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First Price Package Auction with Many Traders

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We study a first price package auction with many buyers and *many sellers* in a decentralized networked market. We show that any equilibrium with profit-target strategies is efficient and the set of the equilibrium payoffs is equal to a bidder optimal core relative to an exogenously given network. We further show that in the bidder optimal core, each buyer earns a less payoff than the VCG payoff, but does a larger payoff than the competitive equilibrium payoff. Finally, we discuss coalition-proofness and stability of networks.

1 Introduction

A package auction is a selling mechanism where each buyer bids on bundles of multiple items, or packages. The theory of the package auction recently plays an important role in the real economy. For example, the U.S. and the U.K. governments sell their bundles of spectrum bands under the guidance of auction theorists (see e.g. Cramton et al. (2006)).

In their seminal paper, Bernheim and Whinston (1986) first analyze a static package auction in which only one seller exists. They show that there exist equilibria where each bidder is truth-telling, and the corresponding equilibrium payoffs are in the bidder-optimal frontier of the core. Ausubel and Milgrom (2002) show that the bidder-optimal frontier of the core is implemented by their dynamic ascending proxy package auction with a single seller. This paper extends the static first price package auction model with a single seller to that with multiple sellers, and show that the results of Bernheim and Whinston (1986) hold true in the auction with multiple sellers.

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Furthermore, this paper studies an auction market where a network structure is embedded. We often observe that a large two-sided market with many traders is networked. Each trader cannot encounter any other trader unless they are linked. Kranton and Minehart (2001) and Corominas-Bosch (2004) characterize a relation to the competitive equilibrium in networked markets using centralized mechanisms.

A natural class of selling mechanisms between multiple buyers and multiple sellers is the class of *centralized mechanisms*. Each centralized mechanism (e.g. the Vickrey-Clark-Groves (VCG) mechanism and the Double auction) assumes the existence of a unique auctioneer or a unique market maker who can collect all messages from all buyers and all sellers, compute an array of trades and prices, and impose them. If the market maker exists, the VCG mechanism implements an efficient allocation and the Double auction mechanism implements the competitive equilibrium, respectively.

In contrast, this paper assumes no market maker. We study a class of *decentralized mechanisms*, where each buyer's message is a collection of separate messages sent to different sellers—one for each seller, and all actions of a particular seller and her final allocation is independent of messages that the buyers send to other sellers (Peters and Severinov (2006)). Our motivations are as follows. First, implementing the centralized VCG mechanism would be difficult because of computational complexity in a large market consisting of multi-buyers and multi-sellers. Second, many real exchange markets (e.g. a wholesaler-retailer market and a manufacturer-supplier market) have no market maker. Third, emergence of market makers is often prohibited by the government from the view of antitrust law.

Peters and Severinov (2006) analyze a decentralized auction with many sellers where buyers have single-unit demands and sellers have single-unit supplies. They show that the existence of a symmetric perfect Bayesian equilibrium resulting in the Vickrey outcome. Anwar et al. (2006) support their prediction by testing data from competing auctions in eBay.

In an auction with multi-object demand and multi-object supply, we show that there exist equilibria where each buyer bids truthfully given a network, and any corresponding equilibrium payoff vector is efficient and in a bidder optimal core relative to a given network. This result is an extension of Milgrom (2004, Theorem 8.7). He further shows that in the auction with a single seller, the equilibrium outcome is unique and equal to the VCG outcome if goods are substitutes for all buyers.

In our auction with multiple sellers, in contrast, we find that in any equilibrium, each buyer earns less payoff than his/her VCG payoff and greater payoff than his/her competitive equilibrium payoff.

The rest of the paper is organized as follows. Section 2 introduces a networked market with many buyers and many sellers, and models a first price package auction. Section 3 provides our main results. We show that the set of equilibrium payoffs is the same as the bidder-optimal core relative to an exogenously given network. Sections 4 investigate relations to the VCG mechanisms and the competitive equilibrium. Section 5 discusses coalition-proofness of equilibria and stability of networks, and Section 6 concludes.

2 Preliminaries

2.1 A Pure Exchange Networked Market

A (pure exchange) networked market consists of b buyers indexed by $i = 1, \dots, b$ and s sellers indexed by $j = 1, \dots, s$. Let I and J be the set of buyers and the set of sellers, respectively. They trade N commodities indexed by $n = 1, \dots, N$. Each commodity n is perfectly divisible. We denote each package or bundle of commodities by $x \in \mathbb{R}_+^N$ and the set of all bundles by X .

Each seller j has endowments $\omega_j \in \mathbb{R}_+^N$ and a valuation function v_j over X . Each buyer i has no endowments and a valuation function v^i over X . Valuation functions v^i and v_j satisfy continuity, quasi-concavity, and *free disposal*, i.e. $v^i(x) \geq v^i(x')$, and $v_j(x) \geq v_j(x')$ for all $x, x' \in X$ with $x \geq x'$. For normalization, let $v^i(0) = v_j(0) = 0$. We assume that v^i , v_j , and ω_j are common knowledge among all buyer i and all seller j (*Complete information*).

The market is networked. Let ij be a link between buyer i and seller j .¹ A set of links $g \subset \{ij\}_{i \in I, j \in J}$ is a (bipartite) network in the networked market. All trading between buyers and sellers are restricted by network $g \subset \{ij\}_{i \in I, j \in J}$. Each pair of buyer i and seller j can trade if and only if they are linked, i.e. $ij \in g$. Let $L^i(g)$ be the set of sellers who are linked to i , and $L_j(g)$ be the set of buyers who are connected to j . We denote an allocation from seller j to buyer i by $x_j^i = (x_{j1}^i, \dots, x_{jN}^i)$. For simplicity, we denote an allocation to j -self by $x_j^0 = \omega_j - \sum_i x_j^i$.²

¹In what follows, the left i designates a buyer and the right j designates a seller.

²In what follows, the superscript i designates buyer i and the subscript j designates seller j .

Definition 1. An allocation $x = (x_1^1, x_2^1, \dots, x_s^b) \in \mathbb{R}_+^{b \times s \times N}$ is *feasible* relative to network g (*g -feasible*) if

$$\begin{aligned} \sum_j \omega_j &\geq \sum_{ij} x_j^i, \\ x_j^i &\geq 0 \text{ for all } ij \in g, \text{ and} \\ x_j^i &= 0 \text{ for all } ij \notin g. \end{aligned}$$

Let $F(g)$ be the set of g -feasible allocations.

We assume that all buyers and all sellers have quasi-linear payoff functions. A Pareto-efficient allocation relative to g is defined as follows:

Definition 2. An allocation $\bar{x}(g) = (\bar{x}_1^1(g), \dots, \bar{x}_s^b(g))$ in the networked market g is *g -efficient* if

$$\bar{x}(g) \in \arg \max_{x \in F(g)} \sum_i v^i(\sum_j x_j^i) + \sum_j v_j(x_j^0).$$

2.2 First price package auction in a networked market

We develop a decentralized mechanism in which each seller j sells j 's endowments using a first price package auction. A first price package auction is organized as follows:

Step 1. Each buyer i bids a payment schedule $t_j^i(x_j^i)$ for $j \in J$ simultaneously, where $x_j^i = (x_{j1}^i, \dots, x_{jN}^i)$ is an allocation from j to i . We assume $x_j^i \geq 0$ and $t_j^i(x_j^i) \geq 0$ for all i, j , and x_j^i . For seller $j \notin L^i(g)$ who does not linked with i , $t_j^i(x_j^i) = \emptyset$.

Step 2. Each seller j allocates commodities $x_j = (x_j^1, \dots, x_j^b) \in \mathbb{R}_+^{b \times N}$, simultaneously. For buyer $i \notin L_j(g)$ who does not linked with j , $x_j^i = (0, \dots, 0) \in \mathbb{R}_+^N$. By the resource constraint, an allocation to j -self $x_j^0 = \omega_j - \sum_i x_j^i \geq 0$.

In Step 1, each buyer i offers a payment schedule $t_j^i(x_j^i)$ to any linked seller $j \in L^i(g)$. Each payment schedule t_j^i depends only on an allocation from j to i , x_j^i , and cannot depend on other x_k^l ($kl \neq ij$). After bidding, each seller j allocates commodities to buyers $x_j = (x_j^0, x_j^1, \dots, x_j^b)$ in Step 2. Finally, each buyer i pays sellers for $x^i = (x_1^i, \dots, x_s^i)$, according to the given i 's bids $t^i = (t_1^i, \dots, t_s^i)$.

Let $t = (t^1, \dots, t^b)$ be a strategy profile of buyers, and $x = (x_1, \dots, x_s)$ be an allocation profile of sellers. The payoff functions for buyer i and seller j over a profile (t, x) are given by

$$\Pi^i(t, x) = v^i\left(\sum_{j \in L^i(g)} x_j^i\right) - \sum_{j \in L^i(g)} t_j^i(x_j^i) \quad (1)$$

$$\Pi_j(t, x) = v_j(x_j^0) + \sum_{i \in L_j(g)} t_j^i(x_j^i). \quad (2)$$

2.3 Profit-Target Strategies

Let $t_j = (t_j^1, \dots, t_j^b)$ be a bidding profile to j . Given t_j , independently of a bidding profile to other sellers t_{-j} , seller j maximizes the payoff function $\Pi_j(t, x)$. Let $X_j^*(t_j) = \arg \max_x \Pi_j(t, x)$. Each seller j chooses $x_j^*(t_j) \in X_j^*(t_j)$ for any t_j . Fix a profile $(x_j^*(t_j))_{j \in J}$ for each bidding profile t . Then, the auction is truncated to normal form game $\Gamma = (t^i, \Pi^i)_{i \in I}$, where a payoff function for buyer i is redefined by

$$\Pi^i(t) = v^i\left(\sum_j x_j^{i*}(t_j)\right) - \sum_{j \in L^i(g)} t_j^i(x_j^{i*}(t_j)). \quad (3)$$

Definition 3. Let $\pi^i = (\pi_1^i, \dots, \pi_s^i)$. For any profile t , buyer i 's bidding strategy $t^i = (t_j^i)_{j \in J}$ is a π^i -profit-target strategy if for all t_j^i , all x_j^i , and $j \in L^i(g)$,

$$t_j^i(x_j^i) = \max\{0, f_j^i(x_j^i) - \pi_j^i\},$$

where

$$f_j^i(x_j^i) = v^i(x_j^i + \sum_{l < j} x_l^{i*}(t_l)) - v^i\left(\sum_{l < j} x_l^{i*}(t_l)\right).$$

Note that $\sum_{j \in J} f_j^i(x_j^{i*}(t_j)) = v^i(\sum_{j \in J} x_j^{i*}(t_j))$. This implies that any π^i -profit-target strategy reveals i 's valuation function *truthfully*. Given i 's allocation $(x_l^{i*}(t_l))_{l < j}$ from sellers $l < j$, buyer i 's payment schedule to seller j is a function defined as the increase of true valuation minus constant target profit π_j^i for any package x_j^i and any seller j . Each buyer i earns the sum of target profit $\sum_{j \in J} \pi_j^i$.

Proposition 1. For a bidding profile of other buyers t^{-i} , take a best response $t^i \in \arg \max_{t^i} \Pi^i(t^i, t^{-i})$.

Let for $j \in J$,

$$\begin{aligned}\bar{\pi}_j^i &= v^i(x_j^{i*}(t_j) + \sum_{l < j} x_l^{i*}(t_j)) - v^i(\sum_{l < j} x_l^{i*}(t_j)) - t_j^i(x_j^{i*}(t_j)) \\ f_j^i(x_j^i) &= v^i(x_j^i + \sum_{l < j} x_l^{i*}(t_j)) - v^i(\sum_{l < j} x_l^{i*}(t_j)).\end{aligned}$$

Then, the $\bar{\pi}^i$ -profit-target strategy is a best response of buyer i to t^{-i} .

Proof. Since $\sum_j \bar{\pi}_j^i = v^i(\sum_j x_j^{i*}(t_j)) - \sum_j t_j^i(x_j^{i*}(t_j))$, buyer i 's payoff for profile t is $\Pi^i(t, t^{-i}) = \sum_j \bar{\pi}_j^i$. Denote by \bar{t}^i , the $\bar{\pi}^i$ -profit-target strategy of i . By the definition, $t_j^i(x_j^{i*}(t_j)) = \bar{t}_j^i(x_j^{i*}(t_j))$ for all i, j , and x_j^{i*} . By $x_j^* \in \arg \max_{x_j} \Pi_j(t, x)$, we obtain $\sum_{i \in L_j(g)} \bar{t}_j^i(x_j^i) + v_j(x_j^0) \leq \sum_{i \in L_j(g)} t_j^i(x_j^{i*}(t_j)) + v_j(x_j^{0*}(t_j))$.

Suppose that $\bar{t}_j^i(x_j^i) = 0$ for some x_j^i . Then, by $0 = \bar{t}_j^i(x_j^i) \leq t_j^i(x_j^i)$, we obtain

$$\bar{t}_j^i(x_j^i) + \sum_{k \neq i} t_j^k(x_j^k) + v_j(x_j^0) \leq t_j^i(x_j^i) + \sum_{k \neq i} t_j^k(x_j^k) + v_j(x_j^0) \leq \bar{t}_j^i(x_j^{i*}(t_j)) + \sum_{k \neq i} t_j^k(x_j^{k*}(t_j)) + v_j(x_j^{0*}(t_j)).$$

Thus, $x_j^{i*}(\bar{t}_j^i, t_j^{-i}) \neq x_j^i$ for all x_j^i with $\bar{t}_j^i(x_j^i) = 0$. This implies that $\bar{t}_j^i(x_j^{i*}(\bar{t}_j^i, t_j^{-i})) = f_j^i(x_j^{i*}(\bar{t}_j^i, t_j^{-i})) - \bar{\pi}_j^i > 0$.

Hence,

$$\Pi^i(\bar{t}_j^i, t_j^{-i}) = v^i(\sum_j x_j^{i*}(\bar{t}_j^i, t_j^{-i})) - \sum_j \bar{t}_j^i(x_j^{i*}(\bar{t}_j^i, t_j^{-i})) = \sum_j \bar{\pi}_j^i = \Pi^i(t^i, t^{-i}). \quad \blacksquare$$

Proposition 1 implies that the profit target strategy is a best response for any profile t^{-i} . The existence of Nash equilibria (NE) with profit target strategies is shown by construction in Proposition 2. We remark that the set of all NE payoff vectors with profit-target strategies is a proper subset of all NE payoff vectors.

Example 1. Consider a networked market with $b = 2$, $s = 1$, $N = 1$, and $g = g^c = \{11, 21\}$. Let $v_1 = 10x_1^1$, $v_2 = 5x_1^2$, and $\omega_1 = 1$. Take any (t_1^1, t_1^2) such that $(x_1^{1*}(t), x_1^{2*}(t)) = (1, 0)$. Given t_1^1 , $\bar{\pi}^2$ -profit-target strategy \bar{t}_1^2 is the 0-profit-target strategy for any best response t_1^2 . Given \bar{t}_1^2 , $\bar{\pi}_1^1$ -profit-target strategy \bar{t}_1^1 is the 5-profit-target strategy for any best response t_1^1 . Thus, in the NE with profit-target strategies $(\bar{t}_1^1, \bar{t}_1^2)$, the payoff vector is $(5, 0, 5)$. By the same argument, we can show that the $(\bar{t}_1^1, \bar{t}_1^2)$ is the unique NE with profit-target strategies. However, take a profile $(t_1^1, t_1^2) = (7x_1^1, 7x_1^2)$. Then $x_1^*(t) = (1, 0)$. This profile yielding the payoff vector $(3, 0, 7)$ is an NE. Thus, $(3, 0, 7)$ supported by an NE is not supported

by any NE with profit-target strategies.

3 Equilibria and Bidder-Optimal Core

To characterize the set of NE payoff vectors with profit-target strategies, we define a cooperative game (M, g, w) in the networked market as follows. Let $M = I \cup J$ be a set of all buyers and all sellers, and $S \subset M$ be a coalition. Fix a network g . Let g_S be a subnetwork for coalition S such that $ij \in g_S$ if $i, j \in S$ and $ij \in g$, and $ij \notin g_S$ otherwise. Denote by $F(g_S)$, the set of g_S -feasible allocations. We define a characteristic function w as for all $S \subset M$,

$$w(S|g) = \max_{x \in F(g_S)} \sum_{i \in S} v^i \left(\sum_{j \in S} x_j^i \right) + \sum_{j \in S} v_j(x_j^0). \quad (4)$$

Obviously, $w(S|g)$ is super-additive in coalition S . Thus, the core is defined as follows. We denote a payoff for i and j by ϕ^i and ϕ_j , respectively.

Definition 4. The core of (M, g, w) is given by

$$\text{Core}(M, g, w) = \left\{ \phi \mid \sum_{i \in I} \phi^i + \sum_{j \in J} \phi_j \leq w(M|g) \right\} \cap \left\{ \phi \mid \sum_{i \in I \cap S} \phi^i + \sum_{j \in J \cap S} \phi_j \geq w(S|g) \forall S \subset M \right\}.$$

By definition, each payoff vector in the core is g -efficient. When $s = 1$, the set of payoffs supported by an NE with profit-target strategies coincides with the bidder optimal core (Milgrom (2004, Theorem 8.7)). The following proposition shows that the above result can be extended to cases $s \geq 2$.

Definition 5. A payoff vector ϕ is *bidder-optimal* relative to g if $\phi \in \text{Core}(M, g, w)$ and there exists no $\phi' \in \text{Core}(M, g, w)$ with $\phi'^i \geq \phi^i$ for all $i \in I$ and $\phi'^i > \phi^i$ for some $i \in I$. The set of bidder-optimal payoff vectors relative to g is the *bidder-optimal core* relative to g .

Proposition 2. Fix a networked market $((v^i, v_j, \omega_j)_{i,j \in M}, g)$.

- (i) Suppose ϕ is bidder optimal relative to g . Then, any profile t of π^i -profit-target strategies yielding the payoff vector ϕ constitutes an NE.
- (ii) Conversely, suppose that profile t of π^i -profit-target strategies yielding the payoff vector ϕ constitutes an NE. Then, ϕ is bidder optimal relative to g .

Proof. Part (i): We show that if ϕ is bidder optimal then any profile t of π^i -profit-target strategies yielding ϕ constitutes an NE. Take a profile t of π^i -profit-target strategies yielding ϕ and let $x^*(t)$ be the corresponding allocation of sellers. Then $\sum_j \pi_j^i = \phi^i$ for all buyer i . By Proposition 1, it suffices to show that there is no deviation to another profit-target strategy \hat{t}^i .

Suppose that buyer $i \in L_j(g)$ deviates to \hat{t}^i with $\hat{\pi}^i = (\pi_j^i + \delta, \pi_{-j}^i)$ for some seller $j \in L^i(g)$ ($\delta > 0$). First, suppose $x_j^{i*}(t_j) = 0$. Then $x_j^{i*}(\hat{t}_j^i, t_j^{-i}) = 0$. Hence, the deviation is not profitable for buyer i .

Second, suppose $x_j^{i*}(t_j) > 0$. If there exists no subcoalition S such that $i \notin S, j \in S$, and $w(S|g) = \sum_{k,l \in S} [\phi^k + \phi_l]$, then there exists $\delta > 0$ such that $(\phi^i + \delta, \phi^{-i}, \phi_j - \delta, \phi_{-j})$ which dominates ϕ for buyers is in $Core(M, g, w)$. This contradicts that ϕ is bidder optimal.

Thus, since ϕ is bidder optimal, there exists a subcoalition S such that $i \notin S, j \in S$, and $w(S|g) = \sum_{k,l \in S} [\phi^k + \phi_l]$ for any buyer i and any seller j . This implies

$$\phi_j = \max_{\{x_j | x_j^k = 0 \forall k \notin S\}} \sum_{k \in I} t_j^k(x_j^k) + v_j(x_j^0).$$

Then, we obtain for such coalition S ,

$$\begin{aligned} \max_{\{x_j | x_j^k = 0 \forall k \notin S\}} \sum_{k \in I} t_j^k(x_j^k) + v_j(x_j^0) &> \phi_j - \delta \\ &= \max_{x_j} \sum_{k \in I} t_j^k(x_j^k) + v_j(x_j^0) - \delta \\ &\geq \max_{\{x_j | x_j^i > 0\}} \sum_{k \in I} t_j^i(x_j^i) + v_j(x_j^0) - \delta. \end{aligned}$$

This inequality shows that it is profitable for seller j to exclude buyer $i \notin S$. Hence, $x_j^{i*}(\hat{t}_j^i, t_j^{-i}) = 0$, and the payoff for i decreases. Therefore, there is no profitable deviation for any buyer i .

Part (ii): To show the converse, suppose that profile t of profit-target strategies and the corresponding $x^*(t)$ constitutes an NE with payoff vector ϕ .

First, we show that $\phi \in Core(M, g, w)$. Suppose $\phi \notin Core(M, g, w)$. Then, there exists coalition S such that $\sum_{i \in I \cap S} \phi^i + \sum_{j \in J \cap S} \phi_j < w(S|g)$. Since each buyer i adopts profit-target strategy, $t_j^i(x_j^i) =$

$\max\{0, f_j^i(x_j^i) - \pi_j^i\}$. Then, we obtain for such S ,

$$\begin{aligned}
\sum_{j \in J \cap S} \phi_j &= \sum_{j \in J \cap S} \max_{x_j} \sum_{i \in I} t_j^i(x_j^i) + v_j(x_j^0) \\
&\geq \sum_{j \in J \cap S} \max_{\{x_j | x_j^i = 0 \forall i \notin S\}} \sum_{i \in I} t_j^i(x_j^i) + v_j(x_j^0) \\
&\geq \sum_{j \in J \cap S} \max_{\{x_j | x_j^i = 0 \forall i \notin S\}} \sum_{i \in I \cap S} f_j^i(x_j^i) - \pi_j^i + v_j(x_j^0) \\
&= \left[\sum_{j \in J \cap S} \max_{\{x_j | x_j^i = 0 \forall i \notin S\}} \sum_{i \in I \cap S} f_j^i(x_j^i) + v_j(x_j^0) \right] - \sum_{i \in I \cap S} \phi^i \\
&= \left[\max_{\{x_j | x_j^i = 0 \forall i \notin S\}} \sum_{i \in I \cap S} v^i(\sum_{j \in J} x_j^i) + \sum_{j \in J \cap S} v_j(x_j^0) \right] - \sum_{i \in I \cap S} \phi^i \\
&= w(S|g) - \sum_{i \in I \cap S} \phi^i > \sum_{j \in J \cap S} \phi_j.
\end{aligned}$$

The fifth equality follows that $x_j^i = 0$ for all $j \notin S$ and $\sum_{j \in J} f_j^i = v_i(\sum_{j \in J} x_j^i)$. This is a contradiction.

Hence, $\phi \in \text{Core}(M, g, w)$.

Next, we show that ϕ is bidder optimal. Let \bar{x} be the corresponding allocation to ϕ . Suppose that ϕ is not bidder optimal. Then, there exists (i, j) and $\delta > 0$ such that $\tilde{\phi} = (\phi^i + 2\delta, \phi^{-i}, \phi_j - 2\delta, \phi_{-j}) \in \text{Core}(M, g, w)$. Suppose that buyer i deviates to $\hat{\pi}^i$ -profit-target strategy such that $\hat{\pi}^i = (\pi_j^i + \delta, \pi_{-j}^i)$ with $\sum_l \pi_l^i = \phi^i$, denoted by $\hat{t}^i = (t_j^i, t_{-j}^i)$. Then, we obtain

$$\begin{aligned}
\max_{x_j} \hat{t}_j^i(x_j^i) + \sum_{k \neq i} t_j^k(x_j^k) + v_j(x_j^0) &\geq \hat{t}_j^i(\bar{x}_j^i) + \sum_{k \neq i} t_j^k(\bar{x}_j^k) + v_j(\bar{x}_j^0) \\
&= \phi_j - \delta > \phi_j - 2\delta.
\end{aligned}$$

Let S^j be any coalition such that $l \in S^j$ for all $l \leq j$ and $l \notin S^j$ for all $l > j$. Since $(\phi^i + 2\delta, \phi^{-i}, \phi_j - 2\delta, \phi_{-j})$ is in $\text{Core}(M, g, w)$,

$$\begin{aligned}
\phi_j - 2\delta &\geq \max_{\{S^j | i \notin S^j\}} w(S^j|g) - \sum_{k \in I \cap S^j} \phi^k - \sum_{l \in J \cap S^j \setminus \{j\}} \phi_l \\
&= \max_{\{S^j | i \notin S^j\}} \left[\max_x \sum_{k \in I \cap S^j} v^k(\sum_{l \leq j} x_l^k) + \sum_{l \leq j} v_l(x_l^0) \right] - \sum_{k \in I \cap S^j} \phi^k - \sum_{l < j} \phi_l \\
&\geq \max_{\{S^j | i \notin S^j\}} \left[\max_{x_j} \sum_{k \in I \cap S^j} v^k(x_j^k) + \sum_{l < j} \bar{x}_l^k + v_j(x_j^0) + \sum_{l < j} v_l(\bar{x}_l^0) \right] - \sum_{k \in I \cap S^j} \phi^k - \sum_{l < j} \phi_l.
\end{aligned}$$

Substituting $\phi^k = \sum_{l \in S^j} \pi_l^k$ and $\phi_l = \sum_k [v^k(\tilde{x}_l^k + \sum_{m < l} \tilde{x}_m^k) - v^k(\sum_{m < l} \tilde{x}_m^k) + v_l(\tilde{x}_l^0) - \pi_l^k]$ into it yields

$$\begin{aligned}
& \max_{\{S^j | i \notin S^j\}} \left[\max_{x_j} \sum_{k \in I \cap S^j} v^k(x_j^k + \sum_{l < j} \tilde{x}_l^k) + \sum_{l < j} v_l(\tilde{x}_l^0) + v_j(x_j^0) \right] - \sum_{k \in I \cap S^j} \phi^k - \sum_{l < j} \phi_l \\
&= \max_{x_j} \max_{\{S^j | i \notin S^j\}} \sum_{k \in I \cap S^j} \left[(v^k(x_j^k + \sum_{l < j} \tilde{x}_l^k) - \pi_j^k - v^k(\sum_{l < j} \tilde{x}_l^k)) + v_j(x_j^0) \right] \\
&= \max_{x_j} \sum_{k \neq i} \left[\max \{ 0, v^k(x_j^k + \sum_{l < j} \tilde{x}_l^k) - v^k(\sum_{l < j} \tilde{x}_l^k) - \pi_j^k \} \right] + v_j(x_j^0) \\
&= \max_{x_j} \sum_{k \neq i} t_j^k(x_j^k) + v_j(x_j^0).
\end{aligned}$$

Therefore, $\phi_j - 2\delta \geq \max_{x_j} [\sum_{k \neq i} t_j^k(x_j^k) + v_j(x_j^0)]$. This inequality implies that seller j 's payoff decreases if j rejects buyer i 's $(\pi_j^i + \delta)$ -profit-target strategy. Thus, since seller j accepts the buyer i 's deviation, it is profitable for buyer i . This contradicts the assumption that profile t is an NE. Hence, ϕ is bidder optimal. ■

Remark 1. Consider the following sequential first price package auction. First, sellers are ordered at random and renamed according to the order. In Stage 1.1, each buyer bids schedules to seller 1. In Stage 1.2, seller 1 decides an allocation. In Stage 2.1, each buyer bids schedules to seller 2, and so on. Since valuations are private, there exists a subgame perfect equilibrium with profit-target strategies which is payoff equivalent to an NE with profit-target strategies of the first price package auction.

By Proposition 2, NE allocation $x^*(t)$ is g -efficient for any g , where t is an NE bidding profile with π^i -profit-target strategies. Thus, any g -efficient outcome yielding a bidder optimal payoff vector is implemented by the first price package auction mechanism for any g .

4 Comparison with Other Market Mechanisms

4.1 Substitutes and VCG outcome

This section studies a relation between the decentralized first price package auction mechanism resulting in the bidder optimal core and the centralized Vickrey-Clark-Groves (VCG) mechanism.

In what follows, we assume that each seller has no valuation over any package (i.e. $v_j(x) \equiv 0$) for

simplification. Milgrom (2004) shows that, whenever $s = 1$, the bidder-optimal payoff vector is unique and coincides with the VCG payoff vector, and then the VCG payoff vector is in the core, if valuations satisfy concavity³ and a substitute condition.⁴

In the VCG mechanism, there is a unique planner. Each buyer i reports valuation \hat{v}^i to the planner (valuations of sellers are known). Then, the planner imposes a g -feasible allocation of commodities and transfer. It is well-known that each buyer i earns i 's marginal contribution $w(M|g) - w(M \setminus \{i\} | g)$ in the truthful equilibrium with dominant strategies of the VCG mechanism. We denote buyer i 's and seller j 's VCG payoffs by $\phi_V^i(g)$ and $\phi_{jV}(g)$, and i 's and j 's bidder optimal payoffs by $\phi_B^i(g)$ and $\phi_{jB}(g)$ for any $g \in G$, respectively. Let $\phi_V = (\phi_V^i, \phi_{jV})_{i,j \in M}$ and $\phi_B = (\phi_B^i, \phi_{jB})_{i,j \in M}$.

Proposition 3. *For any i and any g , $\phi_V^i(g) \geq \max\{\phi_B^i(g)\}$, and $\sum \phi_{jV}(g) \leq \min\{\sum \phi_{jB}(g)\}$.*

Proof. Any payoff vector such that some buyer i 's payoff is strictly greater than i 's marginal contribution is not in core. Thus, i 's VCG payoff $\phi_V^i(g)$ is greater than or equal to i 's maximum payoff $\max\{\phi_B^i(g)\}$ in the bidder optimal core. Since $\sum_{i,j} [\phi_V^i(g) + \phi_{jV}(g)] = \sum_{i,j} [\phi_B^i(g) + \phi_{jB}(g)] = w(M|g)$, we obtain $\sum \phi_{jV}(g) \leq \min\{\sum \phi_{jB}(g)\}$. ■

If Proposition 4 holds with equalities, then the payoff equivalence holds true as $s = 1$. In the following example, equalities hold.

Example 2. Let $I = \{1, 2, 3\}$, $J = \{1, 2\}$, $N = 1$, and $g = g^c$ (complete bipartite graph). Suppose that $\omega_j = 1$ for all j . Each buyer has a valuation function given in Table 1. Each v_i is concave and linear-substitute valuation for $i = 1, 2, 3$. The marginal contribution of buyer 1, 2, and 3 are given by 2, 1, 0, respectively. Hence, $(\phi_V^1, \phi_V^2, \phi_V^3) = (2, 1, 0)$. The bidder optimal payoff vector for buyers is uniquely given by $(\phi_B^1, \phi_B^2, \phi_B^3) = (2, 1, 0)$. This example demonstrates that each buyer $i = 1, 2$ earns i 's marginal contribution, which is the VCG payoff, in the auction.

The next example, however, shows a strict inequity even if the substitute condition holds for buyers.

³We additionally require concavity since commodities are divisible. The equivalence holds true when valuation functions are concave nonlinear-substitute valuations for divisible goods. If we consider multiple indivisible commodities, the strong-substitute property is sufficient. See Milgrom and Strulovici (2009) for details.

⁴Formally, we denote a price vector over commodities N by $p = (p^n)_{n \in N}$ ($p^n \in \mathbb{R}_+$). A demand correspondence for agent i is given by $D^i(p) = \arg \max v^i(\sum_j y_j^i) - \sum_j p \cdot y_j^i$. Valuation v^i is linear-substitute if whenever $p^n \leq \tilde{p}^n$, $p^{-n} = \tilde{p}^{-n}$, and $x \in D^i(p)$, there exists $\tilde{x} \in D^i(\tilde{p})$ such that $x^{-n} \leq \tilde{x}^{-n}$. The linear-substitute and the nonlinear-substitute conditions are equivalent for concave valuations.

	v^1	v^2	v^3
$0 \leq x \leq 1$	$10x$	$9x$	$8x$
$1 < x \leq 2$	$10 + 8(x - 1)$	$9 + 7(x - 1)$	$8 + 6(x - 1)$
$2 < x$	18	16	14

Table 1: Valuation functions for buyers

Example 3. Let $I = \{1, 2\}$, $J = \{1, 2\}$, $N = 1$, and $g = g^c$. Suppose that $\omega_j = 1$ for all j . The valuation function for buyers are also given in Table 1. The marginal contributions of buyer 1 and 2 are 3 and 1 respectively. Thus, $(\phi_V^1, \phi_V^2) = (3, 1)$. However, the set of bidder optimal payoff vector for buyers (ϕ_B^1, ϕ_B^2) is $\{(2, 1)\}$. This example demonstrates that in the auction, each buyer $i = 1, 2$ earns a less payoff than i 's marginal contribution while goods are substitutes for buyer $i = 1, 2$.

Thus, when $s \geq 2$, the payoff equivalence fails even if buyers' valuations are substitute. The VCG payoff vector Pareto-dominates any bidder optimal payoff vector for buyers, and thus is not in the core.

Corollary 1. *Suppose $s \geq 2$. Then, the VCG payoff vector is not in the core, even if buyers' valuations are concave and substitute.*

4.2 Competitive Equilibrium

This section studies a relation to the competitive equilibrium, which is a standard stable outcome when traders are price-takers. In addition, Cripps and Swinkels (2006) show that the centralized Double auction implements the competitive equilibrium. In Example 2, the minimal competitive price $p = 8$ and the corresponding payoff vector for buyers is $(2, 1, 0)$. Since the unique bidder optimal payoff vector for buyers is $(2, 1, 0)$, there is a competitive equilibrium payoff vector in the bidder optimal core. Example 3 also has this relation. However, it fails in the following example.

Example 4. Let $I = \{1, 2, 3\}$, $J = \{1, 2\}$, $N = 1$, and $g = g^c$. Suppose that $\omega_j = 2$ for all j . The valuation function for buyers are also given in Table 1. The minimal competitive price is 7. The corresponding competitive payoff vector for buyers is $(4, 2, 1)$. However, the set of bidder optimal payoff vectors for buyers is $\{(5, 3, 1)\}$. This example demonstrates that each buyer $i = 1, 2, 3$ earns a greater payoff than i 's maximal competitive payoff in the auction.

To investigate the relation formally, we introduce a link-based price vector $(p_j^i)_{i \in I, j \in J}$ into the networked market g . Each $p_j^i(g) \in \mathbb{R}_+^N$ is a price vector between buyer i and seller j . If buyer i buys x_j^i from seller j , then i pays $p_j^i(g)x_j^i$ to j .

Definition 6. $(p_j^i(g), x_j^i(g))_{i \in I, j \in J}$ is a *competitive equilibrium relative to g* (g -competitive equilibrium) if for all i and all j ,

- (i) $(x_1^i(g), \dots, x_s^i(g)) \in \arg \max v^i(\sum_j y_j^i) - \sum_j p_j^i(g)y_j^i$ subject to $y_j^i = 0$ for all $ij \notin g$, and
- (ii) $(x_1^j(g), \dots, x_j^b(g)) \in \arg \max \sum_i p_j^i(g)y_j^i$ subject to $y_j^i = 0$ for all $ij \notin g$ and $\sum_i y_j^i \leq \omega_j$.

Let $\phi_C^i(g)$ and $\phi_{jC}(g)$ be i 's and j 's g -competitive payoffs respectively, and $\phi_C = (\phi_C^i, \phi_{jC})_{i,j \in M}$. The following standard property of the competitive equilibrium holds in the networked market.

Lemma 1. $\phi_C(g) \in \text{Core}(M, g, w)$.

Proof. Fix g . Let $(p_j^i, x_j^i)_{i \in I, j \in J}$ be a competitive equilibrium, and let p_j be a vector with $p_{jn} = \max_{i \in L_j(g)} p_{jn}^i$ for all $n = 1, \dots, N$. Since $(x_j^i)_{i \in I} \in \arg \max_{y_j} \sum_i p_j^i y_j^i$ for all j , we obtain $\sum_i p_j^i x_j^i = p_j \omega_j$ for all j .

Take any coalition S . Suppose that there exists $(\tilde{x}_j^i)_{i,j \in S}$ such that $v^i(\sum_{j \in S} \tilde{x}_j^i) - \sum_{j \in S} p_j \tilde{x}_j^i > \phi_C^i(g)$ and $\sum_{i \in S} p_j \tilde{x}_j^i > \phi_{jC}(g)$ for all $i, j \in S$. Then, $\sum_{i \in S} p_j \tilde{x}_j^i > \sum_i p_j^i x_j^i = p_j \omega_j$ for all $j \in S$. Hence, $\sum_{i,j \in S} p_j \tilde{x}_j^i > \sum_{j \in S} p_j \omega_j$. However, $\sum_{i,j \in S} \tilde{x}_j^i < \sum_{j \in S} \omega_j$ by feasibility. This is a contradiction. Thus, there is no coalition S such that $\sum_{i \in S} \phi_C^i(g) + \sum_{j \in S} \phi_{jC}(g) < w(S|g)$. ■

By definition, we obtain the following straightforwardly.

Proposition 4. *There exists $(\phi_B^i(g))_{i \in I}$ such that $\phi_B^i(g) \geq \phi_C^i(g)$ for all i, g , and $(\phi_C^i(g))_{i \in I}$. Moreover, $\max \{ \sum \phi_{jB}(g) \} \leq \min \{ \sum \phi_{jC}(g) \}$.*

Example 4 shows that a strict inequality holds in some networked market. Thus, the first price package auction mechanism does not implement the competitive equilibrium, and any competitive equilibrium is Pareto-dominated by a bidder optimal outcome for buyers.

5 Discussion

5.1 Coalition-proofness of equilibria

In the auction with a single seller ($s = 1$), Bernheim and Whinston (1986) show that the set of NEs with profit-target strategies yielding a bidder-optimal payoff vector is equal to the set of coalition-proof Nash equilibria. Thus, the bidder optimal core is equal to it. In this section, we extend this result to the auction with multiple sellers, $s \geq 2$.

To show it, we define a component game relative to coalition S with $J \subsetneq S \subsetneq M$ as follows: For any bidding profile t , let $t^S = (t^i)_{i \in S \cap I}$. The component game is given by $\Gamma/t^{M \setminus S} = ((t^i, \tilde{\Pi}^i)_{i \in S})$, where $\tilde{\Pi}^i = \Pi^i(t^{S \cap I}, t^{M \setminus S})$.

Definition 7. Fix a networked market $((v^i, v_j, \omega_j)_{i,j \in M}, g)$.

- (i) In a first price package auction Γ with a single buyer ($b = 1$) and s sellers ($s \geq 1$), profile t^1 is a coalition-proof Nash equilibrium (CPNE) if it is an NE.
- (ii) (a) For a first price package auction Γ with b buyers ($b \geq 2$) and s sellers, profile t^M is self-enforcing if for any coalition S with $J \subsetneq S \subsetneq M$, profile t^S is a CPNE in the component game relative to S , $\Gamma/t^{M \setminus S}$.
- (b) For any first price package auction Γ with a set S of buyers and sellers, profile t^M is a CPNE if it is self-enforcing, and it does not Pareto-dominate by another self-enforcing profile.

Proposition 5. *The bidder-optimal core is equal to the set of CPNE payoff vectors.*

Proof. Fix $s \geq 1$. In any auction with a single buyer ($b = 1$), it is obvious that the unique NE yielding the unique bidder-optimal payoff vector is a CPNE.

Assume that the proposition holds true in any auction with $b = 1, \dots, m$ buyers and s sellers. Consider an auction with $m + 1$ buyers and s sellers. By Proposition 2, for any bidder-optimal payoff vector ϕ , there is an NE t with profit-target strategies yielding ϕ .

We first show that any bidder optimal ϕ is supported by a CPNE. Take an NE t with profit-target strategies yielding bidder optimal ϕ . Since t is a profile of profit-target strategies, for any S and any

seller j , in the component game $\Gamma/t^{M \setminus S}$,

$$x_j^*(t^S, t^{M \setminus S}) \in \arg \max_{\sum_i x_j^i \leq \omega_j} v_j^0(x_j^0) + \sum_i v^i(x_j^i) + \sum_{l < j} x_l^{k^*}(t^S, t^{M \setminus S}).$$

Thus, in component game $\Gamma/t^{M \setminus S}$, each seller j chooses the same $x_j^*(t) = x_j^*(t^S, t^{M \setminus S})$ as in the game Γ . Hence, $(\phi^i, \phi_j)_{i,j \in S}$ is bidder optimal in $\Gamma/t^{M \setminus S}$. Since the proposition holds true for $b = 1, \dots, m$, profile t^S is a CPNE in $\Gamma/t^{M \setminus S}$ for any $S \subsetneq M$. Thus, t is self-enforcing. Since ϕ is bidder optimal, t is not Pareto-dominated by another self-enforcing profile. Hence, t is a CPNE. By mathematical induction, any NE t yielding bidder optimal ϕ is a CPNE for any $b \in \mathbb{N}$.

We next show that any CPNE payoff vector is bidder-optimal. Take a CPNE t yielding ϕ . Since the proposition holds true for $b = 1, \dots, m$, payoff vector $(\phi^i, \phi_j)_{i,j \in S}$ is bidder optimal in component game $\Gamma/t^{M \setminus S}$ for any $S \subsetneq M$. Suppose that payoff vector ϕ is not bidder optimal in game Γ . Then, there exists $\hat{\phi}$ that Pareto-dominates ϕ for buyers in some coalition S with $J \subsetneq S \subsetneq M$. Hence, ϕ is not bidder optimal in component game $\Gamma/t^{M \setminus S}$. However, $(\phi^i, \phi_j)_{i,j \in S}$ is bidder optimal in $\Gamma/t^{M \setminus S}$. This is a contradiction. Thus, ϕ is bidder optimal.

By mathematical induction, any CPNE payoff vector is bidder-optimal for any $b \in \mathbb{N}$.

Applying the above argument for any number of sellers $s \in \mathbb{N}$ shows that the bidder optimal core is equivalent to the set of CPNE payoff vectors for any b, s . ■

5.2 Stability of Networks

We have investigated the allocation problem on a given network. This section discusses efficiency and stability of networks, provided that the selling mechanism is the first price package auction. Denote by η^i and η_j , numbers of links of i and j in g respectively. Let L be a link-based transaction cost function to form network g . We assume that $L(g) = \sum_i l^i \eta^i(g) + \sum_j l_j \eta_j(g)$, where l^i and l_j are some constants for buyer i and seller j respectively. Thus, social welfare in network g is given by $W(g) = w(M|g) - L(g)$. The network g is efficient if $g \in \arg \max_{g'} W(g')$.

Let $g + ij$ be a network given by adding link $ij \notin g$ to g , and $g - ij$ be a network given by deleting $ij \in g$ from g . Then if g is efficient, $w(M|g + ij) - w(M|g) \leq L(g + ij) - L(g) = l^i + l_j$ for all $ij \notin g$ and

$w(M|g) - w(M|g - ij) \geq L(g) - L(g - ij) = l^i + l_j$ for all $ij \in g$.

First, we develop a model of unilateral formation of networks. Each buyer i unilaterally forms a link with any seller j with whom i wants to link at cost $l^i > 0$ for all i . For all seller j , $l_j = 0$. A network g is unilaterally stable if there exists a bidder-optimal payoff vector ϕ_B such that

- (i) for $ij \in g$, $\phi_B^i(g) - \phi_B^i(g - ij) \geq l^i$, and
- (ii) for $ij \notin g$, $\phi_B^i(g + ij) - \phi_B^i(g) \leq l^i$.

Proposition 6. *Any efficient network is unilaterally stable.*

Proof. Take an efficient network g and a payoff vector $\phi_B(g)$. By definition, $w(S|g) = w(S|g - ij)$ for any coalition $S \ni i$. By bidder optimality, this implies that for all i and all $ij \in g$, there exist bidder optimal $\phi_B(g)$ and $\phi_B^i(g - ij)$ such that

$$\phi_B^i(g) = \phi_B^i(g - ij) + [w(M|g) - w(M|g - ij)]. \quad (5)$$

Since g is efficient, $\phi_B^i(g) - \phi_B^i(g - ij) = w(M|g) - w(M|g - ij) \geq l^i$. Thus, any efficient g satisfies the condition (i). Applying the same derivation for $ij \notin g$ yields that $\phi_B^i(g) - \phi_B^i(g + ij) = w(M|g) - w(M|g + ij) \geq -l^i$ for all $ij \notin g$. Thus, the condition (ii) is satisfied. Hence, any efficient network is stable. ■

Next, we model a bilateral formation of networks. A link ij is formed if and only if both i and j agree with forming link ij . We allow side-payments between i and j to form link ij . A network g satisfies pairwise stability with side-payments if

- (i) for $ij \in g$, $[\phi_B^i(g) - \phi_B^i(g - ij)] + [\phi_{jB}(g) - \phi_{jB}(g - ij)] \geq l^i + l_j$, and
- (ii) for $ij \notin g$, $[\phi_B^i(g + ij) - \phi_B^i(g)] + [\phi_{jB}(g + ij) - \phi_{jB}(g)] \leq l^i + l_j$.

If an efficient network g is pairwise stable, then the payoff increase of seller j by forming a new link $ij \notin g$ is smaller than the decrease of social welfare; $\phi_{jB}(g + ij) - \phi_{jB}(g) \leq [L(g + ij) - L(g)] - [w(M|g + ij) - w(M|g)] = W(g) - W(g + ij)$. Thus, if $\phi_{jB}(g + ij) - \phi_{jB}(g) > W(g) - W(g + ij)$ for any $ij \notin g$, then efficient network g is not pairwise stable.

Example 5 (Efficient but not pairwise stable network). Suppose that all buyers are symmetric ($v^i = v$ for all i), all sellers are symmetric and have no value on bundles ($v_j \equiv 0$ and $\omega_j = \omega$ for all j), and $b = s$. Let the link cost l be relatively low such that $v(2\omega) - v(\omega) > 2l$. Then, the unique

architecture of an efficient network is $g = \{11, 22, \dots, ss\}$. Given g , each seller j obtains no surplus ($\phi_{jB}(g) = 0$). If seller j forms a link ij with i ($ij \notin g$), then j earns $\phi_{jB}(g + ij) = v(2\omega) - v(\omega)$. Since $\phi_j(g + ij) - \phi_j(g) = v(2\omega) - v(\omega) > 2l$ and $\phi^i(g + ij) - \phi^i(g) = 0$ for any $ij \notin g$, a pair (i, j) forms a new link $ij \notin g$. Hence, any efficient network is not pairwise stable.

A sufficient condition that an efficient network is pairwise stable is given as follows.

Proposition 7. *Suppose the complete network g^c is an efficient network. Then, it is pairwise stable.*

Proof. By (5), there exists $(\phi_B(g))_{g \in G}$ such that $\phi_B^i(g) - \phi_B^i(g - ij) = w(M|g) - w(M|g - ij)$ and $\phi_{jB}(g) - \phi_{jB}(g - ij) \geq 0$ for all g . Since g^c is efficient, $\phi_B^i(g^c) - \phi_B^i(g^c - ij) + \phi_{jB}(g^c) - \phi_{jB}(g^c - ij) \geq l^i + l_j$ for any $ij \in g^c$. Thus, g^c satisfies the condition (i). Since there is no $ij \notin g^c$, the condition (ii) is satisfied. ■

A networked market with no link cost is an obvious example. If $l^i = l_j = 0$ for all i, j , it is obvious that the complete network is efficient and thus pairwise stable. In addition, both right-hand-sides of the conditions (i) and (ii) of pairwise stability are zero. Since $\phi^i(g + ij) \geq \phi^i(g)$ and $\phi_j(g + ij) \geq \phi_j(g)$, the efficient complete network is pairwise stable.

6 Concluding remarks

We have studied the first price package auction in the decentralized networked two-sided market. We show that the results shown by Bernheim and Whinston (1986) hold true in the auction with multiple sellers. There exist equilibria with profit target strategies, where each buyer bids truthfully, and the set of these equilibrium payoff vectors is equal to the bidder optimal core relative to an exogenously given network. However, the payoff equivalence to the VCG outcome does not hold. We show that the bidder optimal payoff vector is Pareto-dominated by the VCG payoff vector for buyers even if commodities are substitutes for all buyers. We also show that the bidder optimal payoff vector Pareto-dominates the competitive equilibrium payoff vector for buyers.

Further investigation will be necessary since our study has the following three limitations. The first is information structure. Throughout the paper, we have assumed complete information among all buyers and all sellers. Buyers and sellers, however, usually have private information for their valuations or endowments in auctions. The second is optimality. We have assumed that all sellers

sell endowments using the first price package auction. However, it would not be an optimal selling mechanism for sellers. The third is strategic complexity. In the first price package auction, each buyer bids a menu, which is a collection of a payment of any possible package. The package auction might be complex rather than a multi-round auction where buyers bid each commodity individually in each round. Analyzing the package auction with private information, the optimal package auction, and the multi-round simple package auction in decentralized two-sided networked markets are left for future research.

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