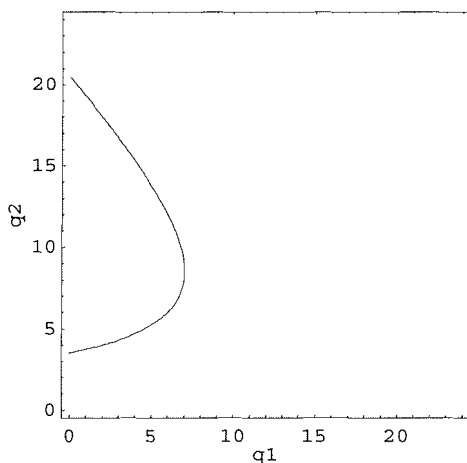


```
Out[59]= - ContourGraphics -
```

**Figure 29. Firm 2's iso-profit curve through the Cournot equilibrium**

Also we can draw the iso-profit curve at which firm 2 gets the half of joint profit maximization.

```
In[60]:= iso2ProfitHalfMonopoly = ContourPlot[pai2,
      {q1, 0, q1Max}, {q2, 0, q2Max}, Contours -> {halfMonopoly},
      ContourShading -> False, PlotPoints -> 100, FrameLabel -> {q1, q2}]
```

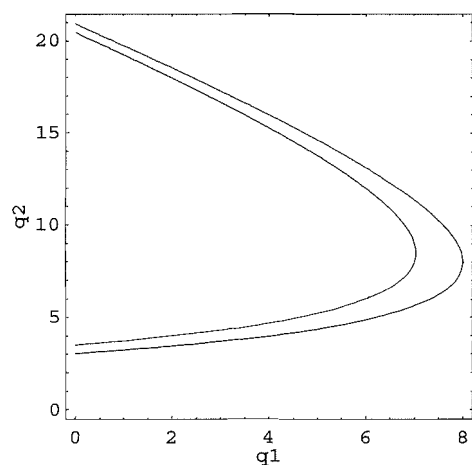


```
Out[60]= - ContourGraphics -
```

**Figure 30. Firm 2's iso-profit curve through shared cooperative solution**

For your reference, we present these two iso-profit curves in one graph. The latter iso-profit curve refers to the iso-profit curve at which firm 2 gets larger profit than at the Cournot equilibrium.

```
In[61]:= iso2CournotHalfMonopoly =
          Show[iso2ProfitCournot, iso2ProfitHalfMonopoly]
```



```
Out[61]= - Graphics -
```

Figure 31. Joint representation of firm 2's two iso-profit curves

## 6. Cournot equilibrium and existence of Pareto superior profits combinations between two firms

Now we can show you the two iso-profit curves through the Cournot equilibrium, two firms' reaction curves, and the Cournot equilibrium in one graph. The Cournot equilibrium profits combination is as follows:

```
In[62]:= {pai1Cournot, pai2Cournot}
```

```
Out[62]= {64, 64}
```

```
In[63]:= textIsoprofit1 = Graphics[Text["Firm 1's isoprofit curve",
                                         {q1Cournot - 3, 0 + 1}]];

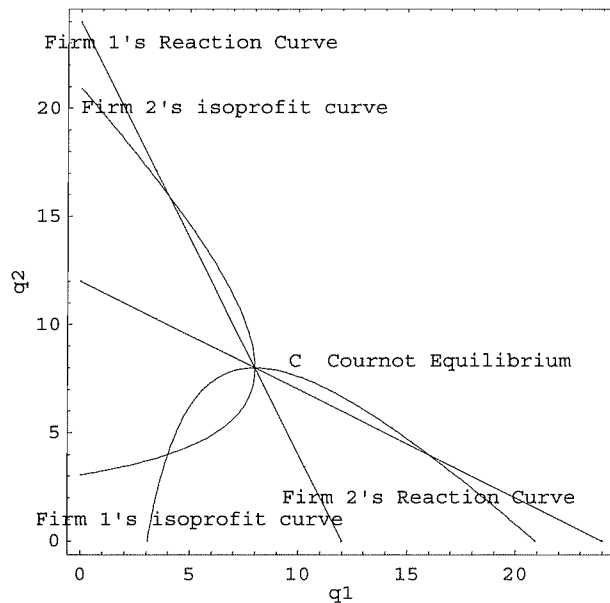
```

```
In[64]:= textIsoprofit2 = Graphics[Text["Firm 2's isoprofit curve",
                                         {0 + 7, q2Max - 4}]];

```



```
In[65]:= grCournotEquilibrium = Show[cournotNashEquilibrium,
    iso1ProfitCournot, iso2ProfitCournot,
    textIsoprofit1, textIsoprofit2]
```



Out [65]= - Graphics -

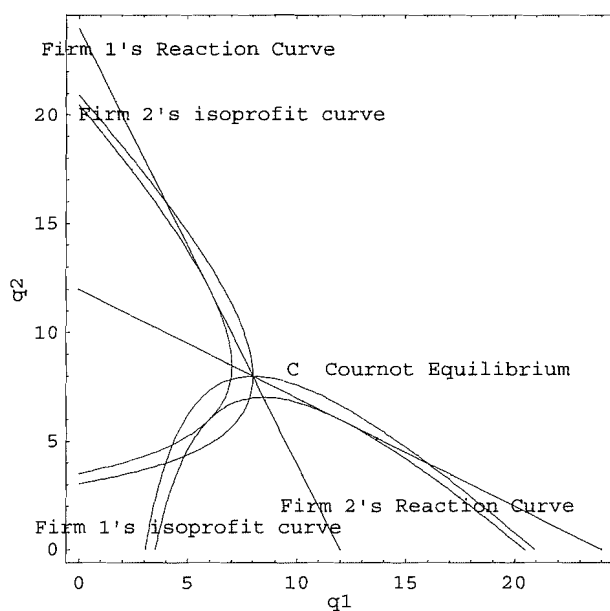
**Figure 32. Two firms' reaction curves, iso-profit curves and the Cournot equilibrium**

We are looking for the profits combination which is preferred to both firms.

Namely, we get and show the Pareto superior profits combination.

The lens-shaped area are the profits combinations which are preferred to both firms because both firm 1 and firm 2 get larger profit in the lens-shaped area than in the Cournot equilibrium.

```
In[66]:= grDominated = Show[cournotNashEquilibrium,
    iso1ProfitCournot, iso2ProfitCournot,
    iso1ProfitHalfMonopoly, iso2ProfitHalfMonopoly,
    textIsoprofit1, textIsoprofit2]
```



```
Out[66]= - Graphics -
```

Figure 33. The Cournot equilibrium and the Pareto dominating allocations

## 7. Concluding remarks

It is convenient to use this file to get the Cournot equilibrium when you can choose any values of parameters in Cournot duopoly model because this paper itself is written as a notebook of **MATHEMATICA**. **What's more** you can transpose this file into HTML format file so that you can create an impressive presentation of getting the Cournot equilibrium by using Microsoft's PowerPoint97.

## References

- Friedman, James W.(1983), *Oligopoly theory*, Cambridge Univeristy Press, 1983.
- Shy, Oz (1998), *Industrial Organization: Theory and Applications*, The MIT Press, Cambridge, Massachusetts, London, England, 1st printing (1995), 4th printing (1998).
- Uzawa, Masaru (1996), "Seeking the Cournot equilibrium and the Stackelberg equilibrium by using *Mathematica*," in Grant Research Report (1995-1996): "Analysis of Mathematical model by using *MATHEMATICA*" (in Japanese), (Otaru University of Commerce), pp.35-56.
- Uzawa, Masaru (1998), "Computer Assisted Learning (CAL) in Economics by using *Mathematica* and Internet Resources on Economics, (in Japanese), *Economic Review* (Otaru University of Commerce), Vol. 48, Nos.2-3, (January 1998), pp.49-74.
- Wolfram, Stephen (1996), The *MATHEMATICA* book, 3rd ed., *Mathematica* Version 3, Wolfram Media/ Cambridge University Press, 1996
- Wolfram, Stephen (1999), The *MATHEMATICA* book, 4th ed., *Mathematica* Version 4, Wolfram Media/ Cambridge University Press, 1999