On Optimal Price Rigidity: An Example(1)

By Masaru Uzawa[1]

0. Introduction

Recently, the quantity adjustment approach has taken the place of the price adjustment approach in microeconomic analysis as well as macroeconomic analysis (See, for example, Barro and Grossman[1976], Benassy[1975a,b], Drèze[1975], Grandmont [1977], Iwai [1974], and Malinvaud [1977]). They use an assumption which is common in Keynesian or disequilibrium theories: *quantities adjust infinitely faster than prices.* This assumption refers to the price rigidity.

There are several interrelated lines of thought that argue for rigid prices.[2] The first is the line of thought associated with Keynesian macroeconomics. The justification for these rigid prices usually relies on some type of transaction-cost argument. Transaction costs pre-

(1) This paper was read at meeting of the Hokkaido Association of Economics, June 23, 1979 at Otaru University of Commerce and at the meeting of the Japan Assosiation of Economics and Econometrics, September 24, 1979 at Tokyo Metropolitan University. Work was supported in part by Research Grant from the Ministry of Education (Theoretical and Econometrical Studies in the Theory of Stagflation) and by Research Grant from the Ministry of Education (Studies in the Theory of Disequilibrium Dynamics and the behavior of the monopolistic firm). I am grateful to Professors E. Fujii, T. Minagawa, H. Nikaido and A. Yatsuka for their valuable comments and helpful suggestions; responsibility for remaining errors is mine.

(2) See Carlton [1979].

Received April 20, 1981.

vent supply and demand shifts from affecting prices. The recent attempts by Baily[1974], Azariadis[1975] and Gordon[1976] to use employee risk aversion to explain wage rigidity would seem to fit in very well the attempts by earlier writers.

A second line of thought related to rigid prices is represented in the writings in industrial organization on "administered" prices and on business pricing policy. The kinky demand curve theory is a good example.(3)

A third thought related to rigid prices appears in the literature on reduced-form econometric price equations. But it would not be at all surprising that an analysis that ignores the complexity of contracting and equates spot supply and spot demand might provide a less than adequate explanation of price movements for many markets.

The reason why firm prefers or employs the quantity adjustment policy (price rigidity) is not yet satisfactorily resolved. There seems to be something answered by way of economic incentives, i. e., profit maximization criterion.

It is well observed that price adjustment costs are relatively greater than quantity adjustment costs in a monopolistic firm in the short run. The cost of detecting the need for a price change, of actually changing price, and of disseminating the information

(3) The theory of the kinky demand curve was advanced independetly and almost simultaneously by Hall and Hitch [1939] and Sweezy [1939]. But, Stigler [1947] writes: "The kinky demand curve theory is silent on monopolies, for the essential feature of retaliation by rivals is absent. Should monopoly prices be rigid, the forces that explain this rigidity (say cost of price changes) may suffice to explain an equal amount of price rigidity under oligopoly. Unless the factors making for monopoly price rigidity do not operate under oligopoly, we can dispence with the kink unless greater rigidity is found in the oligopolistic industries." Our model is an attempt to explain the price rigidity by using the price adjustment costs or cost of price changes.

is so large that frequent price changes are infeasible or unprofitable when the transaction costs exist. Price adjustment costs include both costs of information within the firm and costs with the outside markets, that is, transaction costs. But quantity adjustment costs are often internalized within the monopolistic firm and it will be dismissd here.

We offer an example to show the dominance of the quantity adjustment policy for some range of demand shifting parameter *even if the monopolistic firm can know the exact shift of the market demand curve.* Our example shows the relevance of transaction costs argument to explain the price rigidity. It will be clear that parameters of the market demand curve, demand shifting, cost function, and price adjustment costs play the most important role in determinig whether or not the quantity adjustment policy dominates the price adjustment policy.

Our example shows that for some range of demand shifting, the price rigidity is derived by the optimal behavior of the monopolistic firm. Namely, the optimal price rigidity is obtained. Our example also shows that both the optimum output produced and the optimal price setting cannot, in general, be a continuous function of the demand shifting parameter.

The paper will proceed as follows. In section 1, we will set notation and model. In section 2, we will consider the comparison of two profits and obtain the main results. In section 3, the extension of the model to capacity constraint is briefly discussed. Section 4 will be used for concluding remarks. In Appendix, we will present some generalization of the model.

1. Notation and Model

In the analysis of this paper, we shall consider a highly simplified version of a monopolistic firm behavior in the partial equilibrium framework.

Let us define:

$C(x) = cx^2$: cost function, $c > 0$,

$p = a - bx$: (inverse) demand function, $a > 0$, $b > 0$,

$p = a - b(x - e)$: (inverse) demand function after shifting e, and

x = output, or demand.

We assume that the monopolistic firm producing only one good is in equilibrium in the initial state.[4] Namely, it maximizes the monopolistic profit

$\Pi = px - C(x) = (a - bx)x - cx^2$.

The optimal (monopoly) price p^* and optimum output x^* are, respectively, given as follows:

$$p^* = \frac{a(b + 2c)}{2(b + c)},$$

$$x^* = \frac{a}{2(b + c)}.$$

Now assume that the firm faces the demand shifting e. The monopolistic firm can choose either price adjustment policy with taking price adjustment costs into account or quantity adjustment policy without quantity adjustment costs to maximize his profit after demand shifting. For the sake of simplicity we assume here that only price adjustment takes costs. In particular, it is assumed that

(4) This assumption is a most critical condition for the following discussion.

$A_p(x) = gx^2$: price adjustment costs, $g > 0$.

The reason is simply touched upon in the Introduction. The cost of detecting the need for a price change, of actually changing price and of disseminating the information is so large that frequent price changes are infeasible or unprofitable when the transaction costs exist. Price adjustment costs include both costs of information within the monopolistic firm and costs with the outside markets, that is, transaction costs. But, quantity adjustment costs are often internalized within the monopolistic firm(For example, it is observed that the speed of the assembly lines can be easily changed within a small range without extra costs). So it is well observed that price adjustment costs are relatively greater than quantity adjustment costs in the monopolistic firm in the short run.

In order to compare both profits we begin with the case of

(1) Profit maximization with price adjustment policy,

and then proceed in the case of

(2) Profit maximization with quantity adjustment policy (Price rigidity).

2. Comparison of Two Profits

(1) Profit maximization with price adjustment policy

Since the profit is

$$\Pi_p = px - C(x) - A_p(x)$$
$$= (a + be - ex)x - (c + g)x^2$$

in the case of price adjustment policy, we can obtain the following optimal price $p^{**}(e)$ and optimum output $x^{**}(e)$, respectively:

$$p^{**}(e) = \frac{(a+be)(b+2c+2g)}{2(b+c+g)}$$

and $p^{**}(e) > 0$ for $e \in (-\frac{a}{b}, \infty)$,

$$x^{**}(e) = \frac{a+be}{2(b+c+g)}$$

and $x^{**}(e) > 0$ for $e \in (-\frac{a}{b}, \infty)$.

Then, the maximum profit is

$$\Pi_p(e) = p^{**}(e)\,x^{**}(e) - C(x^{**}(e)) - A_p(x^{**}(e))$$
$$= \frac{(a+be)^2}{4(b+c+g)} \quad \text{for } e \in (-\frac{a}{b}, \infty).$$

The maximum profit curve is depicted in Figure 1.

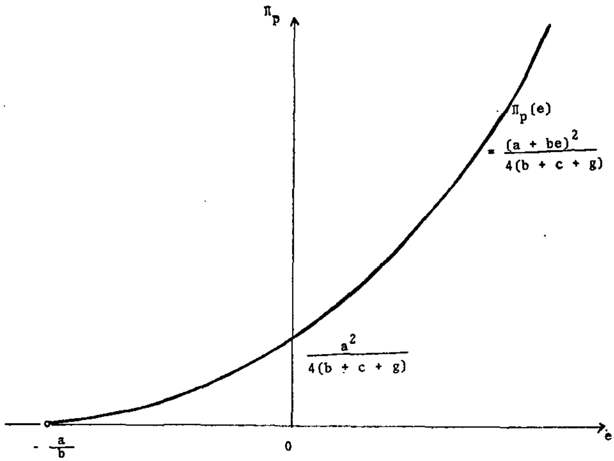

Figure 1.

Remark: The equality of demand for and supply of good holds for all

$$e \in \left(-\frac{a}{b},\ \infty\right).$$

(2) Profit maximization with quantity adjustment policy (Price rigidity)

Since the profit is

$$\Pi_q = p^* x - C(x)$$
$$= p^* x - cx^2$$

in the case of quantity adjustment policy, we can obtain the following optimal (rigid) price p^* and optimum output $x^*(e)$, respectively:

$$p^* = \frac{a(b+2c)}{2(b+c)} \quad \text{and} \quad p^* > 0,$$

$$x^*(e) = \mathrm{Min}\left[\frac{p^*}{2c},\ x^* + e\right]$$

and $x^*(e) > 0$ for $e \in \left(-\frac{a}{2(b+c)},\ \infty\right)$,

where $x^* = \frac{a}{2(b+c)}$.

Therefore we have

$$x^*(e) = \mathrm{Min}\left[\frac{a(b+2c)}{4c(b+c)},\ \frac{a}{2(b+c)} + e\right].$$

(i) If $\frac{ab}{4c(b+c)} \geqq e \geqq -\frac{a}{2(b+c)}$, then

$$x^*(e) = \frac{a}{2(b+c)} + e.$$

Then, the maximum profit is

$$\Pi_q^{(i)}(e) = p^* x^*(e) - C(x^*(e))$$
$$= \frac{a^2}{4(b+c)} + \frac{ab}{2(b+c)} e - ce^2$$

for $e \in \left[-\frac{a}{2(b+c)}, \frac{ab}{4c(b+c)} \right].$

Remark: In this case, equality of demand for and supply of good holds in contrast to the case (ii) discussed later in which there exists positive excess demand for good.

(ii) If $\frac{ab}{4c(b+c)} \leqq e$, then $x^*(e) = \frac{a(b+2c)}{4c(b+c)}$.

Then, the maximum profit is

$$\Pi_q^{(ii)}(e) = p^* x^*(e) - C(x^*(e))$$

$$= \frac{a^2 (b+2c)^2}{16c(b+c)^2}$$

$$= \text{constant for } e \in \left[\frac{ab}{4c(b+c)} \quad \infty \right).$$

Remark: In this case, there exists positive excess demand for good for $e \in (e^\#, \infty)$, where $e^\# = \frac{ab}{4c(b+c)}$.

So the market price would be rising in the long run. But, this situation is out of scope of this paper.

The maximum profit curves $\Pi_q^{(i)}(e)$ and $\Pi_q^{(ii)}(e)$ are depicted in Figure 2.

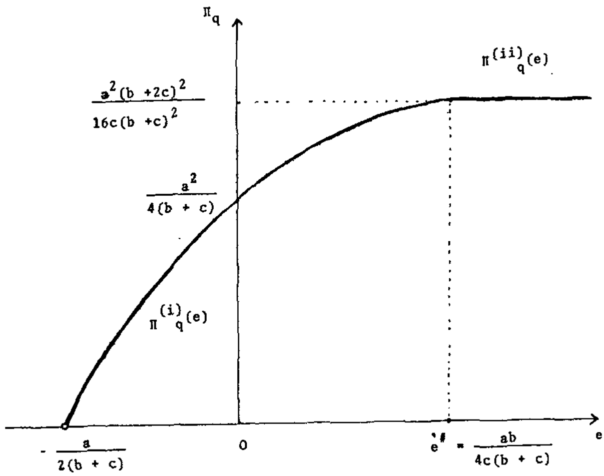

Figure 2.

Now we are in a position to compare $\Pi_p(e)$ with $\Pi_q^{(i)}(e)$ or $\Pi_q^{(ii)}(e)$.

It is easily seen that if g increases, the curve $\Pi_p(e)$ rotates clockwise (See Figure 3).

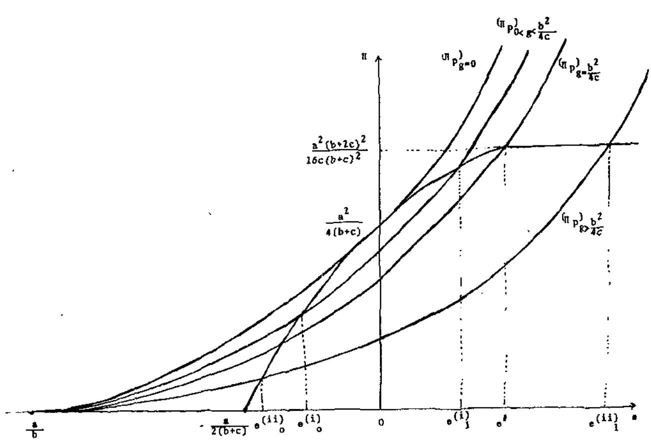

Figure 3.

We obtain $g^{\#}$ such that

$$\Pi_p(e^{\#}) = \Pi_q^{(ii)}(e^{\#}) = \frac{a^2(b+2c)^2}{16c(b+c)^2}$$

holds.

Calculation gives us $g^{\#} = \frac{b^2}{4c}$.

Let us define:

$$I_q^{(i)}(g) = \left\{ e \in \left[-\frac{a}{2(b+c)}, e^{\#} \right] \middle| \Pi_q^{(i)}(e) \geqq \Pi_p(e) \right\},$$

$$I_q^{(ii)}(g) = \left\{ e \in [e^{\#}, \infty) \mid \Pi_q^{(ii)}(e) \geqq \Pi_p(e) \right\}.$$

If $0 < g < g^{\#}$, $I_q^{(ii)}(g) = \phi$ (See Figure 3).

Now define

$$I_q(g) = I_q^{(i)}(g) \cup I_q^{(ii)}(g).$$

Proposition 1

$I_q(g)$ is *increasing with respect to* g *in the sense that*

$$g_1 > g_2 (>0) \Rightarrow I_q(g_1) \supset I_q(g_2).$$

Remark: This Proposition conforms to our intuition (more costly is price adjustment policy, more advantageous is quantity adjustment policy).

Proposition 2

If $0 < g < g^{\#}$, the monopolistic firm facing the exact demand shifting should choose the quantiy adjustment policy in keeping the initial optimal price p^* constant for $e \in [e_0^{(i)}, e_1^{(i)}] = I_q(g)$. ***But, it should choose the price adjustment policy even taking the price adjustment costs into account for***

$$e \in \left(-\frac{a}{b}, e_0^{(i)}\right) \cup \left(e_1^{(i)}, \infty\right)$$

(See Figure 4).

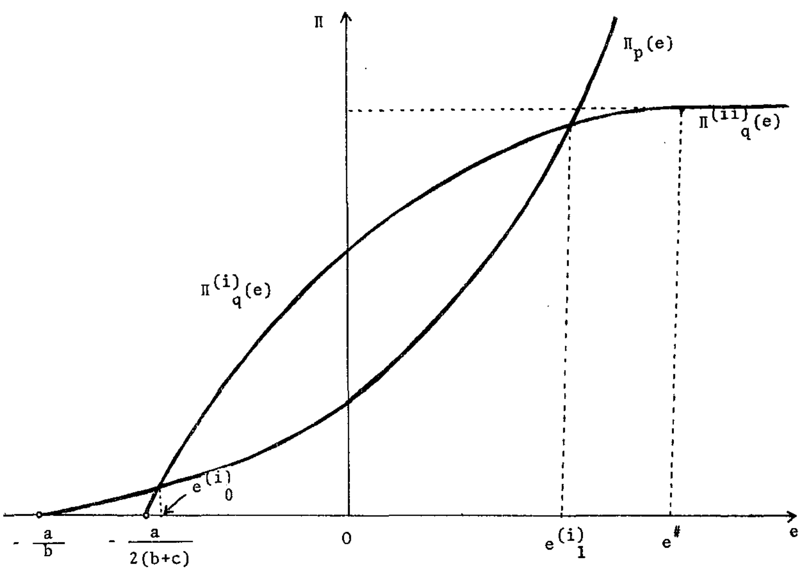

Figure 4.

Proposition 3

If $g > g^{\#}$, the same is true for the monopolistic firm for substituting $e_0^{(ii)}$ and $e_1^{(ii)}$ instead of $e_0^{(i)}$ and $e_1^{(i)}$ (See Figure5).

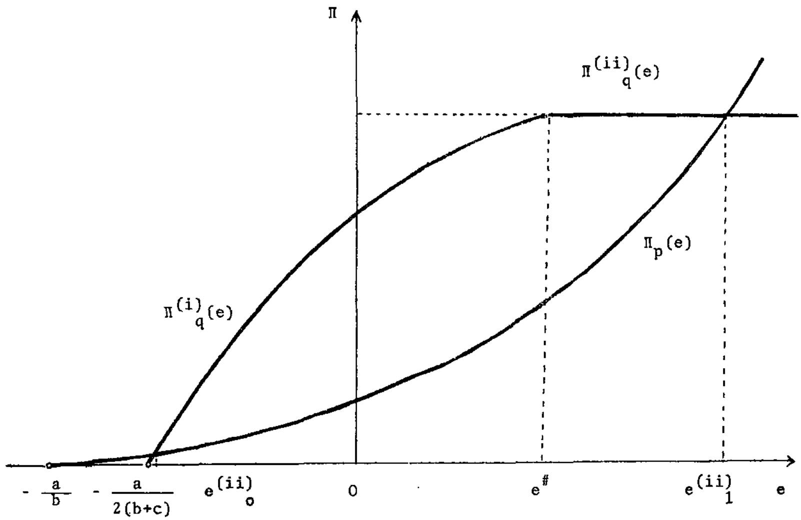

Figure 5

Remark: The interval $[e_0^{(ii)}, e_1^{(ii)}]$ is further divided into two intervals. The condition for its division is whether or not demand for good equals to supply of good.

Proposition 4

If $0 < g < g^{\#}$, the optimum quantity produced is not, in general, a continuous function of the demand shifting parameter e. That is,

$$\begin{cases} x^*(e) = \dfrac{a}{2(b+c)} + e \text{ for } e \in [e_o^{(i)}, e_1^{(i)}] = I_q(g), \\ x^{**}(e) = \dfrac{a+be}{2(b+c+g)} \text{ for } e \in (-\dfrac{a}{b}, e_o^{(i)}) \cup (e_1^{(i)}, \infty) \end{cases}$$

(See Figure 6).

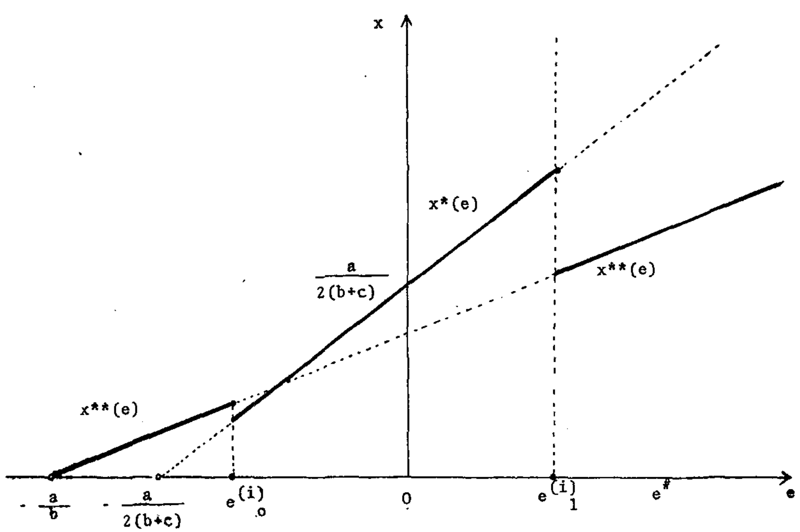

Figure 6.

Proposition 5

If $g > g^{\#}$, the optimum quantity produced is not, in general, a continuous function of the demand shifting parameter e. That is,

$$\begin{cases} x^{*}(e) = \dfrac{a}{2(b+c)} + e & \text{for } e \in [e_{0}^{(ii)}, e^{\#}], \\ x^{*}(e) = \dfrac{a(b+2c)}{4c(b+c)} = \text{constant} & \text{for } e \in [e^{\#}, e_{1}^{(ii)}], \\ x^{**}(e) = \dfrac{a+be}{2(b+c+g)} & \text{for } e \in (-\dfrac{a}{b}, e_{0}^{(ii)}] \cup [e_{1}^{(ii)}, \infty) \end{cases}$$

(See Figure 7).

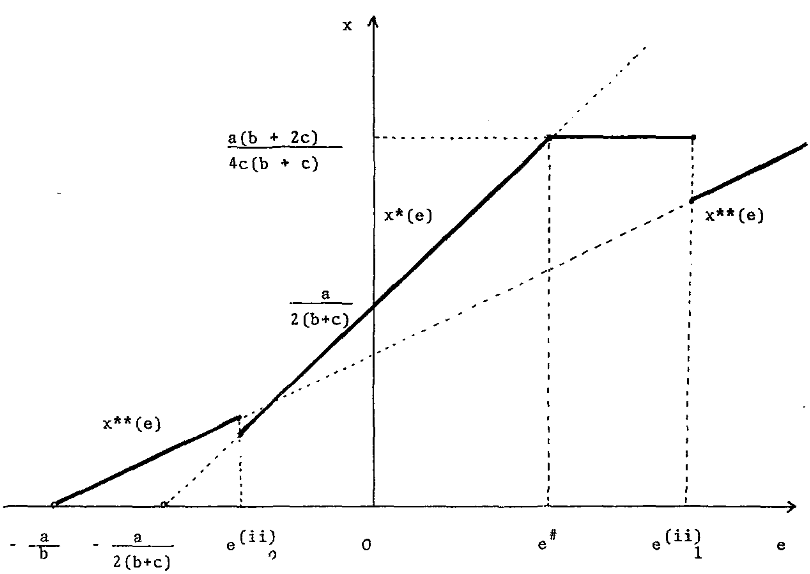

Figure 7.

Propostition 6

If $0 < g < g^{\#}$, the optimal price setting is not, in general, a continuous function of the demand shifting parameter e. That is,

$$\begin{cases} p^{*} = \dfrac{a(b+2c)}{2(b+c)} = \text{constant} \quad \text{for } e \in [e_{o}^{(i)}, e_{1}^{(i)}] = I_{q}(g), \\ p^{**}(e) = \dfrac{(a+be)(b+2c+2g)}{2(b+c+g)} \quad \text{for } e \in (-\dfrac{a}{b}, e_{o}^{(i)}] \cup [e_{1}^{(i)}, \infty) \end{cases}$$

(See Figure 8).

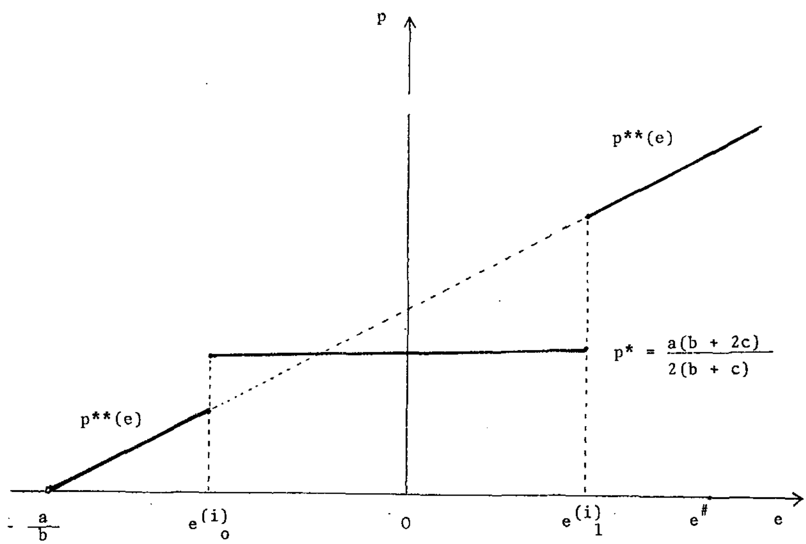

Figure 8.

Proposition 7

If $g > g^{\#}$, the optimal price setting is not, in general, a continuous function of the demand shifting parameter e. That is,

$$\begin{cases} p^* = \dfrac{a(b+2c)}{2(b+c)} = \text{constant for } e \in [e_o^{(ii)}, e_1^{(ii)}], \\ p^{**}(e) = \dfrac{(a+be)(b+2c+2g)}{2(b+c+g)} \text{ for } e \in (-\dfrac{a}{b}, e_o^{(ii)}] \cup [e_1^{(ii)}, \infty) \end{cases}$$

(See Figure 9).

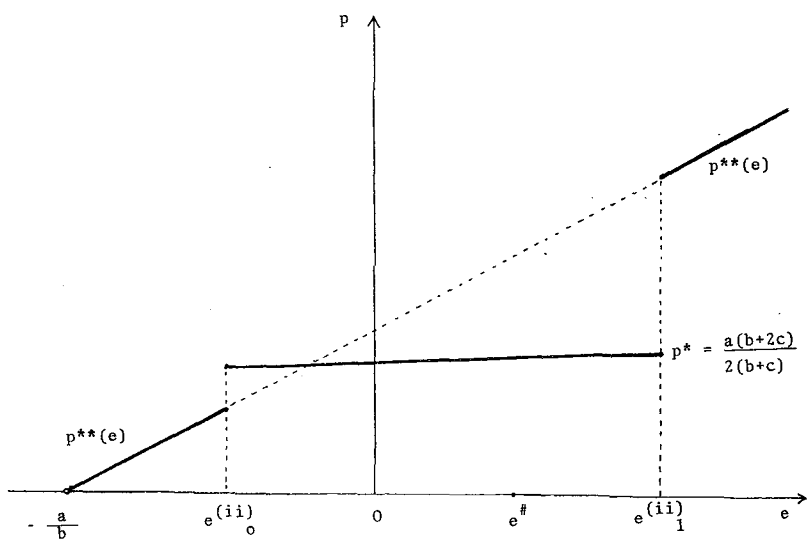

Figure 9.

3. Extension of the Model to Capacity Constraint

We are now in a position to consider the effects of the capacity constraint. This can be done easily without losing general properties of the model. Namely, there exists some interval of demand shifting parameter e for dominance of quantity adjustment policy(***with price rigidity***). So technical exercises are a problem for the interested reader (See Figures 10, 11).

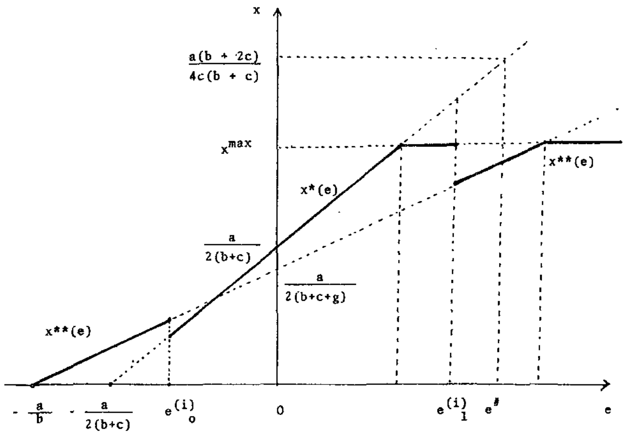

Figure 10

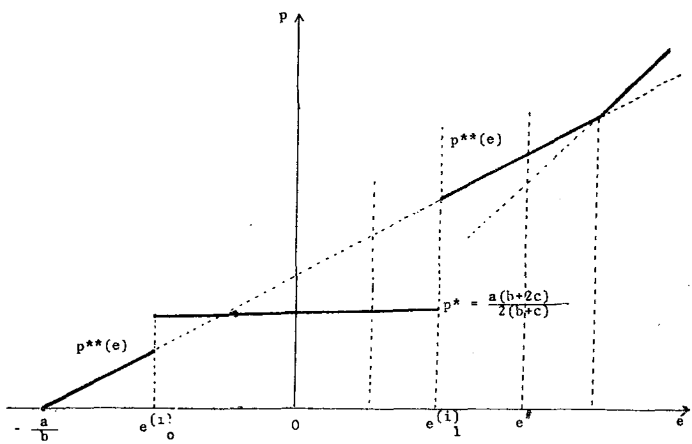

Figure 11

4. Concluding Remarks

We can show a model having a catastrophic optimal adjustment policy. Namely, for some range of the demand shifting parameter e, the monopolistic firm chooses quantity adjustment policy but over the critital points $e_0^{(i)}$ and $e_1^{(i)}$ or $e_0^{(ii)}$ and $e_1^{(ii)}$ he chooses, economically and catastrophically, price adjustment policy *even taking the price adjustment costs into account.* The critical interval of e, $I_q(g)$, for dominacne of *quantity adjustment policy* is an *increasing* function of g, the parameter of the price adjustment costs (See Figure3).

Our model seems to depend crucially on the restricted assumptions, but they can be easily relaxed (See Appendix). Our model would provide a concrete foundation of the fix-price method in microeconomic analysis as well as macroeconomic analysis.

Appendix (Generalization of the Model)

Let us define:

$A_p(x)$: adjustment cost function, $A'(x) \geqq 0$, $A''_p(x) \geqq 0$,

$C(x)$: cost function, $C'(x) > 0$, $C''(x) > 0$,

$p = f(x)$: inverse demand function, $f'(x) < 0$, $f''(x) \geqq 0$,

$p = f(x - e)$: inverse demand function after shifting e, and

x : output, or demand.

We assume that a monopolistic firm producing only one good is in equilibrium in the initial state. Since the profit is

$\Pi = px - C(x) = f(x)x - C(x)$,

the necessary and sufficient conditions for interior maximum are, respectively, as follows:

$$(A-1) \quad \frac{d\Pi}{dx} = f(x) + xf'(x) - C'(x) = 0$$

and

$$(A-2) \quad \frac{d^2\Pi}{dx^2} = 2f'(x) + xf''(x) - C''(x) < 0.$$

We obtain the optimum output x^* which satisfies the both conditions (A−1) and(A−2) and the optimal price setting $p^* = f(x^*)$.

(1) Price adjustment policy

Since the profit is

$\Pi_p = f(x - e)x - C(x) - A_p(x)$,

the necessary and sufficient conditions for the interior maximum are, respectively, as follows:

$$(A-3)\quad \frac{d\Pi_p}{dx} = f(x-e) + xf'(x-e) - [C'(x) + A'_p(x)] = 0$$

and

$$(A-4)\quad \frac{d^2\Pi_p}{dx^2} = 2f'(x-e) + xf''(x-e) - [C''(x) + A''_p(x)] < 0.$$

We obtain the optimum output $x^{**}(e)$ which satisfies the both conditions and the optimal price setting $p^{**}(e) = f(x^{**}(e) - e)$.

Now consider the effect of e on $\Pi_p(e)$, where

$\Pi_p(e) = f(x^{**}(e) - e)\,x^{**}(e) - C(x^{**}(e)) - A_p(x^{**}(e))$.

$$\frac{\partial \Pi_p(e)}{\partial e} = -x^{**}(e)\,f'(x^{**}(e) - e) > 0$$

and

$$\frac{\partial^2 \Pi_p(e)}{\partial e^2} = -[f'(x^{**}(e) - e) + x^{**}(e)\,f''(x^{**}(e) - e)]\,\frac{\partial x^{**}(e)}{\partial e} + x^{**}(e)\,f''(x^{**}(e) - e) \geqq 0.$$

[The first term is non–negative which is derived from (A −3) and $f''(\ .\ ) \geqq 0$ for assumption].

So the shape of $\Pi_p(e)$ is similar to that of example in the text.

(2) Quantity adjustment policy (with price rigidity p^*)

Since the profit is

$\Pi_q = p^*x - C(x)$

in this case, the necessary and sufficient conditions for the interior maximum are, respectively, as follows:

$$(A-5)\quad \frac{d\Pi_q}{dx} = p^* - C'(x) = 0$$

and

$$(A-6)\quad \frac{d^2 \Pi q}{dx^2} = -C''(x) < 0.$$

We have the optimum output $x^*(e)$ as follows:

$x^*(e) = \mathrm{Min}\,[x^* + e, (C')^{-1}(p^*)]$, where $x^*(e) \geqq 0$.

(i) If $x^* + e \leqq (C')^{-1}(p^*)$, then we have

$$x^*(e) = x^* + e.$$

Then, the optimum profit is

$$\Pi_q^{(i)}(e) = p^*(x^* + e) - C(x^* + e).$$

$$\frac{\partial \Pi_q^{(i)}(e)}{\partial e} = p^* - C'(x^* + e).$$

Since $C'(x)$ is an increasing function with respect to x, we have

$C'(x^* + e) < C'((C')^{-1}(p^*)) = p^*$.

Hence, we have

$$\frac{\partial \Pi_q^{(i)}(e)}{\partial e} = p^* - C'(x^* + e) > 0$$

and

$$\frac{\partial^2 \Pi_q^{(i)}(e)}{\partial e^2} = -C''(x^* + e) < 0.$$

(ii) If $x^* + e \geqq (C')^{-1}(p^*)$, we have

$$x^*(e) = (C')^{-1}(p^*) = \text{constant}.$$

So the optimum profit is

$$\Pi_q^{(ii)}(e) = p^*[(C')^{-1}(p^*)] - C[(C')^{-1}(p^*)] = \text{constant}.$$

$$\frac{\partial \Pi_q^{(ii)}(e)}{\partial e} = 0.$$

From (i) and (ii) both curves of $\Pi_q^{(i)}(e)$ and $\Pi_q^{(ii)}(e)$ have similar properties of that of example in the text.

One can easily state Propositions similar to that of example in the text and we may refrain from it here.

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丸山直起商学部助教授：昭和56年11月1日～昭和57年2月21日　イスラエル，連合王国，日本学術振興会特定国派遣研究者

内地研究員

福島吉春商学部講師：一橋大学商学部　昭和56年9月1日～昭和57年2月27日

小平裕商学部助教授：一橋大学経済学部　昭和56年5月1日～昭和57年2月27日

佐々蘭短期大学部助教授：神戸商科大学商経学部　昭和56年9月1日～昭和57年2月27日

訂　正

前号掲載論文 M. Uzawa, “On Optimal Price Rigidity: An Example (I)” の（I）を削除します。