Second Thoughts on the Twin Paradox

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Abstract

An argument is presented to show that contrary to the prevailing lore there would be no asymmetrical aging to the twins who separate and reunite after a high speed relative motion.

I. INTRODUCTION

The twin paradox is a vexing question that has been disputed time and again since the inception of relativity theory. According to the prevailing lore, the twin paradox is no paradox at all and asymmetrical aging occurs to the twins. The long history of the controversy,[1] however, indicates that the argument for the majority view is not completely convincing. In fact, a naive question from a nonscience student motivated me to reexamine this argument, and it will be revealed in the present paper that no asymmetrical aging would be possible. This result will be discussed in the light of experimental evidence of the time dilatation effect.

II. ANALYSIS BY THE TWIN AT REST

Following Lord Halsbury,[2] I simplify the twin problem as a

1) A convenient list of references is given in L. Marder, *Time and the Space-Traveller* (George Allen & Unwin, London, 1971).

2) Lord Halsbury, *Discovery* **18**, 174 (1957).

problem of three brothers. Brother A is at rest in an inertial system and brother B moves away from A at constant speed v. Later, B passes brother C, who is approaching A at the same speed. Unless otherwise stated, A, B, and C are supposed to be situated at the origins of inertial systems. The direction of B's motion is taken as the x axes of these systems. A and B set their clocks to zero when they separate. The crux of the problem is how to set the zero of time for C. This can be best illustrated in a two-dimensional space-time diagram, which is shown in Fig. 1. For simplicity, the space axes of B's and C's inertial systems are omitted.

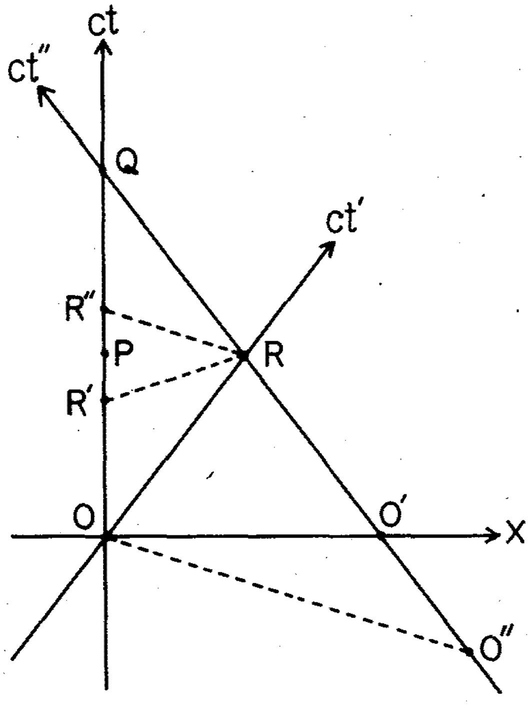

Figure 1. O, R, and Q represent the separation of A and B, the rendezvous of B and C, and the meeting of A and C, respectively. P is the midpoint of the world-line OQ. R ′ is simultaneous with R from B's viewpoint. R ″ and O ″ are simultaneous with R and O, respectively, from C's standpoint.

In a standard argument, the zero of C's time is fixed in such a way that C's time agrees with B's time when B and C meet; C's time is counted from the point O ′ in Fig. 1. In this case, the information conveyed to C from B at R appears to A undistorted; the points O and O ′ are simultaneous, and B's and C's clocks read the same. Therefore, the substitution of C for B poses no difficulty for A. Thus, when C passes A, it appears to A that C's clock stands at $T(1-\beta^2)^{1/2}$, where T is the reading of A's own clock and $\beta = v/c$, with c the speed of light; the traveling brothers' reading is less than the stay-at-home brother's. Throughout this paper, all readings will be given in terms of T, A's proper time interval between O and Q.

III. ANALYSIS BY THE TWIN IN MOTION

Now, I review the whole process from the traveling brothers' standpoint. Also in this case, B's reading at R is $T(1-\beta^2)^{1/2}/2$, which is counted from O. The point R, however, is no longer simultaneous with the point P. Furthermore, the point O is not simultaneous with the point O ′ either from B's standpoint or from C's standpoint; the simultaneity of O and O ′ holds only for brother A. Hence, contrary to the discussion given from A's viewpoint, brother C cannot substitute himself for brother B since it does not appear to C that he was born at the same time as B; the mere fact that C has a different history about his birth disqualifies him for this substitution. (The three brothers are assumed to be born at the zeros of time of their respective inertial systems.) The absence of universal simultaneity of B's and C's birthdays is a direct consequence of the fact that they belong to different inertial systems and were born at different points in space. For the substitution of C for B to make sense physically to C, the zero of C's time must be shifted to O ″ , which appears to C simultaneous with O. In that case, B and C no longer agree at

R; C's clock reads $T(1+\beta^2)/2(1-\beta^2)^{1/2}$ at R,[3] while B's clock reads $T(1-\beta^2)^{1/2}/2$. This has the consequence that, if B leaves his inertial system and joins that of C at R, B's clock must advance upon changing inertial systems by the amount $\Delta\tau = T\ (1+\beta^2)/2\ (1-\beta^2)^{1/2} - T \times (1-\beta^2)^{1/2}/2$. This is based on the following argument.

In addition to the three brothers as defined above, I introduce two more brothers D and E born at O′ and O, respectively; D keeps moving side by side with C, and E moves alongside of B as far as R and then leaves B's inertial system to join C's system and moves with C thereafter. Figure 2 shows the relevant portion of the space-time diagram in an exaggerated fashion. After the event R, brothers C, D, and E are at the same point in space, with their clocks ticking at the

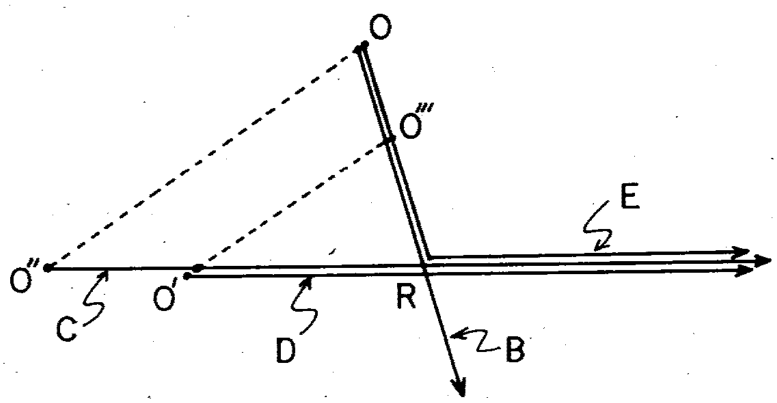

Figure 2. World-lines of four brothers B, C, D, and E. OO″ and O‴O′ are simultaneity lines for C, or D.

same rate. Although, according to the conventional treatment, the readings of D's and E's clocks are the same, D cannot mimic E since his birth point O' does not appear to him simultaneous with E's birth point O. Hence, for a one-to-one correspondence between the age and the

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[3] This can be obtained by noting that $O''R : OR'' = 1 : (1-\beta^2)^{1/2}$, or $O''R$: OR $= 1 : (1-\beta'^2)^{1/2}$, where $\beta' = v'/c$, with v' the relative speed of B and C given by $2v(1+\beta^2)^{-1}$.

clock reading to hold, E's clock must be modified to read differently after R. Since E will appear to D "older" than D by the amount corresponding to OO''', which is equal to O'O'' in his own terms, E's clock must read the same as C's clock. This is the reason for the rapid advancement of E's clock at the changeover point R.

This rapid advancement reflects the fact that the memory B retains of his birth must not be impaired upon changing inertial systems. To my knowledge, no account has been taken of this plain fact in the works published so far. In the traditional treatment, no rapid advancement occurs in B's clock upon changing inertial systems at R while A's clock appears to advance instantaneously from R' to R''. It is interesting to note that the following relation holds in the present treatment:

$$\Delta\tau' = \Delta\tau(1-\beta^2)^{1/2}, \qquad (1)$$

where $\Delta\tau'$ is the interval $R'R''$ Formally, Eq. (1) is the same as the usual formula of time dilatation. However, the roles of $\Delta\tau$ and $\Delta\tau'$ are interchanged here; in the conventional formula, $\Delta\tau$ as appearing in the form of Eq. (1) refers to an inertial system to which the observer belongs and $\Delta\tau'$ refers to *any* moving system whereas in Eq. (1) $\Delta\tau$ refers to the combined system of B and C, i.e., a noninertial system, with $\Delta\tau'$ referring to the inertial system of A.

It thus follows that C's reading at Q is $T(1+\beta^2)/2\ (1-\beta^2)^{1/2}+T \times(1-\beta^2)^{1/2}/2 = T/(1-\beta^2)^{1/2}$; it appears to C that A ages less. Therefore, we are led to the seemingly paradoxical result that it is always the moving brother that ages less either from A's standpoint or from the combined angle of B and C. This result, however, is not a real paradox, the reason being as follows: Since brothers A and C are still in relative motion when they compare their clocks at the point Q, the symmetrical assertion is quite natural rather than absurd.

The inevitable question which arises is what will happen to the reading of C's clock if it comes to a sudden stop. The answer is provided by an argument similar to that for treating the changeover at R. The origin of time for C's clock *after* joining A's system must remain simultaneous with O′. Otherwise, brother C will lose, upon changing inertial systems, his memory that the time of his birth corresponds to the point O′. Hence, C's clock must advance upon coming to a stop at Q by the amount $T-T(1-\beta^2)^{1/2}$, in complete agreement with A's reading. This is the result obtained from A's point of view. A similar result can be obtained from C's viewpoint; A's clock appears to advance, upon joining C's inertial system at Q, by $T/(1-\beta^2)^{1/2}-T$ to read $T/(1-\beta^2)^{1/2}$, which is equal to the reading of C's clock.

I have thus shown that an identical twin who parts company with his brother remaining in an inertial system and later joins him would find that they are the same age though they would appear not to be the same age while in relative motion. This assertion *per se* has been persistently made by a small minority of authors in vain.

IV. CONSISTENCY WITH EXPERIMENT

I now turn to discuss this result in the light of experiments verifying the time dilatation effect. The most relevant experiment is the muon lifetime measurement of Bailey *et al.* [4] They showed with high precision that the muons circling in the CERN storage ring enjoyed

[4] J. Bailey, K. Borer, F. Combley, H. Drumm, F. Krienen, F. Lange, E. Picasso, W. von Ruden, F. J. M. Farley, J. H. Field, W. Flegel, and P. M. Hattersley, *Nature* **268**, 301 (1977).

a lifetime prolonged just as predicted by relativity theory. The usual interpretation that this demonstration confirms the asymmetrical aging effect is rather rash because Bailey *et al.* have ***not*** examined what will happen to the circling muons if they come to a sudden stop. One might argue that such a study was actually made a long time ago by Nereson and Rossi.[5] This, however, is not correct for the following reason. They investigated the effect of stoppage in various absorbers on the lifetime of cosmic ray muons and reported that no effect was observed. More precisely, it is the behavior of muons ***after*** coming to a stop in the absorber that was actually investigated. The muons were found to decay in the same mode as those at rest and no dependence on the absorber was observed.

What is important to the present problem, however, is the decay mode of the muons ***during*** the brief interval of the braking in the absorber. In Ref. 5, counts from the decaying muons in this interval were regarded as spurious for technical reasons and were not considered seriously. Only those counts that were recorded thereafter were analyzed in detail. The counts in the interval in question do not follow the exponential form that was determind from those recorded in the ensuing time interval; they lie much higher than the extrapolated exponential curve. The deviation from the exponential law in the upward direction is consistent with the result of the present paper; the *excess* muons must decay during the brief period of braking because, according to our result, no permanent effect due to the motion should be left on the muons that have come to a stop. The possibility thus arises that this ultra rapid decay is responsible for the above-mentioned deviation.

Finally, I should comment on the flying clock experiment by Hafele

[5] N. Nereson and B. Rossi, *Phys. Rev.* **64**, 199 (1943).

and Keating.[6] They compared four cesium atomic clocks aboard a jet airliner flying around the world with the one left on the earth and found differences of the order $10^{-8} \sim 10^{-7}$ sec. These differences, however, appear to be critically dependent on their procedure of analyzing collected data, thus leaving the possibility that refining this point may offset the reported effect. It is added that I am not alone in questioning the validity of the Hafele-Keating experiment.[7]

In sum, the relevant experimental facts and the result of this paper are *not* in clear contradiction.

ACKNOWLEDGMENT

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[6] J. C. Hafele and R. E. Keating, *Science* **177**, 166 and 168 (1972).
[7] T. Wilkie, *Nature* **268**, 295 (1977).