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Analysis of Value-at-Risk in Shanghai Stock Market using OGARCH-class Models

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Abstract

The purpose of this paper is to detect and propose appropriate models to forecast the Value-at-Risk (VaR) of A-Share index of Shanghai Market. We apply OGARCH-class models (Liu and Morimune (2005)) to estimate the daily VaR, and forecast the one-day-ahead VaR of the log returns of the A-Share index of Shanghai market. By comparison, we show that the OGARCH-class approach outperform the related GARCH-class models. Moreover, we propose some combined models of OGARCH and EVT models. Empirical studies described herein show that these combined models provide better performance than other models used in this paper.

1 Introduction

In this paper, we detect appropriate models to forecast the Value-at-risk (VaR) for the A-Share index of the Shanghai Stock Exchange (for the A-Share index, see Liu and Morimune (2005)). Some OGARCH-class models, including OGARCH and OEGARCH models proposed by Liu and Morimune (2005), are applied to estimate the daily VaR and to forecast the one-day-ahead VaR of the log returns[1] of

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[1]Define the log return as $y_t = \ln(x_t/x_{t-1})$, where x_t denotes the closing price of the tth day.

the A-Share index. For comparison, VaR is simultaneously estimated and forecast using other models. The performances of the models are then evaluated using the results of forecasts and some tests. The results show that the OGARCH approach outperform the related GARCH-class models, which do not account for the effects of spells of shocks (for spells of shocks to see Liu and Morimune, 2005). Moreover, some combined models of OGARCH and EVT models are proposed as a practical approach. Empirical studies described herein show that these combined models provide better performance than other models used in this paper.

The outline of this paper is as follows. First, a definition of VaR is provided in section 2. Section 4 to 6 present a review for estimation and forecasting methods of VaR using the EWMA model, GARCH-class models, EVT models and some joint models of the GARCH-class and EVT models. The way to expand some of these models using our OGARCH-class models is also described. In section 7, new approach is proposed by combining OGARCH and EVT models. Section 8 shows some methods for evaluating and comparing these models. In section 9, VaR is estimated and forecast for the A-Share index using various models described in previous sections. Moreover, the performances of these methods are compared. Finally, a conclusion is given in section 10.

2 Definition of VaR

Several versions of the VaR definition exist. For an asset, the VaR can have two sorts of definitions with respect to the positions, long or short, that an investor holds. Our study specifically address the day-to-day risk of loss. Therefore, a definition of VaR is adopted here for a long position for daily data. For a long position with a small probability a and a time index t, we define VaR as

$$VaR_{t,a} \equiv y_a = \inf \{y|F(y) \geq a\} \tag{2.1}$$

where $F(\cdot)$ is the cumulative distribution function (CDF) of log returns y_t and y_a is the ath quantile of F. For a long position, such VaR corresponds to a threshold. A loss greater than this threshold occurs rarely, with probability equal to a on the tth day. For a short position, the definition of VaR is

$$VaR_{t,a} \equiv y_{1-a} = \inf \{y|F(y) \geq 1-a\}. \tag{2.2}$$

3 Approach with EWMA model

The EWMA models were developed by JP Morgan Chase and Co.; the models are also called RiskMetrics^{TM}. According to Tsay (2002), the EWMA models can be expressed as an IGARCH(1,1) model without a drift. We express the EWMA model as

$$\begin{aligned} y_t &= \mu + \varepsilon_t \\ \varepsilon_t &= \sqrt{h_t} z_t \\ h_t &= (1-\lambda)\,\varepsilon_{t-1}^2 + \lambda h_{t-1}, \end{aligned} \tag{3.1}$$

where z_t is assumed as white noise with mean 0, variance of one. J.P. Morgan suggests the value of λ, 0.94. The unknown parameter, μ, can be estimated using quasi-maximum likelihood estimation (QMLE). The methods for forecasting VaRs resembles those of the GARCH-class models, which will be described in the proceeding section.

4 Approaches with GARCH and OGARCH-class models

A simple method for forecasting VaRs for some assets uses GARCH-class models. Tsay (2002) and Dowd (2006) provide useful examples. The approach using OGARCH-class models is a simple expansion of GARCH-class model approach. We show a common method for both GARCH and OGARCH-class models.

Denote the log return of assets as y_t, commonly, a GARCH or OGARCH-class model can be expressed as

$$\begin{aligned} y_t &= \mu + f(\mathbf{x}_t) + \varepsilon_t \\ \varepsilon_t &= \sqrt{h_t} z_t \\ h_t &= g(\omega_t), \end{aligned} \tag{4.1}$$

where h_t is called volatility. Generally, $\mathbf{x}_t$ is a vector including lags of y_t, and ω_t is a vector including lags of h_t and lags of ε_t, In addition, z_t is an i.i.d. white noise process following a random distribution with cumulative distribution function $F(\cdot)$, with mean 0 and variance of 1, and $f(\cdot)$ and $g(\cdot)$ denote any well-behaved functions. When the log returns of some asset conform to such a model, the VaR of a long position on the tth day is calculated as

$$VaR_{a,t} = \mu + f(\mathbf{x}_t) + z_a\sqrt{h_t} \tag{4.2}$$

where $z_a = \inf\{z|F(z) \geq a\}$, for a small probability: $0 < a < 1$.

To forecast VaR for a GARCH-class model, the unknown parameters must first be estimated. The QMLE method is used here. As a simple example, for an

OGARCH(1,1) model with z_t, a white noise with mean 0 and variance 1, following Liu and Morimune (2005), we can form the expressions

$$\begin{aligned} y_t &= \mu + \varepsilon_t \\ \varepsilon_t &= \sqrt{h_t} z_t \\ h_t &= \alpha_0 + \alpha_1 \exp(\phi \gamma_{t-1}) \varepsilon_{t-1}^2 + \beta h_{t-1}. \end{aligned} \tag{4.3}$$

Once we obtain the estimates of coefficients, $\hat{\mu}, \hat{\alpha}_0, \hat{\alpha}_1, \hat{\beta}, \hat{\phi}$ and $\hat{h}_t$, and assume $z_t \sim N(0,1)$, we can forecast the VaR with $a = 0.05$ for time $t+1$ as

$$\widehat{VaR}_{0.05,t+1} = \hat{\mu} - 1.65 \times \sqrt{\hat{h}_t}, \tag{4.4}$$

where the value -1.65 is used as the 5% quantile of a standard normal distribution, and

$$\sqrt{\hat{h}_t} = \hat{\alpha}_0 + \hat{\alpha}_1 \exp(\hat{\phi} \gamma_{t-1}) \hat{\varepsilon}_{t-1}^2 + \hat{\beta} h_{t-1} \tag{4.5}$$

is the estimation of h_t. The forecasting methods for other GARCH-class models are similar. We show some empirical analyses related to forecasting VaR by OGARCH and OEGARCH models and some other GARCH-class models in section 9.

5 Approach based on EVT

Another popular approach for forecasting VaR is based on a theory called EVT. In this section, we first give a brief review of EVT, which includes theories of generalised extreme-value distribution (GEV) and generalised Pareto distribution (GPD). Basic methods for estimating and forecasting VaR based on EVT are presented next, followed by a description of how to address dependence of data.

5.1 Analyze VaR using GEV

The theory of GEV, which is used to analyze VaR, is based on the Fisher-Tippett theorem (Fisher and Tippett (1928)). Roughly speaking, GEV can be regarded as an asymptotic distribution of the extreme value of some random variable.

5.1.1 Derivation and basic properties

Let $\{x_i\}$, $i = 1, 2, \cdots, n$, be a independent random sample from some underlying distribution F; then define $M_n \equiv \max(x_i)$. If the cumulated distribution function F satisfies some regular conditions, then two sequences, c_n, b_n, and a parameter ξ

exist such that the normalized minimum $(M_n - b_n)/c_n$ asymptotically follows the following distribution

$$
\begin{aligned}
H(x) &\equiv \lim_{n\to\infty} P\left\{\frac{M_n - b_n}{c_n} \le x\right\} \\
&= \begin{cases} \exp\left(-(1+\xi x)^{-1/\xi}\right) & \text{for } \xi \neq 0 \\ \exp\left(-\exp(-x)\right) & \text{for } \xi = 0 \end{cases}
\end{aligned}
\tag{5.1}
$$

for x that satisfies $1+\xi x > 0$. In other words, such F is in the maximum domain of attraction (MDA)[2] of H, which is called the generalised extreme-value distribution (GEV). Jenkinson (1955) gives this presentation of GEV. It is a representation of the extreme value distributions discovered using the Fisher-Tippett theorem (Fisher and Tippett (1928)).

The two sequences c_n and b_n sometimes are called location series and series of scaling factors; the parameter ξ is called shape parameter (see Tsay (2002)). The shape parameter ξ indicates the shape of the tail of the underlying distribution F: the larger the value of ξ the fatter the tail of the F. $\alpha = 1/\xi$ is called the tail index in financial studies. For example, α of a Student t-distribution is its degree of freedom; for a normal distribution $\xi = 0$, $\alpha = \infty$.

Relative to different values of ξ, GEV is separable into three types: the Gumble distribution for $\xi = 0$, the Fréchet distribution for $\xi > 0$, and the Weibull distribution for $\xi < 0$. A summary of the characteristics of the underlying distribution F of these three types is tabulated in Table 5.1:

Table 5.1: Summary and Example of ETV.

Type	ξ and α	Right tails of F	Example of F
Gumble	$\xi = 0, a = \infty$	exponential tails	normal, gamma
Fréchet	$\xi > 0, a < 0$	fat tails	student's t, Pareto
Weibull	$\xi < 0, a > 0$	truncated at $x = 1/\xi$	uniform, beta

From Table 5.1, the Frechet type is seen to correspond to an underlying distribution F which has fat tails. Numerous studies have revealed that the distributions of log returns of financial assets have a fat tail. Therefore, the Frechet type will be examined, particularly as we analyze VaR for financial assets.

Differentiating eq. (5.1), one can derive the probability density function (PDF)

[2] For details of MDA, see Embrechts et al. (2001).

relative to H as follows

$$h(x) = \begin{cases} (x\xi+1)^{-\frac{1}{\xi}-1} \exp\left[-(x\xi+1)^{-\frac{1}{\xi}}\right] & \text{for } \xi \neq 0 \\ \exp\left[-x-\exp(-x)\right] & \text{for } \xi = 0 \end{cases} \tag{5.2}$$

where x satisfies $1+\xi x > 0$. In our study, we specifically examine only the VaR of long positions. For that reason, the concern is on the left tail of the underlying distribution F, the GEV for normalized minimum, $\underline{M}_n \equiv \min(x_i)$ is important. Using simple transformations, one can derive the formula for $\underline{M}_n$ from that of M_n:

$$\begin{aligned} H(x) &\equiv \lim_{n\to\infty} P\left\{\frac{\underline{M}_n - b_n}{c_n} \geq x\right\} \\ &= \begin{cases} 1-\exp\left(-(1-\xi x)^{-1/\xi}\right) & \text{for } \xi \neq 0 \\ 1-\exp(-\exp(x)) & \text{for } \xi = 0 \end{cases} \end{aligned} \tag{5.3}$$

and density function of $\frac{\underline{M}_n - b_n}{c_n}$

$$f(x) = \begin{cases} (1-\xi x)^{-1/\xi-1} \exp\left[-(1-\xi x)^{-1/\xi}\right] & \text{for } \xi \neq 0 \\ \exp\left[x-\exp(x)\right] & \text{for } \xi = 0 \end{cases} \tag{5.4}$$

for detail see Tsay (2002)[3]. From eq. (5.4), we can derive the PDF of $\underline{M}_n$ as follows:

$$g(x|c_n, b_n, \xi) = \begin{cases} \frac{1}{c_n}\left(1-\xi\frac{\underline{M}_n - b_n}{c_n}\right)^{-1/\xi-1} \exp\left[-\left(1-\xi\frac{\underline{M}_n - b_n}{c_n}\right)^{-1/\xi}\right] & \text{for } \xi \neq 0 \\ \frac{1}{c_n}\exp\left[\frac{\underline{M}_n - b_n}{c_n} - \exp\left(\frac{\underline{M}_n - b_n}{c_n}\right)\right] & \text{for } \xi = 0 \end{cases}, \tag{5.5}$$

where $1-\xi\frac{\underline{M}_n - b_n}{c_n} > 0$.

5.1.2 Estimation

Using PDF, as shown in equation (5.5), one can perform maximum likelihood estimation (MLE) to estimate the unknown parameters of GEV. However, for a sample $\{x_i\}, i = 1, 2, \cdots, n \times m.$, only one minimum exists. With only one observation, MLE does not work. Tsay (2002) presents a means to solve this problem. He divide the samples into m blocks with equal sample size, n, and obtains an minimum, $\underline{M}_{n,k}$, for $k = 1, 2, \cdots, m$, for every block. Assuming the minima of m blocks as

[3]The definition of ξ is different from the definition of k in Tsay (2002).

an i.i.d. sample from GEV, one can use this sample to estimate parameters of GEV by MLE. The log-likelihood function is

$$\ln l\left(\underline{M}_{n,1}, \cdots, \underline{M}_{n,m} | c_n, b_n, \xi_n\right) = \sum_{k=1}^{m} \ln\left\{g\left(\underline{M}_{n,k} | c_n, b_n, \xi_n\right)\right\},$$

where $g\left(\cdot | c_n, b_n, \xi_n\right)$ is the PDF of the block minimum.

5.1.3 Selection of n and m

An empirical problem exists on MLE of GEV. For different values of n we will we obtain different estimates of c_n, b_n and ξ_n. For c_n and b_n , this does not seem serious because the true values of c_n and b_n are vary with n. However, the true value of ξ does not vary; consequently, the selection of the size of blocks n and the sample size of $\underline{M}_{n,k}$, m, becomes very important. Because a large value of n can raise the quality of the minimum, and because a large value of m give us a large sample to increase the performance of MLE, we want to use large n and m to the greatest possible degree. Nevertheless, we must confront the restriction of the total sample size of x_i, which is $n \times m$. In empirical studies, we must take a balance for n and m. According to the predictive performance, one way to solve this problem is to first estimate unknown parameters with various n and m; thereafter, as the final estimates, we can select the estimates relative to the n and m which correspond to the most predictive performance.

5.1.4 Estimation and forecasting of VaR

From equation (5.4), one can derive the ath quantile of GEV for minima

$$\underline{M}_a = \begin{cases} b_n + \frac{c_n}{\xi_n}\left\{1 - \left(-\ln\left(1-a\right)\right)^{-\xi_n}\right\} & \text{for } \xi_n \neq 0 \\ b_n + c_n \ln\left\{-\ln\left(1-a\right)\right\} & \text{for } \xi_n = 0, \end{cases}$$

then use the relationship

$$a = P\left(\underline{M}_{n,k} \leq \underline{M}_a\right) = 1 - \left\{1 - P\left(x_i \leq \underline{M}_a\right)\right\}^n.$$

Thereby, it is apparent that the VaR, which corresponds to probability $a^* \equiv P\left(x_i \leq \underline{M}_a\right)$, is calculated as

$$VaR_{a^*} = \begin{cases} b_n + \frac{c_n}{\xi_n}\left\{1 - \left(-n\ln\left(1-a^*\right)\right)^{-\xi_n}\right\} & \text{for } \xi_n \neq 0 \\ b_n + c_n \ln\left\{-n\ln\left(1-a^*\right)\right\} & \text{for } \xi_n = 0. \end{cases} \quad (5.6)$$

For detail to see Tsay (2002) and Dowd (2006).

For forecasting, we use the VaR that was estimated using the sample up to time t as the predicate of VaR for one time interval ahead, time $t+1$.

5.2 Analyze VaR by GPD

Embrechts, Klüppelberg, and Mikosch (2001) show us another EVT approach. This approach is based on GPD. . Assuming $\{x_i\}$, $i = 1, 2, \cdots, n$. is an i.i.d. sample from an underlying distribution F, and F is in the MDA of H. Consequently,

$$F_u(x) \approx G_{\xi, b(u)}(x)$$

where

$$F_u(x) = P(X - u < x | X > u)$$

and

$$G_{\xi, c(u)}(x) = \begin{cases} 1 - \left(1 + \xi \frac{x}{c}\right)^{-1/\xi} & \text{for } \xi \neq 0 \\ 1 - \exp\left(\frac{-x}{c}\right) & \text{for } \xi = 0 \end{cases}$$

is the generalised Pareto distribution (GPD).

Smith (1987) derives the density function of GPD for $\xi \geq 0$[4] as

$$f(x) = \frac{1}{c}\left(1 + \xi \frac{x}{c}\right)^{-1/\xi - 1}, c \geq 0.$$

Choose a high threshold u, and for for $x_i > u$, define $z_i \equiv x_i - u$, $i = 1, 2, \cdots, N_u$. Therein, N_u is the number of x_i larger than u. Then, regarding $\{z_i\}$, $i = 1, 2, \cdots, N_u$, as a sample from GPD, one can estimate unknown parameters by MLE using the log-likelihood function as

$$\ln l(z_1, \cdots, z_{N_u} | c, \xi) = -N_u \ln(c) - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^{N_u} \ln\left(1 + \frac{\xi z_i}{c}\right),$$

where $\xi \geq 0$, $c \geq 0$.

For a short position, following Embrechts, Klüppelberg, and Mikosch (2001), once $\hat{\xi}$ and $\hat{c}$, the estimates of ξ and c, are obtained, then the VaR for level a can be estimated as

$$\widehat{VaR}_a = u + \frac{\hat{c}}{\hat{\xi}}\left\{\left[\frac{n}{N_u}(1 - \alpha)\right]^{-\hat{\xi}} - 1\right\}. \tag{5.7}$$

For forecasting, because the method resembles that for GEV, its description is omitted here.

For a long position, using eq. (5.7), we can calculate $\widehat{VaR}_a(y)$ for a new series $\{y_i\} \equiv \{-x_i\}$, yielding $\widehat{VaR}_a(x) = -\widehat{VaR}_a(y)$ for $\{x_i\}$.

[4] We only concern on the case for $\xi \geq 0$, which correspond to distributions with fat tails.

5.3 EVT for dependent data

The EVT described above operates under an i.i.d. assumption. Can we apply such inference of EVT directly to extreme values of non-i.i.d. data as well? The answer is 'No'. However, for some classes of strictly stationary processes, the answer is 'Yes', for which we can calculate VaR using the same method as that based on EVT of i.i.d. data.

According to Embrechts, Klüppelberg, and Mikosch (2001), if an i.i.d. sequence $\{x_i\}, i = 1, 2, \cdots, n$, and a strictly stationary sequence $\{y_t\}$ $t = 1, 2, \cdots, n$ have same marginal distribution, then we have

$$\frac{M_{n,x} - b_n}{c_n} \xrightarrow{d} H \Leftrightarrow \frac{M_{n,y} - b_n}{c_n} \xrightarrow{d} H^{\theta} \tag{5.8}$$

where c_n, b_n and H are the location series, the series of scaling factors and GEV, respectively. In addition, θ is called extremal index. Using simple algebra, one can show that $H^{\theta}(x) = H(cx + d)$ for some $c > 0$, $d \in \mathbb{R}$, which means that choosing some appropriate series as c_n and b_n, one can obtain H^{θ}, which is in the same form of H. In other words, if $\{x_i\}$ has a GEV, H, then $\{y_t\}$ also has a GEV, H^{θ}, which takes the same form as H, the difference is only that of parameters. Therefore, we can perform MLE and forecast the VaR for no-i.i.d. data by the same manner as for i.i.d. data described above.

6 Joint approach

McNeil and Frey (2000) present a procedure to estimate and forecast VaR for a heteroscedastic financial time series. They fit a GARCH-class model to data using QMLE, then apply GPD to the standardized residuals of the GARCH-class model. We define the standardized residual for a GARCH-class models as

$$e_t^* \equiv \frac{\hat{\varepsilon}_t}{\sqrt{\hat{h}_t}}, \tag{6.1}$$

where $\hat{\varepsilon}_t$ and $\hat{h}_t$ are the estimate of ε_t and h_t. The procedure for this joint approach is as follows:

1. Estimate unknown parameters of a particular GARCH-class model for log returns using QMLE.

2. Calculate residuals of these GARCH-class models according to the parameters estimated in step 1.

3. Calculate the standardized residuals e_t^* following eq. (6.1).

4. Apply GPD to the standardized residuals e_t^*.

5. Calculate the predictive values of VaRs according to eq. (5.7).

In step 1, to perform QMLE (see Weiss (1986)), only an i.i.d. assumption for disturbance, z_t, is required. The distribution of disturbance does not need to be assumed as a specific distribution. Using QMLE, one can obtain consistent estimators with normality under some reasonably regular conditions. For the OGARCH-class models, we can construct joint models in a similar manner.

Using some examples of empirical study, McNeil and Frey (2000) show that such a joint approach outperforms approaches which use GARCH-class models or GPD alone because this joint approach can take account of the dynamic structure by the GARCH-class models and the fat tail characteristic of the standardized residuals by GPD.

We also construct joint models of GARCH-class and OGARCH-class models using GEV models. The procedure for estimation is similar to that of joint approach of GARCH and GPD models. In order to save space, a description of this similar procedure is omitted here.

7 A practical combined approach

We propose a new approach by combine OGARCH model and EVT models to improve the performance of VaR models with respect to saving cost of risk management. One can construct combined models of other GARCH-class models and EVT models similarly. This approach can be expanded by using other GARCH-class models. The estimator of VaR for the left tail with level a on tth day for this approach is defined as

$$\widehat{VaR}_{C,a,t} \equiv \max\left(\widehat{VaR}_{O,a,t}, \widehat{VaR}_{E,a,t}\right), \tag{7.1}$$

where $\widehat{VaR}_{O,a,t}$ and $\widehat{VaR}_{E,a,t}$ respectively denote the predictive value of VaR by OGARCH-class models and EVT models approaches with level of VaR, a. The procedure for this combined approach is as follows:

1. Estimate unknown parameters of a particular OGARCH-class model for log returns.

2. Apply EVT, GEV or GPD, to log returns and estimate unknown parameters by MLE.

3. Forecast VaR according to the parameters estimated in step 1 and 2 to obtain $\widehat{VaR}_{O,a,t}$ and $\widehat{VaR}_{E,a,t}$.

4. Calculate the predictive values of $\widehat{VaR}_{C,a,t}$ using eq. (7.1).

The idea of this approach is very simple. However, for investors who want to limit cost of risk management or take some measures to meet BIS regulations (the Basel Committee on Banking Supervision (1995) and the Basel Committee on Banking Supervision (1996)), this approach is extremely useful. It incorporates advantages of both OGARCH and EVT models. We will argue what these advantages are and provide some evidence for this point by presenting some results of empirical studies in section 9.

8 Evaluation methods

Many methods exist to evaluate the performance of various VaR models. They have an identical character, using the predictive value for previous days. Therefore, these methods are called backtests by some researchers. First, three popular likelihood ratio type backtests are reviewed. Secondly, we show how to evaluate models according to the daily capital requirement.

8.1 Likelihood ratio tests for coverage probability

Christoffersen (1998) proposed three likelihood ratio tests for coverage probability to evaluate VaR model performance. The coverage probability is $1-a$ in our setup, where a, the level of VaR, is set by researchers or investors. The first LR test test has a null hypothesis, $\mathrm{H}_0 : a^* = a$, and alternative hypothesis, $\mathrm{H}_1 : a^* \neq a$, where a^* is the probability of unconditional coverage failure. a^* can be estimated as $\hat{a}^* = m/n$, where n is the total number of days in the predictive interval. Also, m is the number of violations: the number of observations which are smaller than the related predictive values of VaRs. The LR statistic takes the following form:

$$\mathrm{LR}_{uc} = -2\ln\frac{\left[(1-a)^{n-m}(a)^m\right]}{\left[(1-\hat{a}^*)^{n-m}(\hat{a}^*)^m\right]}, \tag{8.1}$$

it converges in law to χ^2 with one degree of freedom.

The second is for testing whether the violations are time-independent. The null hypothesis is independent. For a random sample $\{x_t\}$, n and $\widehat{VaR}_t = x_t^*$,

$t = 1, 2, \cdots, n$,

$$a_{01} = P\left(x_t \geq x_t^* | x_{t-1} < x_{t-1}^*\right)$$
$$a_{11} = P\left(x_t \geq x_t^* | x_{t-1} \geq x_{t-1}^*\right).$$

The statistic is as follows:

$$\mathrm{LR}_{ind} = -2\ln \frac{\left[(1-\hat{a}^*)^{n_{00}+n_{10}} (\hat{a}^*)^{n_{01}+n_{11}}\right]}{\left[(1-\hat{a}_{01})^{n_{00}} \hat{a}_{01}^{n_{01}} (1-\hat{a}_{11})^{n_{10}} \hat{a}_{11}^{n_{11}}\right]}, \tag{8.2}$$

which converges in law to χ^2 with one degree of freedom. In LR_{ind}

$$\hat{a}_{01} = n_{01} / (n_{00} + n_{01})$$
$$\hat{a}_{11} = n_{11} / (n_{10} + n_{11})$$

are the estimates of a_{01} and $a_{11}M$, respectively, and

$$n_{00} = \sum_{t=2}^{n} I\left(x_{t-1} < x_{t-1}^*, x_t < x_t^*\right) \tag{8.3}$$

$$n_{01} = \sum_{t=2}^{n} I\left(x_{t-1} < x_{t-1}^*, x_t \geq x_t^*\right) \tag{8.4}$$

$$n_{10} = \sum_{t=2}^{n} I\left(x_{t-1} \geq x_{t-1}^*, x_t < x_t^*\right) \tag{8.5}$$

$$n_{11} = \sum_{t=2}^{n} I\left(x_{t-1} \geq x_{t-1}^*, x_t \geq x_t^*\right) \tag{8.6}$$

where $I(\cdot)$ is the indicator function.

The last is a joint test of coverage and independence. The statistic is a combination of LR_{uc} and LR_{ind}:

$$\mathrm{LR}_{\mathrm{cc}} = \mathrm{LR}_{\mathrm{uc}} + \mathrm{LR}_{ind}, \tag{8.7}$$

which converges in law to χ^2 with two degrees of freedom. Coverage and independence can be tested jointly using LR_{cc}.

8.2 Daily capital requirement

Another evaluation method is based on the BIS rule regarding daily capital requirements. According to the Basel Committee (the Basel Committee on Banking Supervision (1995) and the Basel Committee on Banking Supervision (1996)), banks are allowed to use internal models to forecast their daily VaRs. According to these predictive values of VaRs, daily capital requirements are charged to banks. the Basel Committee on Banking Supervision (1995) states that a daily capital requirement must be set as the higher of the previous day's predictive value of VaR, or an average of predictive values of VaRs on last 60 days times a scaling factor k, which is usually large than 3. The daily capital requirement can be expressed as the following:

$$DCR_t = \max\left(\left|\widehat{VaR}_{t-1}\right|, k\left|\frac{1}{60}\sum_{i=1}^{60}\widehat{VaR}_{t-i}\right|\right). \tag{8.8}$$

Therein, $|\cdot|$ represents the absolute value.

In order to encourage banks to refine their VaR models, Basel Committee (the Basel Committee on Banking Supervision (1996)) proposed a rule of penalty. Banks with internal models that engender numerous violations must be charged a high daily requirement. A penalty is imposed as an increase in scaling factor k. That increase is relative to the number of violations. According to the Basel Committee on Banking Supervision (1996), Table 8.1 presents details related to the penalty rule for a case based on 250 daily observations of predictive VaRs.

Table 8.1: BIS Rule

Zone	Number of violations	Increase in k
	0	0
	1	0
Green	2	0
	3	0
	4	0
	5	0.4
	6	0.5
Yellow	7	0.65
	8	0.75
	9	0.85
Red	10	1

The information in this table is from Basel Committee(1996).

According the rule described above, one can calculate values of daily capital requirements, using scaling factor k derived based on Table 8.1. The mean value of daily capital requirements is useful to evaluate various VaR models. We abbreviate it to MDCR hereafter.

9 Empirical analysis

Liu and Morimune (2005) show that the OGARCH model is useful for analyzing the log returns of A-Share of the Shanghai Stock Exchange. As an empirical study, in this section, we forecast VaRs for log returns of the A-Share index of the Shanghai Stock Exchange. For comparison, we forecast VaRs using other GARCH-class models, EVT models, joint models and the combined models as well. As a preliminary analysis, before forecasting the VaRs, we examine the data and estimation results of some of these models in the proceeding subsection.

9.1 Data and preliminary analysis

The data are the daily log returns of the A-Share index, as used in Liu and Morimune (2005). To check whether we need to apply EVT or joint models to A-Share index, we analyze the characteristics of the standardized residuals e_t^*. For example, the standardized residuals e_t^* of the OGARCH(1,1) model, are calculated as follows

$$\begin{aligned} e_t^* &= (y_t - \hat{\mu}) \Big/ \sqrt{\hat{h}_t} \\ \hat{h}_t &= \hat{\alpha}_0 + \hat{\alpha}_1 \exp\left[\hat{\phi}\gamma_{t-1}\left(y_{t-1} - \hat{\mu}\right)^2\right] + \hat{\beta}h_{t-1}, \end{aligned} \tag{9.1}$$

where $\hat{\mu}$, $\hat{\alpha}_0$, $\hat{a}_1$, $\hat{\phi}$ and $\hat{\beta}$ are the estimates of parameters.

We calculate the e_t^* for GARCH-class and OGARCH-class models for the A-Share data using the estimates in Tables 9.1 and 9.2. For these estimations, the sample period of the A-Share index extends from 5 January, 2001 to 29 September, 2006.

Table 9.1: OGARCH and GARCH Estimation

Parameter	μ	α_0	α_1	β	ϕ	$\log L$
			A share			
GARCH						
Coeff	-0.0069	0.0849^{**}	0.1254^{**}	0.8361^{**}		-2297.93
SE	0.0332	0.0267	0.0336	0.0332		
OGARCH						
Coeff	-0.0227	0.0918^{**}	0.0625^{*}	0.8353^{**}	0.2916^{**}	-2290.67
SE	0.0333	0.0264	0.0278	0.0338	0.1047	

Note: The sample period of the A-Share extends from 5 January, 2001 to 29 September, 2006. * and ** indicate significant coefficient at 5% and 1%, respectively.

Table 9.2: OEGARCH and EGARCH Estimation

Parameter	μ	α_0	α_1	β	κ	ϕ	$\log L$
				A-Share			
EGARCH							
Coeff	-0.0155	0.0280^{**}	-0.0596^{*}	0.2115^{**}	0.9654^{**}		-2280.02
SE	0.0316	0.0100	0.0281	0.0412	0.0141		
OEGARCH							
Coeff	-0.0674^{*}	0.0308^{**}	-0.0420	0.1551^{**}	0.9615^{**}	0.1390	-2278.71
SE	0.0315	0.0101	0.0276	0.0435	0.0145	0.0834	

Note: The numbers of observations is 1381 for A-Share. * and ** indicate significant coefficient at 5% and 1%, respectively.

The summary statistics of e_t^* for several GARCH-class and OGARCH-class models are shown together with the statistics of the log returns, y_t, in Table 9.3. Observing the second line in Table 9.3, we can see that after standardization by conditional volatilities, the kurtoses of e_t^* of these models more closely approximate 3, the kurtosis of standard normal distribution, than 8.42, the kurtosis of log returns. Nevertheless, the kurtoses of the four models are all larger than 5, which indicates that e_t^* of these models follow distributions with fat tails.

We draw QQ-plots for e_t^* against a standard normal distribution and log returns y_t. The graphs are shown in Figures 9.1 and 9.2. Figure 9.2 shows that the tails of e_t^* are less fat than that of y_t. Therefore, it can be concluded that GARCH-class models can cancel fatness of the tails to some extent. However, Figure 9.1 indicates that these standardized residuals e_t^* do not conform to a standard normal distribution; instead, they apparently follow distributions with fat tails. Consequently, applying a GARCH-class model with a standard normal disturbance approach to the A-share index seems to involve some problems. We will examine application of other models described above to analyze VaR in subsequent sections.

9.2 Estimation results

For GARCH-class models and OGARCH-class models, we can find estimation results in Liu and Morimune (2005). In this subsection, we present estimation results of EVT models and EVT models for standardized residuals in the joint approach. The sample period is set to be from 6 January, 1998 to 29 September, 2006 with sample size $n = 2106$. The estimation methods used here are described in section 5. Two cases of GEV model are estimated: the size of blocks n for one case is set as 21, and for the other case, n is 63. The two cases are respectively denoted as GEV21 and GEV63. According to Tsay (2002), the value 21 corresponds to the number of trading days in one month, and 63 to a quarter, approximately. For GPD, to guarantee a sufficient large sample size of block minimum, we decide the value of u according to sample size of block minimum. Two cases of GPD are estimated relative to different values of u:

- case 1,
$$u = \{u|\text{card}\{y_t|y_t > u, t = 1, 2, \cdots, n\} = 180\} \tag{9.2}$$
- case 2,
$$u = \{u|\text{card}\{y_t|y_t > u, t = 1, 2, \cdots, n\} = 100\}, \tag{9.3}$$

where card(A) represents the cardinal number of the set A, which returns the number of elements in set A. We respectively denote these two cases as GPD 180 and GPD 100. The estimation results of these models are tabulate in Table 9.4.

Table 9.3: Summary statistics of standardized residuals.

	Mean	Skewness	Kurtosis
Log Return	−0.01	0.64	8.42
GARCH	−0.02	0.27	5.62
OGARCH	−0.01	0.32	5.69
EGARCH	−0.01	0.28	5.47
OEGARCH	0.03	0.26	5.39

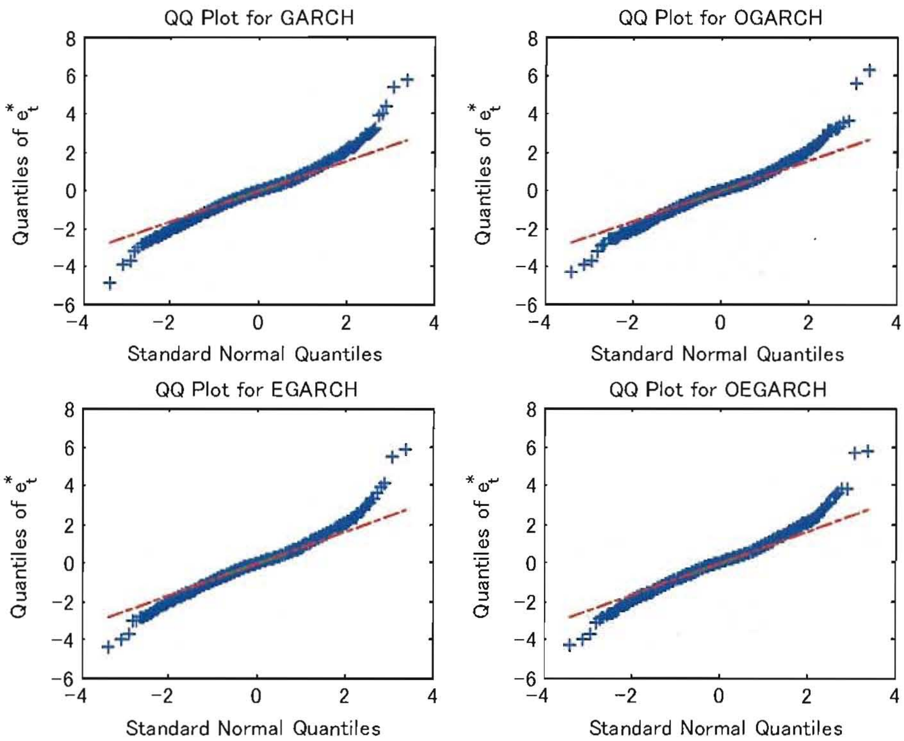

Figure 9.1: QQ plots for e_t^* aginst standard normal.

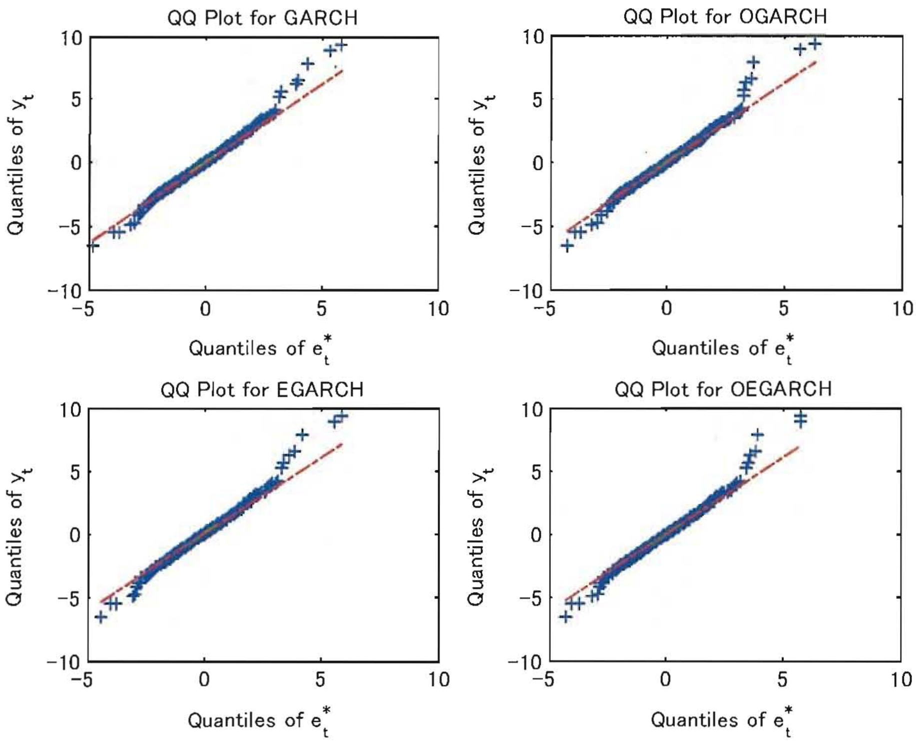

Figure 9.2: QQ plots of e_t^* aginst y_t.

When the estimates hold normality, we can perform a Student's[5]. For both cases of GEV models, the estimates of c and b are significant with confidence level 5% against the null hypotheses $c = 0$ and $b = 0$. However, the estimates of ξ are not significant with a confidence level 5% against the null hypothesis $\xi = 0$. For GPD models, we obtain similar results: the estimates of c are significant but those of ξ are not.

Estimation results of ETV models for the series e_t^* of eight joint models are shown in Table 9.5. For GEV, the sample size in each block are set as $n = 21$, for GPD the value of threshold is set as $u = \{u|\text{card}\,\{y_t|y_t > u, t = 1, 2, \cdots, n\} = 180\}$. All estimates are significant except for ξ, the problem of significance of ξ becomes more serious.

In estimations mentioned above, there is a fact cause problem: the total sample size of log returns is not so large, we can not expect high quality of the sample, minima or y_t lager than u, used in the estimations. Another possible reason is that, when the true value of ξ is overly near to 0, the power of the test will worsen. A conclusion cannot be inferred based only on these t tests of coefficients. We need to check and compare models using some backtests, which include three coverage test and comparisons by number of violations, mean absolute predictive VaR values (MVaR) and MDCR. Those will be performed in the proceeding subsection.

9.3 Backtests

We use the first 1806 observations from 6 January, 1998 as a sample to forecast the VaR of the 1807th day using the methods described in previous sections. Thereafter, maintaining the length of rolling windows as 1806, we forecast VaR for the proceeding 300 days. Two series with 300 predictive VaRs with probabilities 5% and 1% are forecasted.

The models are evaluated using several: the three LR type backtests and evaluation methods according to the number of violations, MVaRs, and MDCRs.

9.3.1 LR tests

We calculate the three statistics, LR_{uc}, LR_{ind} and LR_{cc}, for various models with VaR level $a = 5\%$ and 1%. The results are tabulated in Table 10.1 and Table 10.2. The statistics all show that no tests can reject the null hypothesis at significance levels of 5% or 1%. In other words, only according to these tests, we can not conclude which models are better.

[5] For asymptotic theory on ETV, see Smith (1985).

Table 9.4: Estimation Results of ETV Models

	c	SD	b	SD	ξ	SD
GEV21	0.8807	0.0725	-1.9242	0.1084	0.1103	0.0746
GEV63	0.8211	0.1464	-2.6549	0.1665	0.3975	0.2485
GPD180	0.7443	0.0905			0.1339	0.0858
GPD100	0.6733	0.1119			0.2549	0.1526

SD denotes stadard deviation.

Table 9.5: Estimation Results of Joint Models

GEV21	c	SD	b	SD	ξ	SD
GARCH	0.6090	0.0514	-1.6435	0.0774	0.0080	0.0834
OGARCH	0.6026	0.0514	-1.6358	0.0760	0.0214	0.0872
EGARCH	0.5874	0.0515	-1.6182	0.0729	0.0350	0.1025
OEGARCH	0.5864	0.0502	-1.6104	0.0723	0.0344	0.1073
GPD180	c	SD			ξ	SD
GARCH	0.5633	0.0668			0.0397	0.0875
OGARCH	0.5677	0.0672			0.0333	0.0876
EGARCH	0.5352	0.0658			0.0637	0.1053
OEGARCH	0.5239	0.0642			0.0764	0.1090

The top panel shows the results of the joint models composed of GEV21 and various OGARCH and GARCH class models. The bottom panel shows the results of joint models composed of GPD180 and various OGARCH and GARCH class models. SD denotes stadard deviation.

9.3.2 Evaluate models using other measures

To evaluate the relative performances of VaR models for A-Share, we use the number of violations, MVaR, and MDCR as measures. We infer that numerous violations reflect low performance. Moreover, usually, investors wish to manage risk with a small amount of reservation; banks find it onerous to conform to high capital requirements. For those reasons, low MVaR and MDCR indicate high performance of the model to them. Models will be evaluated in these respects.

We show here how to calculate MDCRs. Using the first 250 of the 300 predictive values of VaRs as a sample, we can calculate the value of scaling factor k, then calculate the value of the daily capital requirement of 251th day following equation (8.8). Maintaining the length of rolling windows as 250 and using the same method, we obtain values of daily capital requirements for the period from the 251st to 300th day. Consequently, the mean value of these 50 daily capital requirements, MDCRs, are obtained.

The first lines in Tables 10.3 and 10.4 show numbers of violations; the second lines show MVaR of 300 days. 10.3 is for the case with level of VaR $a = 5\%$, and 10.4 if for $a = 1\%$. For the case with level of VaR $a = 1\%$, the MDCRs are shown in the last line of Table 10.4.

9.3.3 Comparison

Firstly, we compare the OGARCH-class models with GARCH-class models. From Tables 10.3 and 10.4, it is apparent that MVaRs and MDCRs of OGARCH for the $a = 1\%$ case and MVaRs for the $a = 5\%$ case are less than those of GARCH model. Those of OEGARCH are also less than those of EGARCH. These results illustrate that, by adopting the spells of shocks, γ_{t-1}, into the GARCH-class models, we can obtain high performance in terms of risk management costs and of reducing daily capital requirements.

The results of joint models show a similar conclusion of comparison between GARCH-class models and OGARCH models. The joint models of OGARCH models produce better results than the joint models of GARCH models, and the joint models of OEGARCH models show better results than joint models of EGARCH models. Moreover, the results in Tables 10.3 and 10.4 show that the EGARCH and OEGARCH models outperform related GARCH and OGARCH models. Furthermore, the results in Tables 10.3 and 10.4 also demonstrate that the performance of EWMA is worse than other single models, and the performances of the Joint models composed of EWMA and other models are worse than relative other joint models.

Figures 10.1 and 10.2 depict the plots of predictive VaRs with VaR level $a =$

1%. The solid line in Figure 10.1 shows the values of log returns. The dashed line is a plot of predictive values of VaRs for GARCH model. The dash-dot line is for OGARCH models. The dash-dot line is almost always higher than the dashed line, which indicates that the OGARCH model approach can forecast VaR exactly like the GARCH model approach, but with small mean absolute values of predictive VaRs. Figure 10.1 exhibits further support that the OGARCH model outperformed GARCH model. In addition, to compare the OEGARCH model with the EGARCH model, we show their results in Figure 10.2. The graph depicted in Figure 10.2 supports the arguments related to Tables 10.3 and 10.4: OEGARCH offers superior performance to those of EGARCH models.

Secondly, we compare the GEV models with GPD models according to the three measures: number of violations, MVaRs and MDCR. According to the results of GEV and GPD models in Tables 10.3 and 10.4. The GEV models are apparently better than GPD models.

Thirdly, we compare models with high performances of the GARCH-class models and the EVT models. For the case with $a = 5\%$, although the MVaRs of some of the EVT models, GEV21 and GEV63, are less than that of the OGA-RCH and OEGARCH models, the numbers of violations of these EVT models are large. For the 1% case, the MVaRs of two GEV models are greater than those of OGARCH and EGARCH models, but the mean value of daily capital requirement of GEV21 is less than OGARCH models, and that of the GEV63 model is less than those of GARCH and OEGARCH models. Using these comparison results alone, conclusion about which sort of model is better can be inferred.

Fourthly, comparing joint models with others, the values of MVAR and MDCR in Tables 10.3 and 10.4 demonstrate that the joint models have no better performance than their related single models. This result differs from that obtained by McNeil and Frey (2000). They compare joint models with GARCH-class models and GPD models only using only binomial tests; they find that their joint model is better than a single GARCH-class model or GPD model. However, we can find a Figure in their paper (Fig. 8.), which show that their joint model carries out higher MVAR than other models. Some troubles arise when two models are combined together. A possible reason is that, since the distribution of standardized residuals, e_t^*, is more similar to a standard normal distribution than that of y_t, it becomes difficult to estimate EVT models exactly for e_t^*. We can compare the distributions of e_t^* with that of y_t and standard normal distribution, by observing Table 9.3, and Figure 9.1 together with Figure 9.2 in subsection 9.1.

Lastly, comparing the combined models with other models by MVaRs and MD-CRs, it is apparent that all the combined models outperform almost all other models. In Table 10.4, the MDCRs of some of them are larger than that of the GEV model. However, their MVaRs are less than that of GEV model. In particular, the combined model of OGARCH and GEV models with $n = 63$ (C-GEV63) offers the best performance with respect to MVaR and MDCR. Moreover, MVaR is considerably smaller than the other models.

To explain why the performance of the combined model of OGARCH and GEV models is superior to those of other models, we show plots of predictive VaRs of various models in Figures 10.3 and 10.4. The figures show that the plots of predictive VaRs for OGARCH model fluctuate dynamically, which captures the dynamics of log returns. In contrast, the plots for ETV models are almost horizontal lines, which can not capture the clustering characteristics of volatility. As described by Bekiros and Georgoutsos (2005), the EVT models are unsuitable for forecasting daily VaR; nevertheless, they are suitable for long-term forecasting. In addition, a weak point pertains to GARCH-class models: when GARCH-class models attempt to cover violations in a high-volatility period, they usually overreact and derive a high predictive value of VaR, which will cause a need for extra capital requirements. Observing Figure 10.5 and 10.6, it can be said that the combined models capture the dynamics of log returns, and do not overreact in periods with high volatilities. Consequently, combined models yield low MVaR and MDCR. In other words, they show a high performance with respect to costs of risk management.

10 Conclusion

The OGARCH-class models are applied along with other alternative models to analyze the VaR of A-Share index of Shanghai market. The results of some backtests and evaluations by the number of violations, MVaR and MDCR, show that the OGARCH-class models can provide better performance than the related GARCH-class models: the GARCH model and EGARCH model. The reason is that OGARCH-class models can capture the effects of spells of shocks.

Moreover, the empirical results of comparison between GARCH-class models and ETV models show that GARCH-class models can capture the dynamic structures of log returns. On the other hand, the ETV models can capture long-term characteristic of VaRs.

Furthermore, combined models were proposed and applied to analyze the VaR of the A-Share index. One combined model, which comprises OGARCH-class and GEV models, outperforms all other models. These combined models are appropriate for analyzing VaR of the A-Share index: they are effective with respect to reducing the costs of risk management.

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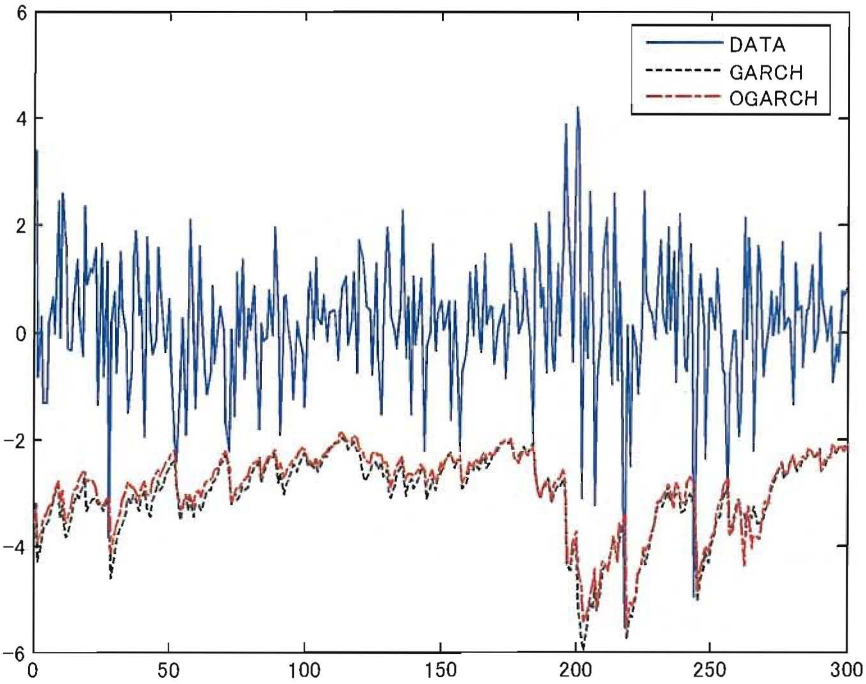

Figure 10.1: plots of VaR for GARCH and OGARCH

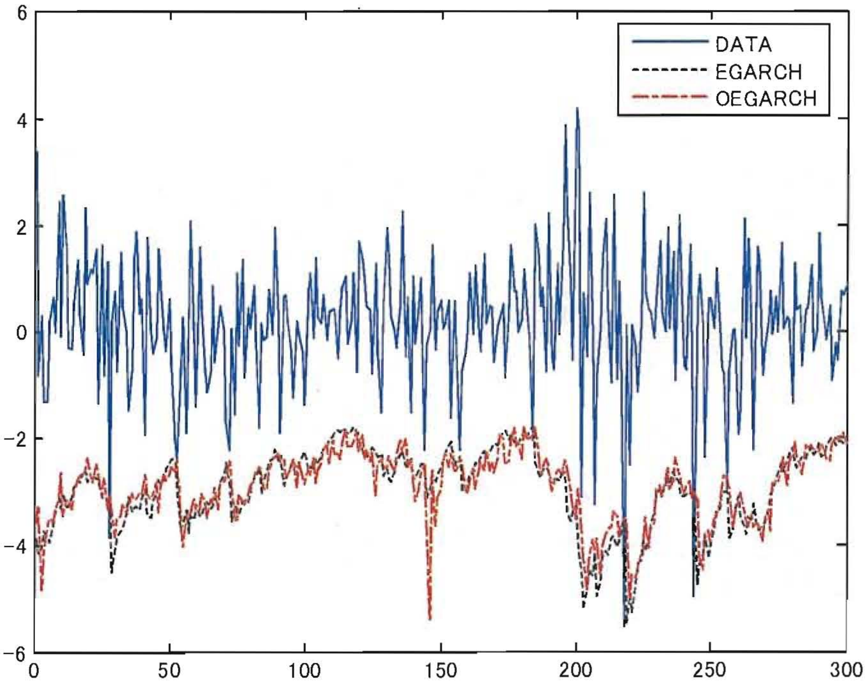

Figure 10.2: Plots of VaR for EGARCH and OEGARCH

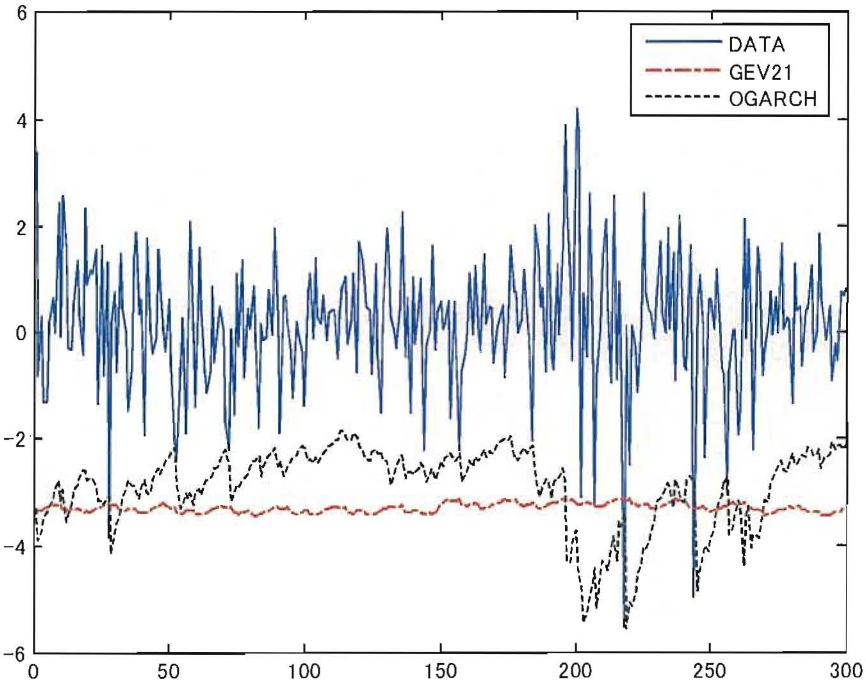

Figure 10.3: Plots of VaR for OGARCH and GEV models. For GEV $n = 21$. $a = 1\%$.

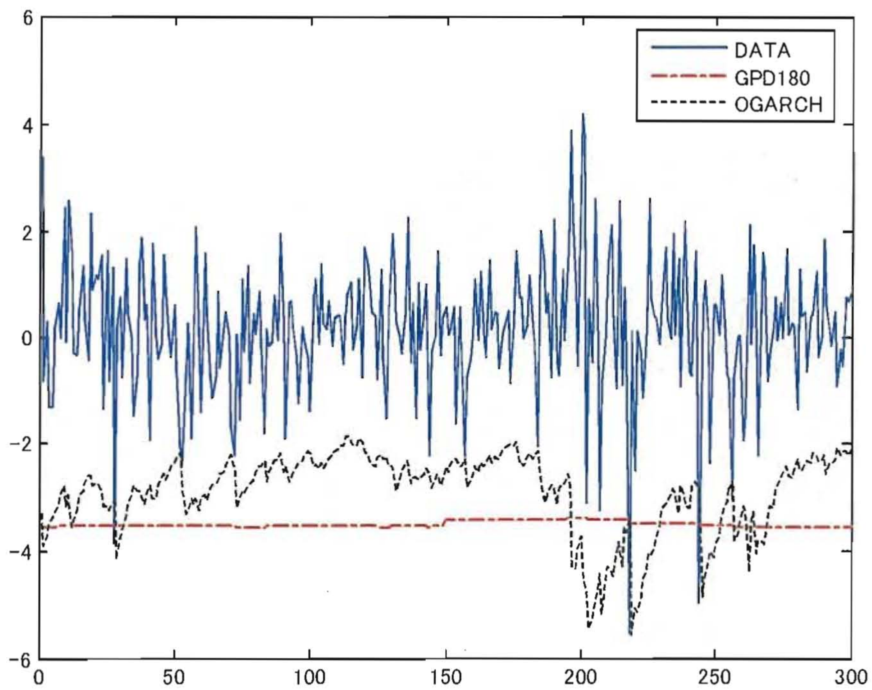

Figure 10.4: Plots of VaR for OGARCH and GPD models. For GPD the plot is the 180 case. $a = 1\%$.

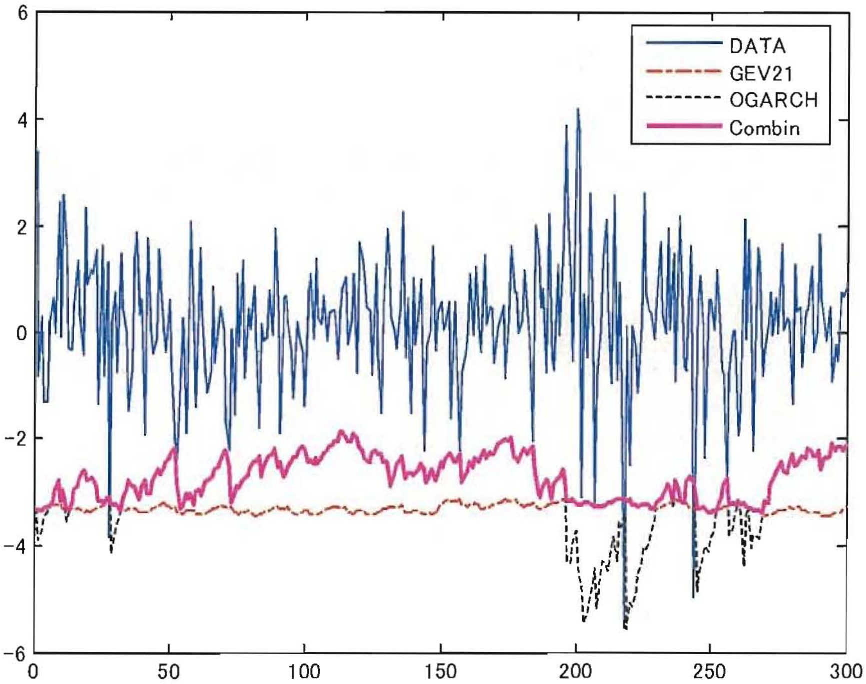

Figure 10.5: Plots of VaR for OGARCH, GEV and combin models. For GEV $n = 21$. $a = 1\%$.

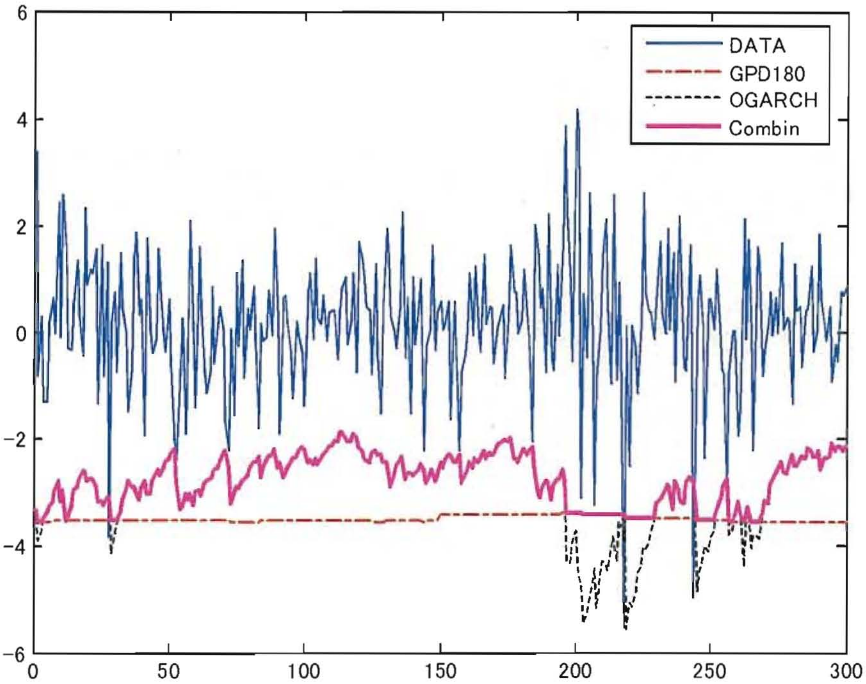

Figure 10.6: Plots of VaR for OGARCH, GPD and combin models. For GPD the plot is the 180 case. $a = 1\%$.

Table 10.1: Results of LR Tests with $a = 5\%$.

Single	R_{uc}	P-value	R_{ind}	P-value	R_{cc}	P-value
GEV21	1.0392	0.3080	0.0431	0.8356	1.0822	0.5821
GEV63	0.2696	0.6036	0.9822	0.3217	1.2518	0.5348
GPD100	0.2934	0.5881	1.1824	0.2769	1.4758	0.4781
GPD180	0.2934	0.5881	1.1824	0.2769	1.4758	0.4781
EWMA	0.0687	0.7932	0.0259	0.8721	0.0947	0.9538
GARCH	0.0717	0.7889	2.0603	0.1512	2.1320	0.3444
OGARCH	0.0717	0.7889	2.0603	0.1512	2.1320	0.3444
EGARCH	0.0717	0.7889	0.1742	0.6764	0.2459	0.8843
OEGARCH	0	1	0.0828	0.7736	0.0828	0.9595
J-GEV21	R_{uc}	P-value	R_{ind}	P-value	R_{cc}	P-value
EWMA	0.2696	0.6036	0.9822	0.3217	1.2518	0.5348
GARCH	0.0717	0.7889	2.0603	0.1512	2.1320	0.3444
OGARCH	0	1	1.6460	0.1995	1.6460	0.4391
EGARCH	0.0687	0.7932	1.2884	0.2563	1.3571	0.5073
OEGARCH	0.0687	0.7932	0.0259	0.8721	0.0947	0.9538
J-GPD180	R_{uc}	P-value	R_{ind}	P-value	R_{cc}	P-value
EWMA	0	1	0.0828	0.7736	0.0828	0.9595
GARCH	0.0717	0.7889	2.0603	0.1512	2.1320	0.3444
OGARCH	0.2696	0.6036	0.9822	0.3217	1.2518	0.5348
EGARCH	0.0687	0.7932	1.2884	0.2563	1.3571	0.5073
OEGARCH	0.0687	0.7932	0.0259	0.8721	0.0947	0.9538

The first panel shows the results of single models. The second and third panels show the results of the joint models constructed of GEV21 with other models, and GPD180 with other models, respectivily.

Table 10.2: Results of LR Tests with $a = 1\%$.

Single Model	R_{uc}	P-value	R_{ind}	P-value	R_{cc}	P-value
GEV21	0	1	0.0608	0.8052	0.0608	0.9700
GEV63	2.3482	0.1254	0.2458	0.6200	2.5940	0.2734
GPD100	0	1	0.0608	0.8052	0.0608	0.9700
GPD180	0	1	0.0608	0.8052	0.0608	0.9700
EWMA	1.1218	0.2895	0.1701	0.6800	1.2919	0.5242
GARCH	0.3048	0.5809	0.1085	0.7418	0.4134	0.8133
OGARCH	0.3048	0.5809	0.1085	0.7418	0.4134	0.8133
EGARCH	1.1218	0.2895	0.1701	0.6800	1.2919	0.5342
OEGARCH	1.1218	0.2895	0.1701	0.6800	1.2919	0.5242
J-GEV21	R_{uc}	P-value	R_{ind}	P-value	R_{cc}	P-value
EWMA	0	1	0.0608	0.8052	0.0608	0.9700
GARCH	0	1	0.0608	0.9700	0.0608	0.9700
OGARCH	0	1	0.0608	0.9700	0.0608	0.9700
EGARCH	0.3048	0.5809	0.1085	0.7418	0.4134	0.8133
OEGARCH	0.3048	0.5809	0.1085	0.7418	0.4134	0.8133
J-GPD180	R_{uc}	P-value	R_{ind}	P-value	R_{cc}	P-value
EWMA	0.3815	0.5368	0.0270	0.8696	0.4085	0.8153
GARCH	0.3048	0.5809	0.1085	0.7418	0.4134	0.8133
OGARCH	2.3482	0.1254	0.2458	0.6200	2.5940	0.2734
EGARCH	0.3048	0.5809	0.1085	0.7418	0.4134	0.8133
OEGARCH	1.1218	0.2895	0.1701	0.6800	1.2919	0.5242
Combin	R_{uc}	P-value	R_{ind}	P-value	R_{cc}	P-value
GEV21	0.3048	0.5809	0.1085	0.7418	0.4134	0.8133
GEV63	2.3482	0.1254	0.2458	0.6200	2.5940	0.2734
GPD180	0.3048	0.5809	0.1085	0.7418	0.4134	0.8133
GPD100	0.3048	0.5809	0.1085	0.7418	0.4134	0.8133

The first panel shows the results of single models. The second and third panels show the results of the joint models constructed of GEV21 with other models, and GPD180 with other models, respectivily. The last panel shows the results of combin models composed of OGARCH$(1,1)$ model with other models.

Table 10.3: Evaluation Results with $a = 5\%$.

Single Model	GEV21	GEV63	GPD180	GPD100	
Violation	19	17	13	13	
MVaR	-1.8408	-1.9136	-2.1257	-2.1222	
Single Model	EWMA	GARCH	OGARCH	EGARCH	OEGARCH
Violation	16	14	14	14	15
MVaR	-2.0857	-2.1600	-2.0591	-2.1006	-2.0671
J-GEV21	EWMA	GARCH	OGARCH	EGARCH	OEGARCH
Violation	17	14	15	16	16
MVaR	-2.0054	-2.0969	-2.0035	-2.0281	-1.9997
J-GPD180	EWMA	GARCH	OGARCH	EGARCH	OEGARCH
Violation	15	14	17	16	16
MVaR	-2.1449	-2.0929	-2.0114	-2.0091	-1.9865

The first two panels show the results of single models. The third and fourth panels show the results of the joint models constructed of GEV21 with other models, and GPD180 with other models, respectivily. Violation and MVAR denote the number of violations and mean absolute predictive VaR value, respectivily.

Table 10.4: Evaluation Results with $a = 1\%$.

Single Model	GEV21	GEV63	GPD180	GPD100	
Violation	3	6	3	3	
MVaR	-3.3040	-2.9637	-3.5058	-3.4210	
MDCR	9.8412	8.8997	10.5238	10.2912	
Single Model	EWMA	GARCH	OGARCH	EGARCH	OEGARCH
Violation	5	4	4	5	5
MVaR	-2.9526	-3.0544	-2.9195	-2.9606	-2.9144
MDCR	11.1713	10.8324	10.5662	10.4346	9.9744
J-GEV21	EWMA	GARCH	OGARCH	EGARCH	OEGARCH
Violation	3	3	3	4	4
Mean VaR	-3.4128	-3.3210	-3.1763	-3.1925	-3.1436
MCR	12.8697	11.6934	11.3542	11.2107	10.7520
J-GPD180	EWMA	GARCH	OGARCH	EGARCH	OEGARCH
Violation	2	4	6	4	5
MVaR	-3.5121	-3.2812	-3.1382	-3.1569	-3.1140
MDCR	13.0304	11.6481	11.4521	11.2290	10.7640
Combin	GEV21	GEV63	GPD180	GPD100	
Violation	4	6	4	4	
MVaR	-2.7401	-2.6479	-2.7839	-2.9606	
MDCR	9.2172	8.6878	9.5567	9.4485	

The first two panels show the results of single models. The third and fourth panels show the results of the joint models constructed of GEV21 with other models, and GPD180 with other models, respectivily. The last panel shows the results of combin models composed of OGARCH(1, 1) model with other models. Violation, MVAR and MDCR denote the number of violations, mean absolute predictive VaR value and mean daily capital requirement, respectivily.