IPO share allocation and conflicts of interest

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Abstract

The underwriter of an IPO has two sources of compensation for its services on behalf of the issuer. One is through a commission (spread), the other is by buying issued shares for itself (or its affiliates) and reselling them in the post-issue market. Profits from the former decrease along with the magnitude of underpricing while profits from the latter increase with it. Faced with these countervailing interests, the present paper analyzes how the underwriter decides upon the pricing and allocation of IPOs.

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1 Introduction

What is an underwriter's remuneration for its IPO services for an issuer? First, the underwriter receives a spread (commission) as a percentage of the issue price. In the U.S., this is frequently around seven percent.[1] Were it not for other sources of profit, the underwriter's profit maximization coincides with the issuer's profit maximization of the IPO. However, there are strong reasons to believe that the underwriter derives profits from underpricing. The underwriter, then, faces a trade-off between two opposing interests, commission and underpricing earnings.

It has been alleged, for instance, that institutional investors accept to pay high commissions in regular share trade with an underwriter financial institution. The investors, in return, receive a favorable treatment (kickbacks) in IPO share allocation and underpricing (Loughran and Ritter (2002)). In the post issue market, the investors make profits from the selling of the shares. Ellis, Michaely, and O'Hara (2002) also finds that contrary to the standard argument of price support, post-issue trade is profitable to the underwriter and points out that the underwriter has an incentive for underpricing, retaining and reselling IPO shares in the after-market. Noting these facts, Biais, Bossaerts, and Rochet (2002) studies the situation where the underwriter colludes with informed investors with whom it deals on a more regular basis than with the issuer. Further, the recent deregulation of the financial sector allows underwriters increasing discretion over share allocation and pricing. They are able to allot increasing proportions of an IPO to themselves or their affiliates.

[1] See Chen and Ritter (2000) and Hansen (2001) for controversies concerning this.

Facing with a trade-off between commission and underpricing earnings, the underwriter seeks to maximize its profit. If attracted by commission gains, the underwriter sets a high issue price, benefiting the issuer but at the expense of investors. If attracted more by the returns from underpricing, it sets a low price and investors are content. To analyze this situation, we adopt the setting of contract delegation. The first time issuer has relatively little expertise in financial affairs, therefore lacking the ability to organize the IPO, which involves information gathering, information offering, advertising, pricing and so forth. The issuing firm delegates the whole IPO to an underwriter, paying the underwriter a commission of a fixed percentage of the issue price. As a seasoned financial institution, the underwriter has sufficient knowledge to enable it to collect and analyze the information possessed by informed investors and estimate the market valuation of the shares to be issued. In full charge of the IPO procedure, the underwriter decides upon the quantity allocation and the price to maximize its own profit.

A common feature among previous studies is that they consider as separate entities only two of the three parties—the issuer, the underwriter and the informed investor—either neglecting one party for simplicity or uniting it with another, implying they pursue common interests by forming a coalition.

Baron (1982) and Baron and Holmstrom (1980) analyze the issuer's optimal incentive contract in the context where there are only an issuer and an underwriter, the latter having better information than the former. Rock (1986), Benveniste and Spindt (1989) and Benveniste and Wilhelm (1990) study the situation in which there are an issuer, an underwriter, informed investors, and uninformed investors but the underwriter is assumed to act completely on the issuer's behalf. Biais, Bossaerts, and Rochet (2002), on the other hand, assume that the underwriter is allied with the informed investor. They investigate the issuer's optimal contract in the context where there are an issuer, uninformed investors and a party which is a coalition of the underwriter and informed investors. In reality, the issuer, the underwriter and informed investors are separate entities concerned for their own well-being. The present paper makes this distinction clear in order to analyze the above mentioned opposing interests.

It is intuitively sensible that when a spread (commission) is large, the underwriter allocates and prices shares in such a way that it makes more profits from the commission by setting a high price. In this case the underwriter's interest coincides with that of the issuer. In contrast, when a spread is small, the underwriter sets a low price to make gains from the underpricing and post-IPO reselling of shares, in accordance with investors' interests. This article in part vindicates this intuition and draws several new empirical predictions from its theoretical results. In passing, we show that it is against the issuer's interest to seek a spread that is too low, since this encourages the underwriter to ally with the investors. The IPO has many facets such as the issue price, marketing of the issue and analyst coverage. If negotiation on the spread is not advantageous, the issuer tries to get better terms on the other dimensions with the underwriter.

The remainder of the article is organized as follows. In the next section, the parties involved are briefly presented. In section 3, the model is formally described. Section 4 sets up the problem as the underwriter's mechanism design. Section 5 concludes the paper.

2 The players

There are three players in the model: a firm, an underwriter and an informed investor.[2] The firm wants to sell a fixed amount of shares on the market for the first time. This firm or issuer is assumed to be unable to do this by itself for the reason described in the introduction.

[2]The term "subscriber" is sometimes used interchangeably below with "investor".

The underwriter, which has greater expertise as a seasoned financial institution, takes on the task of organizing the IPO. It markets, prices and distributes IPO shares to subscribers. In reality, the syndicate of underwriters is often formed by several financial institutions but we assume here that there is just one underwriter.

The informed, often called the "regular", investor is a large investor such as an investment bank, a broker or a securities firm with great expertise on the financial market. Such an investor may well have some information on the post-offering market valuation of IPO equity. The underwriter contacts this investor to seek information during the registration period.

The underwriter here is either a coalition of the underwriter and its affiliated investment bank, or an alliance between the underwriter and its "friendly" investors.[3] As outlined in the introduction, the underwriter might have institutional customers whom it favors in the share allocation in return for expected future profits. Here we make the simplifying assumption that the underwriter considers their profits from the IPO to be its own, in the same way as it regards the affiliated investment bank' profits as its own. The underwriter sets the price and allocates shares among the underwriter coalition and the informed investor, and maximizes the coalition's profits.[4] From now on, we refer to the underwriter coalition as simply the "underwriter". Likewise, strictly speaking, the underwriter allocates shares to the "friendly" investors or its affiliates, but in the following we say that "the underwriter buys shares for itself or allots shares to itself".

In our model, only the informed subscriber possesses private information. In other words, the issuer reveals all its information to the underwriter.[5] Likewise, the un-

[3]Note that the "friendly" investors are completely separate entities from the informed investors already considered.

[4]It is assumed of course that if the underwriter's "friendly" investors or affiliates possess some information, they reveal the information to the underwriter.

[5]Therefore, the issue of signaling by the issuer is disregarded as in Allen and Faulhaber (1989) and Welch (1989).

derwriter reveals to the informed investor all information provided by the issuer and possessed by itself. The issuer in this model is rather unsophisticated. It delegates the whole IPO procedure to the underwriter and pays a fixed percentage of the per share price as a spread.[6] Fully designated to organize the whole IPO process, the underwriter decides upon the quantity allocation and the price of the shares while seeking the informed subscriber's private information.

Given that the underwriter might obtain the private information possessed by the informed subscriber, the issuer might try to force the underwriter to reveal this information through the construction of an incentive commission scheme. However, this article does not consider such a sophisticated issuer. The lack of financial experience needed for a complicated IPO procedure means that the first time issuer is unlikely to be able to construct an optimal commission schedule when dealing with the underwriter, which is a longstanding and experienced financial institution.[7]

In choosing the underwriter, the issuer approaches several financial institutions for issuance conditions and selects one from among them. In this way, the issuer overcomes some of the disadvantage of its inferior financial expertise. It has been argued that competition for underwriting can be fierce. This paper embodies this fact through the setting of the issuer's reservation price. The issuer's choice of the underwriter is made by comparing, above all, the issue prices suggested by competing financial institutions. As a result, the successful underwriter must set an issue price above a certain level to satisfy the issuer. The more competitive the underwriting business, the higher will be the reservation issue price.

[6]Although the linear compensation scheme is assumed for the underwriter, in reality this might not be the case because of the existence of the overallotment right and the warrant right granted to the underwriter. For warrants, see Barry, Muscarella, and Vetsuypens (1991).

[7]Ausubel and Cramton (1998) recognize underwriters' power in the securities market and state that the failure to adopt an auction style system for the IPO is due to institutional inertia resulting from underwriters defending vested interests.

3 The model

This section presents the model formally. All parties concerned, the issuer, the underwriter, the informed investor are risk neutral.[8] The issuing firm goes public to issue a fixed amount of shares which we normalize to 1. As indicated in the previous section, the informational structure of the paper assumes that only the informed investor has private information, which is unobservable by the other parties.

The underwriter sets the per share price p and makes quantity allocation between itself and the informed investor, q_0, q_1 such that $q_0 + q_1 = 1$ and $q_i \geq 0$ for $i = 0, 1$. The underwriter receives from the issuer as compensation for its services the commission of a fixed percentage per share price $0 < a < 1$. The issuer, thus, pays as commission ap in total.

The informed investor has private information on the post-issue value of the shares v, which takes a value in the non-empty interval of positive real numbers, $[\underline{v}, \overline{v}]$. The distribution function of v, $F(v)$ is public information and supposed absolutely continuous. The density $f(v)$ is assumed such that $f(v) > 0$ on the support, $[\underline{v}, \overline{v}]$. The post-IPO per share price is realized as v. Therefore, there is no ex post surprise.

The underwriter maximizes its own profits by deciding upon the share price and the share allocations. It has two sources of profits: it earns a commission ap for the IPO organization, and it can make profits by buying and reselling part of the shares $(v-p)q_0$.

Let us put the upper limit to the amount of shares the underwriter can buy,

$$0 \leq q_0 \leq t$$

[8]As opposed to the other parties, the issuer might more realistically be risk averse. Equation 7 below, then, takes a complicated form and the model becomes intractable. We assume risk neutrality for manageability. Among the references cited, the following assume risk neutrality: Baron (1982), Baron and Holmstrom (1980), Benveniste and Spindt (1989), Benveniste and Wilhelm (1990), and Biais, Bossaerts, and Rochet (2002).

where $0 < t < 1$. In the U.S., the amount of shares that the underwriter can allocate to its affiliates is limited. Even without a definite legal restriction on the quantity of the underwriter's purchase, it may fear that by allotting too few shares to informed institutional investors, it would impair future business relationships with them or attract suspicious attention of the regulatory agency. The existence of t may be justified also by the argument of the availability of the underwriter's funds for the IPO. The underwriter is keen to diversify its portfolio so that it is ready to put limited resources for the purchase of shares of a particular IPO. As a result, the larger is the post-issue valuation and thus the issue price, the fewer shares the underwriter wants to purchase for itself.

During the registration period, the underwriter markets IPO shares and collects information about the market acceptance or the price valuation of the shares. In the setting of this paper, it translates into the underwriter's construction of the revelation mechanism. Let us concentrate on the direct mechanism (Myerson (1979)). Formally, the underwriter proposes to the informed subscriber the map

$$(q_1(v), p(v)) : [\underline{v}, \overline{v}] \to [0,1] \times R, \tag{1}$$

where $q_1(v)$ is a quantity alloted to the informed subscriber and $p(v)$ is the per share price.

If the informed subscriber with information v chooses the contract for $\tilde{v}$, its profit is

$$u_1(v, \tilde{v}) := (v - p(\tilde{v}))q_1(\tilde{v}).$$

If the informed subscriber with information v selects the contract for its true information v, its profit is

$$u_1(v) := (v - p(v))q_1(v). \tag{2}$$

Unable to force the informed subscriber to disclose its information, the underwriter has to make a contract which induces it to reveal its information at will. We thus define the implementable contract.

Definition 1. *The contract* $(q_1(v), p(v))$ *is implementable if and only if for any* $v, \tilde{v} \in [\underline{v}, \overline{v}]$,

$$u_1(v) \geq u_1(v, \tilde{v}).$$

As is standard in the mechanism design literature, we turn the implementable contract into the manageable form which permits us to formalize the maximization problem.

Lemma 1 (incentive compatibility)**.** *If the contract* $(q_1(v), p(v))$ *is implementable, the following two conditions are satisfied;*

$$q_1(v) \text{ is non-decreasing,} \tag{3}$$

$$q_1(v) = \dot{u}_1(v) \qquad a.e.^9 \tag{4}$$

Conversely, given $q_1(v)$ *and* $u_1(v)$ *which satisfy* (3) *and* (4)*, the implementable contract* $(q_1(v), p(v))$ *can be constructed, by putting*

$$p(v) = v - \frac{u_1(v)}{q_1(v)}. \tag{5}$$

Proof. See Rochet (1985). □

As seen in (5), if $q_1 = 0$, p is not well defined but we will see that this is of no concern. Before proceeding further, we mention a simple fact deduced from the above

[9] a.e. stands for almost everywhere and the dot the derivative.

lemma, which will be made use of in the formulation of a participation constraint.

Lemma 2. *The price of the implementable contract* $(q_1(v), p(v))$ *is non-decreasing.*

Proof. Let us $(q_1(v), p(v))$ be an implementable contract. Posit that $v < v'$ and suppose contrary to the lemma that $p(v) > p(v')$. Then we have

$$u_1(v) = (v - p(v))q_1(v) < (v - p(v'))q_1(v) < (v - p(v'))q_1(v').$$

The second inequality follows from Condition (3) of Lemma 1. It is seen, however, that the left hand and right hand contradict the definition of implementability 1. □

Let us now turn our attention to participation constraints. It is not enough that the underwriter manages to get the informed investor to tell the truth. The informed investor must be ensured of at least a certain level of utility for its participation; otherwise it will not participate in the IPO[10]. For instance, the informed investor will naturally demand more than the utility that other similar investment opportunities allow. Also, as in Rock (1986), it might take some cost for an investor to get informed. Then, the informed investor will ask the cost to be covered by reservation utility. Hence, we take the following participation constraint for the informed investor:

$$c \leq u_1(v)$$

where c is a positive constant. From the incentive compatibility conditions, (3) and (4), this participation condition can be transformed into

$$c \leq u_1(\underline{v}). \tag{6}$$

[10] It might be more convincing to put the participation condition as $r \leq \frac{v-p(v)}{q_1(v)}$ where r is a yield rate of other financial products but, for simplicity, the present article adopts a simpler condition.

Unlike much of the literature on asymmetric information, we need another participation constraint, that for the issuer. As was explained in Section 2, the issuer chooses the underwriter by comparing its minimum issue price with those provided by other financial institutions. If the issue price is too low, the issuer will simply cancel the IPO. Likewise, the issuer might have some investment project to finance. Then, the issuer will ask for a higher price to provide funds for the project. Accordingly, we write the participation constraint for the issuer as

$$d \leq p(v).$$

where d is a positive constant. Since only implementable contracts are being considered, this condition is equivalent, by means of Lemma 2, to

$$d \leq p(\underline{v}).$$

Moreover, by Equation (5) in Lemma 1, this can be written as

$$d \leq \underline{v} - \frac{u_1(\underline{v})}{q_1(\underline{v})}. \tag{7}$$

This is the form that we shall use as the participation constraint for the issuer.

It is necessary to make some assumptions in order that there may exist implementable contracts which satisfy (6) and (7). When they are satisfied, it follows directly that

$$c \leq u_1(\underline{v}) \leq (\underline{v} - d)q_1(\underline{v}).$$

Owing to the assumption $0 \leq q_0 \leq t$, q_1 takes a value in $[1-t, 1]$. Therefore we make the following assumption so that any value in this interval satisfies the two participation constraints.

Assumption 1.

$$c < (\underline{v} - d)(1 - t).$$

Were this assumption not met, the two participation constraints might be so stringent that q_1 might not be able to take some values in $[1 - t, 1]$. For instance, if both the issuer and the informed investor ask for unrealistically high reservation utility, the above condition will not be met and the two participation constraints are never satisfied at once.

4 The underwriter's decision making

Recall that $q_0 + q_1 = 1$ and therefore we have

$$0 \leq 1 - q_1 \leq t.$$

The underwriter maximizes its expected profit under the incentive constraints and the participation constraints;

$$\max_{q_1, p, u_1} \int_{\underline{v}}^{\overline{v}} \Big(ap(v) + \big(v - p(v)\big)\big(1 - q_1(v)\big)\Big) f(v)dv$$

s. t.

(3), (4), (6), (7),

$1 - t \leq q_1(v) \leq 1.$

The objective function consists of the profits from the commission and from share reselling on the after-market.[11]

[11]The assumption that all the IPO profits of the affiliates or friendly investors accrue to the under-

We eliminate p from the objective function by virtue of the incentive compatibility lemma and set the problem as that of optimal control. Once the optimal q_1 and u_1 are found, p can be retrieved by Equation (5).

We make q_1 and u_1 state variables. We can transform $1 - t \leq q_1 \leq 1$ by the monotonicity of q_1 (see Lemma 1) into

$$1 - t \leq q_1 (\underline{v}), \quad q_1 (\overline{v}) \leq 1.$$

Likewise, with regard to Condition (3), we introduce a control variable z

$$z := \dot{q}_1 \quad \text{a.e}, \quad z \geq 0.$$

Now we formulate the maximization problem as that of optimal control, which we denote by $\mathcal{P}$.

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writer might seem unrealistic and it may be more sensible to suppose that only part of the profits go to the underwriter. In this case, however, we merely have to rescale a.

$$\max_{z,q_1,u_1} \int_{\underline{v}}^{\overline{v}} (av - (a-1)\frac{u_1}{q_1} - u_1) f(v) dv \tag{8}$$

s. t.

$$\dot{u}_1 = q_1 \quad \text{a.e,} \tag{9}$$

$$\dot{q}_1 = z \quad \text{a.e,} \tag{10}$$

$$0 \leq z, \tag{11}$$

$$c \leq u_1(\underline{v}), \tag{12}$$

$$d \leq \underline{v} - \frac{u_1(\underline{v})}{q_1(\underline{v})} \tag{13}$$

$$1 - t \leq q_1(\underline{v}), \tag{14}$$

$$q_1(\overline{v}) \leq 1. \tag{15}$$

Theorem 1. *The solution of the maximization problem $\mathcal{P}$ is as follows:*

if $a > t$,

$$q_1(v) = 1 - t,$$

$$u_1(v) = (1-t)(v - \underline{v}) + c,$$

$$p(v) = \underline{v} - \frac{c}{1-t};$$

if $a = t$,

$$q_1(v) = 1 - t,$$

$$u_1(v) = (1-t)(v - \underline{v}) + u_1(\underline{v}),$$

$$p(v) = \underline{v} - \frac{u_1(\underline{v})}{1-t},$$

where $u_1(\underline{v})$ *satisfies the end conditions* (12) *and* (13), *namely* $c \leq u_1(\underline{v})$ *and* $d \leq \underline{v} - \frac{u_1(\underline{v})}{1-t}$;

if $a < t$,

$$q_1(v) = 1 - t,$$

$$u_1(v) = (1 - t)(v - d),$$

$$p(v) = d.$$

Proof. See the appendix. □

Let us look into the features of the theorem.

Result 1. *Whatever the relation between* a *and* t *(i.e.* $a \geq t$ *or* $a < t$*), the underwriter always buys the largest amount of shares,* t*.*

In general, the underwriter is attracted by two divergent incentives. If setting a high price, it earns more commission but must pay more for the shares it purchases. By setting a low price, it makes less commission profits but gains from reselling the shares it has bought. Intuition, therefore, tells us that with a high commission, the underwriter will set a high price and refrain from buying many shares. By contrast, with a low spread, the underwriter profits from setting a low price and buying a large number of shares, although it earns less commission.

Contrary to this intuition, Result 1 shows that the underwriter always allocates itself as many shares as possible. The reason is, as seen below, due to the existence of underpricing.

Let us see the inflexibility of the quantity allocation from another point of view.

Result 2. *The underwriter's share allocation is independent of the informed subscriber's private information* v*; namely, regardless of the realized value* v*, the underwriter purchases as many as possible* t *and distributes the rest to the informed subscriber* $1 - t$*.*

The rigidity of the quantity allocation translates into that of the price.

Result 3. *The issue price is insensitive to the informed subscriber's private information, share value v.*

Since the price and quantity are related by Equation (5), it is obvious that the rigid quantity allocation leads to the inflexible price.

Result 4. *Given v, the price for $a > t$ is higher than that for $a < t$.*

Proof. It is obvious from Theorem 1 and Assumption 1. □

The question is how the two prices are different in the two cases despite the identical quantity allocation. It follows–see the theorem–from which participation condition is binding of $u_1(\underline{v}) \geq c$ and $d \leq \underline{v} - \frac{u_1(\underline{v})}{q_1(\underline{v})}$.

When the spread is large($a > t$), the underwriter sets a high price for commission earnings, which benefits the issuer at the expense of the investor. The price is indeed set at such a high level that the informed subscriber's participation constraint $u_1(\underline{v}) \geq c$ is binding. In this case, the underwriter can be viewed as taking sides with the issuer.

Contrariwise, with a small spread($a < t$), the underwriter sets a lower price, attracted by earnings from the reselling of shares on the aftermarket. The price is set so low as to make binding the issuer's participation constraint $d \leq \underline{v} - \frac{u_1(\underline{v})}{q_1(\underline{v})}$. The underwriter, in this case, shares the same interests as the informed investor at the expense of the issuer.

Result 5. *Underpricing persists always and increases in t. Moreover, when $a \geq t$, underpricing strictly increases with t. When $a < t$, underpricing is constant in t.*

Proof. By definition, underpricing is $v - p(v)$. For $a \geq t$, it is immediate from Theorem 1. For $a < t$, the result follows from Assumption 1. □

At the extreme when t tends toward 0, underpricing still exists and it is due to informational rent for the informed investor as in Benveniste and Spindt (1989) and

Benveniste and Wilhelm (1990). The participation constraint (6) is the source of this. However, underpricing is larger when the underwriter is allowed to purchase shares for itself. With higher t, the underwriter is attracted by returns from the purchase of shares. This is why underpricing increases in t if $a > t$. When $a < t$, on the other hand, underpricing cannot increase with t since the price is already at the lowest possible level, the issuer's reservation price d.

This result translates into the following in a general context.[12] When the underwriter finds it profitable to let institutional investors (or just its affiliates) make money in the IPO for future business, it does not benefit the issuer to negotiate hard on the spread, which, on the contrary, harms it by pushing the former towards the investors. It is in the issuer's interest to bargain for terms other than the spread among many dimensions of the IPO contract.

One can sensibly think that with more discretion in allocation, the underwriter is more likely to make future profits with investors by giving favorable treatment in the IPO. To keep the underwriter on its side, the issuer has to accept a higher spread for the underwriter. Consistent with our result, in the U.S. where the underwriter is allowed considerable discretion, a much higher spread is observed than in other countries (Chen and Ritter (2000)).

Beatty and Ritter (1986) shows that in Rock (1986)'s setting with the winner's curse of uninformed investors, more uncertainty leads to more underpricing. Precisely, they demonstrate that underpricing is greater the larger the support of v is on the condition that v follows the uniform distribution and $E(v)$ stays identical. It evident in their article that these assumptions are essential for their result. They assume that uninformed investors must be taken in for the IPO. In order to participate with more uncertainty, the investors have to be compensated more for possible ex post loss. One

[12] Recall that the underwriter is a coalition with friendly investors or affiliates in our context.

can therefore see the crucial nature of uniformity and unvaried $E(v)$ for Beatty and Ritter (1986)'s result. If one of the two conditions is untrue, loss from some v's can be counterbalanced by gains from other v's and larger uncertainty on v does not necessarily lead to greater underpricing.

Although in a different setting, we show that the result of Beatty and Ritter (1986) holds here without a condition on distribution and $E(v)$.

Result 6. *In the case of $a \geq t$, the larger the support of $f(v)$ is, the larger underpricing is whereas in the case of $a < t$, it stays the same. More precisely, when $a \geq t$, underpricing is greater the smaller $\underline{v}$ is and when $a < t$, underpricing stays the same.*

Proof. We shall merely have to examine $v - p(v)$ from the theorem. One can see that it only depends upon $\underline{v}$. The result is obvious. □

Note that our result is meaningful only as long as the enlargement of the support does not violate the participation constraints. If it does, we have to adjust c and d to restore the satisfaction of the constraints and it loses a sense to compare underpricing with different supports of v.

In contrast to Beatty and Ritter (1986), here, greater underpricing is due to informational rent, i.e. the informed investor's participation constraint, which only depends upon $\underline{v}$. With more uncertainty (larger support of $f(v)$), the share valuation $\underline{v}$ becomes smaller and still the investor's reservation utility must be assured. Therefore, with smaller $\underline{v}$, the price must be lower and underpricing greater. This is what happens when $a \geq t$. However, when $a < t$, underpricing stays identical. The reason is that the issue price is bounded above by the informed investor's and below by the issuer's participation constraints. In the case of $a < t$, the price is already at the lowest level, the issuer's reservation price d and cannot go lower according to the change of $\underline{v}$. Naturally, if the issuer lowers the reservation price as uncertainty increases, underpricing becomes larger in this case as well.

5 Concluding remarks

To conclude the paper, we relate our results to existing empirical works and also derive new empirical predictions. Among financial institutions, underwriting business is competed for on various fronts: the minimum issue price, the spread, business advising, analyst coverage, post-issue price support and so forth. We have seen that it is not in the issuer's interest to make the underwriter reduce the spread excessively. Doing so, the former might push the underwriter towards investors and make it take side with them. Chen and Ritter (2000) argues that the actual spread observed in American IPOs is too high on account of anticompetitiveness between financial institutions while Hansen (2001) contends to the contrary. The present paper adds another argument on this issue, based on incentives for the underwriter: the spread must be large enough for the underwriter to stay faithful to the issuer.

To test fully the predictions of the present paper, one needs to know how shares are allocated by the underwriter among subscribers, especially between "friendly" investors (or its affiliates) and normal investors. In many countries, there is no requirement to report on the details of share allocation, not to mention the identities of subscribers to whom shares have been distributed. Even with an underwriter's internal report on share distribution, it might be difficult to distinguish friendly investors from other investors. In contrast, an underwriter's affiliated investors are relatively easy to identify.

Hanley and Wilhelm (1995) is the first work to look into the internal report of underwriters. They examined a long-standing argument concerning American IPOs that institutional investors were favored by underwriters in the share distribution of underpriced issues. Their finding is that institutional investors are allocated a large part of shares in both underpriced and overpriced issues. They, however, fell short of investigating how shares are distributed between institutional investors.

The present paper, lacking a factor of post-issue price surprise, predicts that friendly investors receive a larger proportion in outperforming IPOs or those issued in a hot period. Using public data, Ritter and Zhang (2006) recently make a finding consistent with our prediction. They study how U.S. underwriters allocate IPO shares to their affiliated mutual funds. They report that in the U.S., underwriters allocate more shares of hot IPOs to their affiliated investors.

Johnson and Marietta-Westberg (2005) study the effects of universal banking, specifically financial institutions with underwriting and asset management divisions. They also report, consistent with our prediction, that underwriting institutions buy more shares in the IPOs where they are lead underwriters.

Along with regulatory reforms of financial sectors throughout the world, there are fewer and fewer fire walls between underwriters and institutional investors. The underwriter has more discretion to allocate IPO shares to its affiliates. Our result predicts that the underwriter's share allocation to its affiliates increases with this deregulation. Ritter and Zhang (2006) confirms that over time, underwriters are allocating more and more shares to their affiliated funds.

Finally, we note new empirical predictions which deserve future research. The paper predicts that the share allocation is stable between the underwriter (including its friends) and other institutional investors. Although it is hard to identify friendly investors, we can examine whether the proportion of the share allocation is unvarying between the underwriter's affiliates and other investors. Further, the underwriter's share allocation is insensitive to parameter values and only depends upon the underwriter's maximum share percentage. This varies according to the IPO size since the underwriter's available fund for a particular IPO is determined by its size due to portfolio diversification. One can expect therefore that the share allocation pattern is identical across IPOs of a similar size (or value).

We know that when the spread is large, the underwriter happens to share the same interest with the issuer. We therefore infer that underpricing is smaller the larger the spread. In other words, there will be a negative relationship between the magnitude of underpricing and the spread.

With increasing universal banking, more and more financial institutions have an asset management division and an underwriting division. The underwriter's capacity to purchase t translates into the size of its asset management division or the number of its financial affiliates. We predict that larger underpricing is observed among underwriters that have more affiliates or a bigger asset management division.

As large financial institutions, underwriters diversify their portfolio. In large-scale IPOs, the total share value is very large. As a result, the underwriter's purchase limit t is lower and more easily reached.[13] Therefore, the case of $a > t$ is more likely in large IPOs whereas in small IPOs, $a < t$ is more probable. Underpricing should therefore be more significant in small IPOs than in large ones.

From Result 6, it is predicted that firms with lower uncertainty rely more on the spread to compensate the underwriter. Firms with higher uncertainty should remunerate the underwriter more through large underpricing.

A The proof of Theorem 1

Let us set λ_0, λ_1, λ_2 as adjoint variables and we have the Hamiltonian,

[13] Recall that t is the percentage of shares that the underwriter can purchase.

$$H\left(u_1, q_1, z, \lambda\right) = \lambda_0 \big(av - (a-1)\,\frac{u_1}{q_1} - u_1\big) f + \lambda_1 q_1 + \lambda_2 z$$

$$\text{where}\, \lambda := \left(\lambda_0, \lambda_1, \lambda_2\right).$$

u_1 and q_1 are absolutely continuous state variables and z is a measurable control variable.

The necessary conditions for optimality can be written as follows. First there are $\lambda_0 =$ 0 or 1 and non-negative real numbers α_i for $i = 1, \ldots, 4$ such that $(\lambda_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4) \neq 0$. In addition, there are absolutely continuous adjoint variables λ_1 and λ_2 and the following conditions hold:

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial u_1} = \lambda_0((a-1)\frac{1}{q_1} + 1) f \qquad \text{a.e,} \tag{16}$$

$$\dot{\lambda}_2 = -\frac{\partial H}{\partial q_1} = \lambda_0 \frac{(1-a) u_1 f}{q_1^2} - \lambda_1 \qquad \text{a.e;} \tag{17}$$

As the transversality conditions, we have

$$\lambda_1(\underline{v}) = -\alpha_1 + \alpha_2, \qquad \lambda_1(\overline{v}) = 0, \tag{18}$$

$$\lambda_2(\underline{v}) = -\alpha_2(\underline{v} - d) - \alpha_3, \qquad \lambda_2(\overline{v}) = -\alpha_4, \tag{19}$$

and also

$$\alpha_1(u_1(\underline{v}) - c) = 0, \tag{20}$$

$$\alpha_2(q_1(\underline{v})(\underline{v} - d) - u_1(\underline{v})) = 0, \tag{21}$$

$$\alpha_3(q_1(\underline{v}) - (1 - t)) = 0, \tag{22}$$

$$\alpha_4(1 - q_1(\overline{v})) = 0; \tag{23}$$

In addition, z has to maximize $H(u_1, q_1, z, \lambda)$ a.e. with optimal u_1 and q_1. Hence $\lambda_2 \leq 0$.

Lemma 3. $\lambda_0 = 1$

Proof. Suppose that $\lambda_0 = 0$. Then $\lambda_1 = 0$ and $\alpha_1 = \alpha_2$ from (16) and (18). It follows that $\alpha_1 = \alpha_2 = 0$; for if $\alpha_1 = \alpha_2 \neq 0$, it must be that $u_1(\underline{v}) - c = 0$ and $q_1(\underline{v})(\underline{v} - d) - u_1(\underline{v})) = 0$. This is impossible from Assumption 1. Now we know that λ_2 is a non-positive constant from (17). In fact it must be that $\lambda_2 = 0$. Suppose to the contrary. Then the Hamiltonian maximizing z is zero and accordingly q_1 is a constant. On the other hand, since we have $\lambda_2(\underline{v}) = -\alpha_3$ and $\lambda_2(\overline{v}) = -\alpha_4$ from the transversality conditions, it must hold that $q_1(\underline{v}) = 1 - t$ and $q_1(\overline{v}) = 1$. This is contradictory to q_1 being constant. Now that we have found that $\lambda_2 = 0$, it follows from the terminal conditions that $\alpha_3 = \alpha_4 = 0$. Accordingly, we have $(\lambda_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4) = 0$, which is a contradiction.

□

We split the analysis into three cases (1) $a > t$, (2) $a = t$, (3) $a < t$.

(1) *The case of* $a > t$.

From (16), we see that $\dot{\lambda}_1$ is strictly increasing with respect to q_1. Recall $1-t \leq q_1 \leq 1$ and substitute $q_1 = 1-t$ into (16). Then we have $\left(\frac{a-1}{1-t}+1\right) f > 0$ a.e. Accordingly, we have

$$\dot{\lambda}_1 > 0 \quad \text{a.e.}$$

From this and the transversality conditions, it follows that

$$\lambda_1 \begin{cases} < 0 & \text{in } [\underline{v}, \overline{v}), \\ = 0 & \text{at } \overline{v}. \end{cases} \tag{24}$$

From the first inequality and the transversality condition (18), we have $\alpha_2 < \alpha_1$. If $0 < \alpha_2$, then it must follow that $u_1(\underline{v}) - c = 0$ and $q_1(\underline{v})(\underline{v} - d) - u_1(\underline{v}) = 0$. This is impossible from Assumption 1. Therefore we have

$$0 = \alpha_2 < \alpha_1.$$

This leads from (20) to

$$u_1(\underline{v}) = c.$$

From λ_1 and (17), it follows that $\dot{\lambda}_2 > 0$ a.e. and thus

$$\lambda_2 \begin{cases} < 0 & \text{in } [\underline{v}, \overline{v}), \\ \leq 0 & \text{at } \overline{v}. \end{cases} \tag{25}$$

It follows that the Hamiltonian maximizing z is almost everywhere zero and thus q_1 is constant. From the transversality condition, we have $\lambda_2(\underline{v}) = -\alpha_3$, which is negative. We conclude from (22) that $q_1(\underline{v}) = 1-t$ and thus

$$q_1 = 1 - t.$$

Now we find u_1 from (9) and p from (5):

$$u_1 = (1-t)(v - \underline{v}) + c, p = v - \frac{(1-t)(v - \underline{v}) + c}{1-t} = \underline{v} - \frac{c}{1-t}.$$

(2) *The case of $a = t$.*

As we have $\dot{\lambda}_1 \geq 0$ a.e. in the previous case, we have

$$\lambda_1 \leq 0.$$

Then $\dot{\lambda}_2 > 0$ a.e. follows from (17) and we have (25).

We deduce that $z = 0$ a.e. and thus q_1 is a constant.

Indeed, we can obtain that

$$q_1 = 1 - t.$$

Proof. Suppose to the contrary. Then, it follows that

$$\lambda_1 \begin{cases} < 0 & \text{in } [\underline{v}, \overline{v}), \\ = 0 & \text{at } \overline{v}. \end{cases} \tag{26}$$

From the transversality condition, we have $\lambda_1(\underline{v}) = -\alpha_1 + \alpha_2 < 0$. It can be deduced that $0 = \alpha_2 < \alpha_1$; for we know by Assumption 1 that $u_1(\underline{v}) - c = 0$ and $q_1(\underline{v})(\underline{v} - d) - u_1(\underline{v}) = 0$ cannot hold simultaneously.

Now we obtain from the transversality condition and (26), $\lambda_2(\underline{v}) = -\alpha_3 < 0$. Then it follows from (22) that $q_1(\underline{v}) = 1-t$ and with the fact that q_1 is a constant, $q_1 = 1-t$. This is a contradiction.

$\square$

Now we know $\dot{\lambda}_1 = 0$ a.e. and

$$\lambda_1 = 0.$$

It follows from (18) that $\lambda_1(\underline{v}) = -\alpha_1 + \alpha_2 = 0$. By Assumption 1, $u_1(\underline{v}) - c = 0$ and $q_1(\underline{v})(\underline{v} - d) - u_1(\underline{v})) = 0$ do not hold simultaneously, which leads to

$$\alpha_1 = \alpha_2 = 0.$$

To find u_1, we have only to find $u_1(\underline{v})$. It cannot be determined by (20) and (21) because of the value of α_1 and α_2. Indeed if we substitute $q_1 = 1-t$ and $(1-t)(v-\underline{v})+u_1(\underline{v})$ into the objective function of Problem $\mathcal{P}$, it is seen that $u_1(\underline{v})$ is irrelevant to the maximization of the objective function. Therefore the optimal u_1 is written as

$$u_1 = (1-t)(v - \underline{v}) + u_1(\underline{v})$$

such that $u_1(\underline{v})$ satisfies Condition (12), $c \leq u_1(\underline{v})$ and Condition (13), $d \leq \underline{v} - \frac{u_1(\underline{v})}{1-t}$.

(3) *The case of* $a < t$. First, we will prove that

$$q_1 \leq 1 - a.$$

Proof. Let us prove that it is impossible to have $q_1 > 1 - a$ on the whole interval.

Suppose so and then we obtain (24) from (16) and (18)

Therefore from (18), we have $\lambda_1(\underline{v}) = -\alpha_1 + \alpha_2 < 0$ and thus it follows from Assumption 1, (20) and (21) that

$$0 = \alpha_2 < \alpha_1.$$

From (17), we now have $\dot{\lambda}_2 > 0$ and (25).

Consequently q_1 is constant and $\alpha_3 = 0$ from (22). It leads to $\lambda_2(\underline{v}) = 0$ by (19). This contradicts (25). We have demonstrated that $q_1 > 1 - a$ is impossible.

Now that we have found that there is a point v at which $q_1(v) \leq 1 - a$, let us prove that $q_1 \leq 1 - a$ on the whole interval $[\underline{v}, \overline{v}]$. Suppose that there is a point where $q_1(x) > 1 - a$. Then since q_1 is continuous and non-decreasing, there is a point y such that $q_1(y) = 1 - a$ and $y < x$. Moreover, it holds that $\dot{\lambda}_1 \geq 0$ a.e. on $[y, \overline{v}]$ and thus $\lambda_1 \leq 0$ on the same interval and in turn $\dot{\lambda}_2 > 0$ a.e. on this interval from (17). Accordingly we have

$$\lambda_2 < 0 \quad \text{in}\, [y, \overline{v}).$$

Therefore, on this interval, $z = 0$ and q_1 is constant. It follows that $q_1 = 1 - a$, which is contradictory.

□

We obtain from (16) that

$$\lambda_1 \geq 0.$$

This leads from (18) to $\lambda_1(\underline{v}) = -\alpha_1 + \alpha_2 \geq 0$. Again, by Assumption 1, (20) and (21), it is deduced that $\alpha_2 \geq \alpha_1 = 0$.

Indeed we can establish

$$\alpha_2 > \alpha_1 = 0.$$

If $0 = \alpha_2$, considering that $\dot{\lambda}_1 \leq 0$ from (16), we have $\lambda_1 = 0$ and $\dot{\lambda}_1 = 0$ a.e. Again from (16), $q_1 = 1 - a$ a.e. and thus everywhere by absolute continuity. Then from (22) and (23), $\alpha_3 = \alpha_4 = 0$. However, with $\lambda_1 = 0$, we have $\dot{\lambda}_2 > 0$ a.e. This is a contradiction.

Now we can conclude from (21) that $q_1(\underline{v})(\underline{v} - d) - u_1(\underline{v}) = 0.$ and from (5), $p(\underline{v}) = \underline{v} - \frac{u_1(\underline{v})}{q_1(\underline{v})} = d.$

It has been proved that if $a < t$,

$$q_1(v) \leq 1 - a, \qquad p(\underline{v}) = d.$$

We further improve on this result. We proceed to solve the maximization problem $\mathcal{P}$ with condition (13) replaced by the following condition

$$d \leq p(\overline{p}) = \overline{v} - \frac{u_1(\overline{v})}{q_1(\overline{v})}. \tag{27}$$

and at the end verify that the original participation constraint (13), $d \leq \underline{v} - \frac{u_1(\underline{v})}{q_1(\underline{v})}$ is satisfied.

Why this replacement can be done is intuitively explained in the following way. Setting a high price, the underwriter gains more commission but pays more for the shares it purchases. Setting a low price, it makes less commission but gains more by reselling. Therefore, the underwriter will set a high price when the spread is large and refrain from buying shares. On the other hand, with a small spread, it will make profits by underpricing. The result of case $a > t$ shows that even in that case, the underwriter buys the maximum. It is then natural to think that in the case of $a < t$ the underwriter should also buy the largest amount t. Then q_1 is constant by monotonicity and so is p

and we can replace the original participation constraint by the new one.

All of the necessary conditions of Problem $\mathcal{P}$ are carried over here except those from (18) to (23), which we replace by

$$\begin{aligned}
\lambda_1(\underline{v}) &= -\alpha_1, & \alpha_1(u_1(\underline{v}) - c) &= 0,\\
\lambda_1(\overline{v}) &= -\alpha_2, & \alpha_2(q_1(\overline{v})(\overline{v} - d) - u_1(\overline{v})) &= 0,\\
\lambda_2(\underline{v}) &= -\alpha_3, & \alpha_3(q_1(\underline{v}) - (1-t)) &= 0,\\
\lambda_2(\overline{v}) &= \alpha_2(\overline{v} - d) - \alpha_4, & \alpha_4(1 - q_1(\overline{v})) &= 0,
\end{aligned}$$

Lemma 4. $\lambda_0 = 1$.

Proof. Suppose that $\lambda_0 = 0$ and then we obtain that λ_1 is a non-positive constant.

Actually, λ_1 is a negative constant. To see this, let us suppose $\lambda_1 = 0$. Then we obtain that $\lambda_2 < 0$ and thus $z = 0$ from the maximization of the Hamiltonian. We see that q_1 is constant from (10). On the other hand, $\lambda_2 < 0$ leads to $q_1(\underline{v}) = 1 - t$ and $q_1(\overline{v}) = 1$ by the terminal conditions. This is a contradiction to q_1 being constant.

Now, since $\dot{\lambda}_2 = -\lambda_1 > 0$, we have $\lambda_2 < 0$ in $[\underline{v}, \overline{v})$, which leads from the terminal condition to $q_1(\underline{v}) = 1 - t$. In addition, $z = 0$ in $[\underline{v}, \overline{v})$.
Thus from (10), we obtain

$$q_1 = 1 - t \quad \text{in}\, [\underline{v}, \overline{v}]\,.$$

Since λ_1 is a negative constant, we have $u_1(\underline{v}) = c$ and $u_1(\overline{v}) = q_1(\overline{v})(\overline{v} - d)$. We also obtain by (9) that

$$u_1(v) = (1-t)(v - \underline{v}) + u_1(\underline{v}) = (1-t)(v - \underline{v}) + c.$$

At the same time, u_1 must satisfy $u_1(\overline{v}) = (1-t)(\overline{v} - d)$. As a result, it must be satisfied that

$$(1-t)(\overline{v}-d) = (1-t)(\overline{v}-\underline{v}) + c.$$

This is impossible due to Assumption 1.

□

Lemma 5.

$$\lambda_1 \leq 0.$$

Proof. We will prove $\lambda_1 \leq 0$ by contradiction. Let us suppose there exists v_1' such that $\lambda_1(v_1') > 0$. Then there is in the neighborhood of v_1' such v that $v \neq \underline{v}$ and $\lambda_1(v) > 0$ and that there exists $\dot{\lambda}_1$ at v, because λ_1 is absolutely continuous. Now we can suppose $\lambda_1(v) > 0$. Then there is in $[\underline{v}, v]$ a non-negligible set S at which point $\dot{\lambda}_1$ exists by absolute continuity and $\dot{\lambda}_1 > 0$. For if there is not, $\dot{\lambda}_1 \leq 0$ a.e. in $[\underline{v}, v]$ and

$$\lambda_1(v) = \int_{\underline{v}}^{v} \dot{\lambda}_1(s)ds + \lambda_1(\underline{v}) \leq 0.$$

Thus if we take $y \in S$, $\dot{\lambda}_1(y) = \big(\frac{a-1}{q_1(y)} + 1\big)f(y) > 0$. It follows that $q_1(y) > 1-a$, which leads to $q_1(y) > 1-a$ due to the monotonicity of q_1. Therefore,

$$\dot{\lambda}_1(v) = \big(\frac{a-1}{q_1(v)} + 1\big)f(v) > 0.$$

Again, by the monotonicity of q_1, it is true that $\dot{\lambda}_1 > 0$ a.e. in $[y, \overline{v}]$. Then we have, for $x \geq v$,

$$\lambda_1(x) = \int_{v}^{x} \dot{\lambda}_1(s)ds + \lambda_1(v) > 0.$$

Therefore,

$$\lambda_1(\overline{v}) = \int_{v}^{\overline{v}} \dot{\lambda}_1(s)ds + \lambda_1(v) > 0.$$

This is a contradiction to $\lambda_1(\overline{v}) \leq 0$.

$\square$

Now we obtain, as in the other two cases, (25) and thus $q_1 = 1 - t$.

The rest is similar to the other cases and we actually obtain the results of the theorem.

It only remains to verify that the original participation constraint (13), $d \leq \underline{v} - \frac{u_1(\underline{v})}{q_1(\underline{v})}$ is indeed satisfied. Immediately we can see that it is.

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