FACTOR PRICE RIGIDITIES AND COMPARATIVE ADVANTAGE*

Hideki I. Funatsu

1. Introduction

It was six decades ago that a serious policy debate on the employment effect of import tariff took place. When the world economy began to show the severe downward trend, Keynes was one of advocates who insisted on the merit of import tariff to ease the unemployment problem. He argued that nominal wage rates were downward rigid because some employments were protected by contract. Therefore, some drastic remedies were necessary even though they were known to be second best solution. Robbins (1931) was the one who opposed to the idea of tariff protection for solving the unemployment problem on the basis of several convincing reasons. One of them quoted below is the straightforward but insightful application of comparative advantage theory in the presence of unemployment.

> ……, the attempt to do so [cure unemployment] by means of protective tariffs is open to a very grave objection. For surely if it is

* This research was completed when the author was research scholar at London School of Economics. The author is grateful to participants of research student seminar for their helpful comments. An earlier version of this paper was presented at the 1988 annual conference of the Japan Society of Economics and Econometrics.

desired to increase employment by stimulating industry, it is also desirable to increase it by stimulating the *right* industries, that is to say those industries in which our comparative advantage are greatest. Now this is precisely what the protective tariff fails to do. For, obviously, by definition, it stimulates those industries which need protection, those industries, that is to say, in which our comparative advantage are most questionable: ……

Although the world economy has not experienced a depression like the one which Keynes and Robbins had to face, protectionist sentiments loom when unemployment rate rises in a recession period. It is important to examine the validity of comparative advantage theory in the presence of underutilization of productive factors due to factor price rigidities.

The purpose of this study is to investigate into the relation between factor price rigidities and comparative advantage in the factor proportions model of international trade by using the duality theory. In the previous literature, Brecher (1974, 1981) and Batra and Seth (1977) study the small dimension model of general equilibrium which incorporates an economy-wide minimum wage rate. In general, basic results derived under the assumption of fully flexible factor prices have to be altered considerably and there are some paradoxical cases of interest. Later, Schweinberger (1978) extends the Brecher's model to large dimension fix-factor price model and discusses the effect of wage subsidies to employment. Neary (1985) further reshapes the model and points out the formal similarities between the theory of direct investment and the theory of minimum wage rates. Although the Neary's work han been extended into the two-country framework by Svensson (1984) in the context of international factor mobility, it has not been extended to explore the explicit relation between the level of factor price rigidities and comparative advantage. The present paper tries to fill this gap in the literature by applying the technique utilized by Dixit and Woodland (1982) to the fix-price factor general equilibrium model.

Dixit and Woodland derives an explicit relation between factor endowment and comparative advantage in a multi-factor, multi-product context by confining their attention into a small neighborhood of an autarky equilibrium. They consider two identical countries which have their factor endowments in the same proportions. Then, let one country's factor endowment change and see what happens to the autarky good prices. Since nothing happens to the other countries there will be a difference in relative prices of goods which causes commodity trade.

The present paper takes a similar step of reasoning. At first, the autarky price vector is related to the endowment vector of flex-price factors and fix-factor prices. Then we consider two identical countries which have the same factor endowment and the same factor price rigidities. There is no incentive for trade between two countries. By changing the fix-factor price slightly, we studies how the net export vector will be affected.

This paper also demonstrates some interesting results for a small country in the course of discussion. Especially, the Brecher's paradoxical result in the small dimension case is generalized. It will be shown that an increase in endowment of the flex-price factor may reduce employments of some fix-price factor. Also demonstrated in the context of a small country is the effect of protection on the employments of fix-price factors. On the contrary to the usual assertion of protectionism, protection may reduce employments of fix-price factors as Robbins correctly argued sixty years ago.

The plan of this paper is as follows. Section 2 spells out the basic model of production side. Some interesting results from reciprocity relations due to duality are presented in Section 3. Section 4 derives the main results on the relation between comparative advantage and factor price rigidities by introducing a simple demand side. Finally, Section 5 states concluding remarks.

2. The Production Side

In order to describe the production side of the economy, it is convenient to start discussion with a small country case. Consider a small open economy which produces internationally traded goods with nontraded factors. Prices of some factors are assumed to be exogenously fixed so that there are always excess supply or underutilization of some factors of production. As for the technology, the production possibility set of the economy is assumed to be convex and to exhibit constant returns to scale. Other than the rigidities of some factor of production, perfect competition prevails in all good and factor markets. According to Schweinberger (1978) and Neary (1985), the production process of such an economy is conveniently described as the maximization of value added of flex-price factors.

Define π as the value added function.

$$\pi(p, v, w) \equiv \max[px - wL\,;\ F(x, L, v) \leq 0] \qquad (1)$$

where x is the output vector, v is the endowment vector of flex-price factors, w is the fix-price vector, p is the product price vector, L is the employment vector of fix-price factor, and F is the implicit production possibility set which is convex. The value added function π is assumed to be twice differentiable with respect to each argument. As discussed by Neary (1985), this assumption requires among other things that the number of flex-price factors is at least as great as that of goods produced in the economy.

The next step is to relate the value added function π with the standard revenue function or sometimes referred to as the GNP function. Let G denote the maximum GNP attained by utilizing given factor endowments, and a production technology at given product prices.

$$G(p, v, L) \equiv \max[px : F(x, L, v) \leq 0] \tag{2}$$

From the first order conditions for maximization,

$$w = G_L(p, v, L) \tag{3}$$

where G_L denotes the first partial derivatives with respect to L.

Solving equation (3) for L yields demand functions for fix-price factors.

$$L = \phi(p, v, w) \tag{4}$$

By using equation (4), the value added function π is related to unconstrained revenue function G as follows;

$$\pi(p, v, w) = G[p, v, \phi(p, v, w)] - w\phi(p, v, w).$$

By the virtue of envelope theorem, the first partial derivatives of the value added function are calculated as follows;

$$\pi_p = G_p + (G_L - w)\phi_p = G_p = x, \tag{5}$$

$$\pi_w = (G_L - w)\phi_w - \phi = -L, \tag{6}$$

$$\pi_V = G_V + (G_L - w)\phi_V = G_V = r \tag{7}$$

where r denotes the price vector of flex-price factors.

Equations (5) to (7) are crucial to derive results in the following sections. Before finishing this section, it is convenient to include the definition which states the relation among factor prices and good prices.

Definition 1.

The ith good is called friend (enemy) to the jth flexi-price factor if $\partial r_j/\partial p_i > 0 (<0)$.

Definition 2.

The kth fix-price factor is called friend (enemy) to the jth flexi-price factor if $\partial r_j/\partial w_k > 0 (<0)$.

Needless to say, these definitions identify the second cross partial derivatives with respect to the value added function π. In the small dimension model, the friend-enemy relations among goods and factors are explicitly related to the definition of factor intensity. The two good three factor case is presented in Appendix.

3. Reciprocity Relations

The assumption of twice differentiability with respect to the value added function π gives us three pairs of reciprocity relations. Each one leads to interesting results.

3-1 Generalization of Brecher's Result

The first interesting result in this model is about the effect of an endowment change in a flexi-price factor on employments of the fix-price factor. Brecher (1981) demonstrated a seemingly paradoxical case that capital accumulation might reduce employment of labor in the two-good, three-factor minimum wage economy which is a special case of the present model. The first reciprocity relation easily generalizes the Brecher's result.

Due to the twice differentiability of π, differentiation of equations (6) and (7) yield

$$-\partial L/\partial V = \pi_{wv}' = \partial r/\partial w', \tag{8}$$

where a prime denotes the transpose of the matrix.

Proposition 1.

(a) An increase (decrease) in the endowment of the jth flexi-price factor reduces (increases) employment of the kth fix-price factor if and only if they are friend.

(b) An increase (decrease) in the endowment of each flexi-price factor may reduce (increase) employments of some fix-price factors but cannot reduce (increase) employments of all the fix-price factors while it can increase (reduce) all of them.

proof.

Part (a) is already established. From the duality relation,

$$r(p, v, w)v = \pi(p, v, w) \tag{9}$$

Partially differentiating both sides with respect to the vector w yields

$$(\partial r/\partial w)\cdot v = \pi_w = -L \tag{10}$$

From equation (10) it follows that each fix-price factor may be friend to some flexi-price factors but cannot be friend to all of them. This fact combining with Part (a) proves Part (b).

Proposition 1 has an interesting implication on frequently discussed debate on the effect of direct investment on domestic employments. Under the present setting of the model, direct investment from abroad always increases at least one kind of employment in the economy and can increase all kinds of employments. However, it may sometimes reduce some employments. In the worst possible case, reductions may outweigh a rise in other category of jobs so that total employment of the economy may shrink. By the same token, outflow of domestic capital may not be perilous for domestic employments. There is a possible employment increase due to capital outflow. This fact may well calm down an excessive pessimist who insists on hollowlization of domestic industry due to outflying direct investment. The effect of direct investment on employment requires a careful observation on the relation between wage rates and profit rates in various forms of capital.

3-2 Rybczynsky-effect

The second reciprocity relation is well known one between the Rybczynski effect and the Stolper-Samuelson effect. Differentiating equations (5) and (7) with respect to v and p obtains

$$\partial x/\partial v = \pi_{pv} = \pi_{vp}' = \partial r/\partial p' \qquad (11)$$

For the ith flexi-factor and the jth good, we have

$$\partial x_j/\partial v_i = \partial r_i/\partial p_j \qquad (12)$$

Proposition 2.

An increase in the endowment of the ith flexi-price factor increases (decreases) the output of the jth good if and only if the jth good is friend (enemy) to the ith flexi-price factor.

Despite underutilization of fix-price factors, the Rybczynski theorem in the many dimension case continues to hold with respect to the flexi-price factors. As it will be shown later, relative abundance in endowments of the flexi-price factors is the source of comparative advantage in the present model. However, the sign of the Rybczynski term may not be same as that of the no factor rigidity case. This can be seen by relating π_{pv} to the unconstrained Rybczynski effect.

$$\pi_{pv} = G_{pv} + G_{pL} \cdot \phi_v \tag{13}$$

For the ith flexi-factor and the jth good,

$$\partial x_j/\partial v_i = \overline{\sum_k x_j/\partial v_i} + \overline{\sum_k \partial x_i/\partial l_k} \cdot \partial L_k/\partial v_i \tag{14}$$

where a bar denotes the unconstrained Rybczynski effect.

As it was noted in the case of international factor mobility studied by Svensson (1984), the effect of an endowment change in the flexi-price factor on the output depends on the direct unconstrained Rybczynski effect and the indirect effect through a change in the employment of fix-price factors. If all the fix-price factors are enemy to the ith flexi-price factors (so that all $\partial L_k/\partial v_i$ are positive) and the signs of unconstrained Rybczynski terms of all the fix-price factors are same as those of the ith flexi-price factor, the sign of the constrained Rybczynski term of the ith flexi-price factor is same as that of the unconstrained term. But, in general, there is no guarantee for them to be same. This implies that the pattern of trade flow based on the relative abundance of flexi-price factors will be different from one in which there is no factor price rigidity.

3-3 Protection and Employment

The third reciprocity relation is derived by partially differentiating equations (5) and (6).

$$\partial x/\partial w = \pi_{pw} = \pi_{wp}' = -\partial L/\partial p'$$

Relate the constrained matrix to the unconstrained matrix, and we have

$$\pi_{pw} = G_{pL} \cdot \phi_w$$

Therefore, for the kth fix-price factor and the jth good, we have

$$\partial x_j/\partial w_k = \sum_m \overline{\partial x_j/\partial L_m} \cdot \partial L_m/\partial w_k = -\partial L_k/\partial p_j.$$

In the context of a small country case, the relation above gives us a useful insight on employment effects of protectionism. In general, a rise in P_j by protecting the jth industry may or may not increase the employment of the kth fix-price factor. The result depends on the unconstrained Rybczynski terms of all the fix-price factors and complementary relations among fix-price factors. In a special case where only one factor price is rigid, the sign of $\partial L_k/\partial p_j$ is same as that of $\overline{\partial x_j/\partial L_k}$ because own substitution term $\partial L_k/\partial w_k$ is always negative. Hence, in an economy-wide minimum wage economy, protection of the jth industry raises employment of minimum wage labor if and only if the unconstrained Rybczynski term of the jth industry with respect to the fix-price factor in the Rybczynski sense, protection of that industry reduces employment.

Thus, protectionsism aiming at improving an employment situation could be self-hurting in the present model. This is exactly what Robbins argued in the beginning of depression sixty years ago. Protection of industry which does not have comparative advantage in a full employment situation cannot increase employment.

In order to summarize the result in this subsection and also to facilitate the discussion in the next section, we introduce the following definition on the sign of $\partial L_k/\partial p_i$ which is crucial in the relation between factor price rigidity and comparative advantage.

Definition 3.

The jth good is called friend (enemy) to the kth fix-price factor if $\partial L_k/\partial p_i > 0 (< 0)$.

Proposition 3.

(a) An increase in the price of the kth fix-factor price increases (decreases) the output of the jth good if and only if the jth good is enemy (friend) to the kth fix-factor price.

(b) If only one factor price is rigid, then the jth good is friend (enemy) to that factor if and only if the unconstrained Rybczynsky term of the jth good with respect to that fix-price factor is positive (negative).

4. Sources of Comparative Advantage

This section demonstrates that factor price rigidities along with endowments of flexi-price factors are sources of comparative advantage in this model. The approach taken here is similar to Dixit and Woodland (1982) who studied the generalized Hecksher-Ohlin theorem in the absence of factor price rigidities. First, we relate autarky commodity prices with endowment and factor price rigidities and then study a trading equilibrium between two identical countries.

4-1 Autarky Commodity Prices

Suppose that the demand side of the economy is described by the following one consumer Marshallian demand function;

$$c = D(p, y) \tag{15}$$

where c is the consumption vector and y denotes the national income.

Autarky equilibrium conditions in goods market are simply

$$x = c. \tag{16}$$

Replacing the national income y with GNP function and using (5), we have

$$\pi_p(p, v, w) = D[p, G(p, v, L)] \tag{16'}$$

Totally differentiating (16'), we have

$$\pi_{pp}dp + \pi_{pv}dv + \pi_{pw}dw = D_p dp + D_y(xdp + rdv + wdL).$$

Applying the Slutzky decomposition,

$$\pi_{pp}dp + \pi_{pv}dv + \pi_{pw}dw = sdp + D_y(rdv + wdL) \tag{17}$$

where s denotes the Slutzky matrix.

As it was done by Svensson (1984) in the context of international capital mobility, we employ the normalization rule that neutralizes the effect from the size of the country. One way to normalize the income effect is to hold the utility level constant throughout experiments. This normalization rule is justified on the ground that we investigate the problem only in a small neighborhood of an autarky equilibrium.

Let E and u denote the expenditure function and the utility level, respectively. If all the income are spent on consumption, then

$$E(p, u) = G[p, v, \phi(w, p, v)] \tag{18}$$

Total differentiation yields;

$$E_p dp + E_u du = G_p dp + G_v dv + G_L dL$$

Evaluating at the autarky equilibrium, $E_p = G_p$. Therefore,

$$E_u du = r dv + w dL.$$

Our normalization rule $du = 0$ implies;

$$r dv + w dL = 0.$$

Then, equation (17) can be written as

$$(\pi_{pp} - s) dp = -(\pi_{pv} dv + \pi_{pw} dw).$$

Premultiply both sides by dp, and we have

$$dp(\pi_{pp} - s) dp = -dp\pi_{pv} dv - dp\pi_{pw} dw.$$

Since the l. h. s. is quadratic form in a positive semi-definite matrix minus a negative semi-definite matrix,

$$dp\pi_{pv} dv \leq 0 \text{ at a given level of w,} \tag{19}$$

and

$$dp\pi_{pw} dw \leq 0 \text{ at a given level of v.} \tag{20}$$

Interpretation of the bilinear form above are wellknown from Dixit and

Woodland (1982). Using definitions on the friendship relation between commodity prices and factor prices, results are summarized as follows.

Proposition 4.

(a) At a given level of factor price rigidities, if goods are friend to flexi-price factors, autarky prices of goods are on the average negatively correlated with endowments of flexi-price factors.

(b) At a given level of endowments of flexi-price factors, if goods are enemy to fix-price factors, then autarky good prices are on the average negatively correlated with prices of fix-price factors.

4-2 Trading Equilibrium

As it was argued by Drabicki and Takayama (1979), the autarky price vector is sometimes a poor predictor of comparative advantage. Therefore, it is necessary for an analysis on trade patterns to deal explicitly with the equilibrium conditions for a trading world. Following the approach taken by Dixit and Woodland (1982) and Svensson (1984), we can relate trade patterns with endowments of the flexi-price factor and fix-factor prices. Since the result on the endowment effect is same as that in Svensson, we demonstrate only the result on the factor price rigidities.

Suppose that there is a world which consists of two exactly identical nations in all the respects including the factor endowment level and factor price rigidities. Naturally, they have the same autarky equilibrium price vector. Now suppose that the fix-price factor vector of the home nation change by a small amount and then we see how the excess supply of goods, namely net export vector will be affected by a change in factor price rigidities.

The net export functions are defined as

$$z(p, v, w) \equiv x - c.$$

Let * denote the foreign country. The equilibrium conditions for free trade are

$$z(p, v, w) + z^*(p, v^*, w^*) = 0 \tag{21}$$

Throughout the argument, $v = v^*$ is fixed. Therefore, total differentiation of (21) yields;

$$z_p dp + z_w dw + z_p^* dp = 0$$

Since $z_p = z_p^*$ at the initial equilibrium,

$$2z_p dp + z_w dw = 0.$$

Therefore, a change in the free trade vector can be written as

$$dz = z_p dp + z_w dw = (1/2) \cdot z_w dw.$$

Taking the transpose and post-multiplying z_w dw in the both side, we have

$$dz \cdot Z_w \cdot dw \geq 0.$$

Utilizing the normalization rule in the previous sub-section,

$$z_w = \pi_{pw}.$$

Thus,

$$dz \cdot \pi_{pw} \cdot dw \geq 0$$

Proposition 5.

At the given endowments of flexi-price factors, if the goods are enemy to fix-price factors, then prices of fix-price factors are on the average positively correlated with the net export vector.

5. Concluding Remarks

This paper has studied the relation between factor price rigidities and comparative advantage. The main finding is that factor price rigidities themselves are sources of comparative advantage while endowments of flexi-price factors continue to be source of comparative advantage in a slightly modified fashion. The result has an important implication on the empirical studies of the trade patterns. Since there are nations which experience underutilization of some productive resources for a certain period of time, one must be careful about applying the factor abundant theory in order to predict trade patterns. The trade pattern predicted in the absence of any rigidity could be markedly different in the presence of factor price rigidities.

Finally, some limitation of the present analysis is noted. Since the analysis was carried out only around the small neighborhood of an autarky equilibrium, all the results are local one. Effects of large changes in factor endowments and factor price rigidities are desirable but an explicit result is difficult to come by. Secondly, this paper utilizes the normalization rule to omit the effects from the size of country. However, the income effect is important especially when we consider the gain from trade.

Appendix

Special Case : Two-Good Three-Factor Case.

An interesting special case of the present model is the case in which the economy produces two-goods with three-factors. The full employment case has been extensively studied by Batra and Casas (1976), Ruffin (1981), Suzuki (1983), Jones and Easton (1983) and Thompson (1985). The small dimensionality enables us to relate the definition of enemy-friendship among goods and factors with the familiar intensity conditions. This appendix demonstrates the explicit relations between two definitions by using the Jones's type of specification on the two-sector analysis.

Let a_{ji} denote the input-output ratio. Under the constant returns to scale technology, a_{ji} does not depend on the level of production and known to be homogeneous degree zero in all factor prices.

$$a_{ji}=\overline{a_{ji}}(r_1,\ r_2,\ w)$$

where $j=1,\ 2,\ w$ and $i=1,\ 2$.

Free entry and exit under perfect competition leads each sector to zero profit in an equilibrium.

$$a_{11}r_1+a_{21}r_2+a_{w1}w=p_1 \tag{A-1}$$

$$a_{12}r_1+a_{22}r_2+a_{w2}w=p_2 \tag{A-2}$$

Total differentiations of Equations (A-1) and (A-2) together with the cost minimization conditions yield

$$\theta_{11}\hat{r}_1+\theta_{21}\hat{r}_2=\hat{p}_1-\theta_{w1}\hat{w} \tag{A-3}$$

$$\theta_{12}\hat{r}_1+\theta_{22}\hat{r}_2=\hat{p}_2-\theta_{w2}\hat{w} \tag{A-4}$$

where θ_{ij} denotes the jth factor's share in the ith sector.

For Definition 1.

Solving (A-3) and (A-4) simultaneously yields

$$\hat{r}_1/\hat{p}_1=\theta_{22}/|\theta|,\ \hat{r}_2/\hat{p}_1=-\theta_{12}/|\theta|,\ \hat{r}_1/\hat{p}_2=-\theta_{21}/|\theta|,\ \hat{r}_2/\hat{p}_2=\theta_{11}/|\theta|,$$

where $|\theta|\equiv\theta_{11}\theta_{22}-\theta_{21}\theta_{12}=\theta_{22}\theta_{12}\left(\frac{\theta_{21}}{\theta_{22}}-\frac{\theta_{w1}}{\theta_{w2}}\right)/|\theta|$.

Let the first flex-price factor called capital and the second called land. The positive sign of $|\theta|$ means that the first industry is relatively capital intensive to the second industry. Then, according to Definition 1, the first good is friend to capital and enemy to land while the second good is enemy to capital and friend to land. Needless to say, both Stolper-Samuelson theorem and Rybczynski theorem hold for flex-price factors.

For Definition 2.

Again solving (A-3) and (A-4) simultaneously yields

$$\frac{\hat{r}_1}{\hat{w}}=\theta_{w2}\theta_{22}\left(\frac{\theta_{21}}{\theta_{22}}-\frac{\theta_{w1}}{\theta_{w2}}\right)/|\theta|$$

$$\frac{\hat{r}_2}{\hat{w}}=\theta_{w2}\theta_{12}\left(\frac{\theta_{w1}}{\theta_{w2}}-\frac{\theta_{11}}{\theta_{12}}\right)/|\theta|$$

There are three cases according to factor intensity conditions.

Case 1. $\frac{\hat{r}_1}{\hat{w}}<0,\ \frac{\hat{r}_2}{\hat{w}}<0$ if and only if $\frac{\theta_{21}}{\theta_{22}}\lessgtr\frac{\theta_{w1}}{\theta_{w2}}\lessgtr\frac{\theta_{11}}{\theta_{12}}$

Case 2. $\frac{\hat{r}_1}{\hat{w}}>0,\ \frac{\hat{r}_2}{\hat{w}}<0$ if and only if $\frac{\theta_{w1}}{\theta_{w2}}\lessgtr\frac{\theta_{11}}{\theta_{12}}\lessgtr\frac{\theta_{21}}{\theta_{22}}$

Case 3. $\frac{\hat{r}_1}{\hat{w}}<0,\ \frac{\hat{r}_2}{\hat{w}}>0$ if and only if $\frac{\theta_{11}}{\theta_{12}}\lessgtr\frac{\theta_{21}}{\theta_{22}}\lessgtr\frac{\theta_{w1}}{\theta_{w2}}$

According to factor intensity definitions proposed for three-factor cases by Ruffin, labor is enemy to both capital and land if and only if labor is middle factor. Labor is friend to capital and enemy to land if and only if capital is middle factor. Labor is enemy to land and friend to capital if and only if land is middle factor. Thus one should be able to see that the Brecher's paradoxical case does not happen if labor is middle factor.

For Definition 3.

Full employment conditions for each sector are written as follows:

$$a_{11}x_1 + a_{12}x_2 = v_1 \tag{A-5}$$

$$a_{21}x_1 + a_{22}x_2 = v_2 \tag{A-6}$$

$$a_{w1}x_1 + a_{w2}x_2 = L < L^* \tag{A-7}$$

where L^* is the endowment for labor.

Total differentiations of (A-5), (A-6), and (A-7) together with (A-3) and (A-4) yield the following system of equations.

$$\begin{pmatrix} \alpha_{11} & \alpha_{12} & 0 & \lambda_{11} & \lambda_{12} \\ \alpha_{21} & \alpha_{22} & 0 & \lambda_{21} & \lambda_{22} \\ \alpha_{w1} & \alpha_{w2} & -1 & \lambda_{w1} & \lambda_{w2} \\ \theta_{11} & \theta_{21} & 0 & 0 & 0 \\ \theta_{12} & \theta_{22} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{r}_1 \\ \hat{r}_2 \\ \hat{L} \\ \hat{x}_1 \\ \hat{x}_2 \end{pmatrix} = \begin{pmatrix} \hat{v}_1 - \alpha_{1w}\hat{w} \\ \hat{v}_2 - \alpha_{2w}\hat{w} \\ -\alpha_{L2}\hat{w} \\ \hat{p}_1 - \theta_{w1}\hat{w} \\ \hat{p}_2 - \theta_{w2}\hat{w} \end{pmatrix}$$

where λ_{ji} denotes the ith sector's share of the jth factor and α_{jk} the economy wide substitution between two factors. Let σ_{jk} denote the Allen elasticity substitution. Then, $\alpha_{jk} \equiv \sum_{i=1}^{2} \lambda_{ji}\theta_{kj}\sigma^{i}_{jk}$.

Let D denote the determinant of the coefficient matrix.

$D = (\lambda_{11}\lambda_{22} - \lambda_{21}\lambda_{12})(\theta_{12}\theta_{21} - \theta_{22}\theta_{11}) < 0$. Cramer's rule gives us:

$$\hat{L}/\hat{p}_1 = [(\theta_{21}\alpha_{w2} - \theta_{22}\alpha_{w1})(\lambda_{11}\lambda_{22} - \lambda_{21}\lambda_{12}) + (\theta_{22}\alpha_{11} - \theta_{21}\alpha_{12})(\lambda_{w1}\lambda_{22} - \lambda_{21}\lambda_{L2}) + (\theta_{21}\alpha_{22} - \theta_{22}\alpha_{21})(\lambda_{w1}\lambda_{12} - \lambda_{11}\lambda_{w2})]/D$$

$$\hat{L}/\hat{p}_2 = [(\theta_{21}\alpha_{11} - \theta_{11}\alpha_{12})(\lambda_{21}\lambda_{L2} - \lambda_{w1}\lambda_{22}) + (\theta_{21}\alpha_{11} - \theta_{11}\alpha_{12})(\lambda_{11}\lambda_{w2} - \lambda_{w1}\lambda_{12}) + (\theta_{11}\alpha_{w1} - \theta_{11}\alpha_{w2})(\lambda_{11}\lambda_{22} - \lambda_{21}\lambda_{12})]/D$$

In addition to factor intensity conditions, restrictions on the economy wide substitution are necessary to determine the sign.

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