Dynamic Tax Incidencein a Two-Class Economy*

Jun-ichi Itaya

Abstract

The main purpose of this paper is to re-examine the results of long-run tax incidence in a heterogeneous agents' growth model. To this end, we adopt an intertemporal optimizing approach to provide rigorous microfoundations of the savings behavior of individuals in a Kaldorian two-class growing model. Earlier literature in this field has mainly used a neoclassical growth model with fixed savings ratios, and shows that the results of tax incidence crucially depend on various structural parameters of the model, such as the fixed savings ratios, the growth rate of population, and the production function, and that by substituting labor income taxes for capital income taxes, workers also bear a larger part of the burden. By contrast, in our model the savings ratios of the two classes are endogenously determined by intertemporal optimizing behavior of agents. Furthermore, contradictive results would emerge; under some mild assumptions on the production functions workers are better off under an imposition of capital income taxation.

* I wish to thank John H. Boyd III, Masaaki Homma, and participants of seminars in University of Rochester, Otaru University of Commerce, and Hokkaido University for their helpful comments. However, I am solely responsible for any remaining errors. An earlier version of this paper was presented at the 1986 annual conference of the Japan Society of Economics and Econometrics.

1. Introduction

This paper re-examines the long-run incidence of capital income taxes in a Kaldorian two-class model studied by Homma (1981), using a dynamic non-cooperative game approach. It is well known that there are two methods to investigate the problem of tax incidence; static tax incidence and dynamic tax incidence. In the former, Harberger (1962) and Atkinson and Stiglitz (1980) have demonstrated that, with fixed capital stocks, the burden of general factor taxes is entirely borne by the factor on whose income the tax has been levied.

In the latter, after the contribution of Feldstein (1974), many authors, including Grieson (1975), Boadway (1979), Yeh (1979), and Homma (1981), found that in the long-run the above conclusions do not necessarily hold true and that a substantial part of the burden of factor taxes (capital income taxes) is borne by another factor (labor) whose income is not taxed, due to dynamic capital stock adjustments. Especially, Homma extended their models to a Kaldorian two-class growing economy initiated by Pasinetti and gave the detailed long-run consequences of increasing capital income taxes as follows:

(i) *The differential tax change results in the tax burden in the private sector in the sense that it reduces private capital (wealth) accumulation.*

(ii) *The burden is more than* 100 *percent borne by capitalists in case* Ⅰ $(f'<n/s_c)$ *and more than* 50 *percent in case* Ⅱ $(n/s_c<f'<n/s_a)$, *while the burden is more than* 50 *percent shifted to workers in case* Ⅲ $(n/s_a<f'<n/s_w)$ *and more than* 100 *percent in case* Ⅳ

$(n/s < f')$[1].

(iii) *A higher or lower rate of capital income taxes tends to increase or decrease the tax shifting from capitalists to workers.*

However, these contributions, including Homma's, share some common methodological shortcomings. First, the models they used are based on traditional neoclassical one-sector growth models. Thus the savings behavior underlying these models is arbitrarily specified, so that the savings ratio is typically fixed at a particular rate. As a result, these models lack explicit microfoundations that endogenously determine the savings behavior of agents over time. This postulate, furthermore, is inappropriate from a microeconomic viewpoint, because the savings decisions are sensitively affected by relevant tax changes through intensive effects.[2] Secondly, authors, such as Feldstein, Yeh and Homma, often made the assumption that the savings ratio of capitalists is greater than that of workers. This assumption plays a crucial role in their comparative statics exercises. Despite its importance, however, they did not give any satisfactory theoretical rationale for it.[3]

The present paper is an attempt to correct these shortcomings. To this end, we shall adopt an intertemporal optimizing approach to derive the savings behavior of agents over time. The savings ratio therefore becomes an endogenous variable, depending on future as well as current economic variables, such as market prices and poli-

---

1) See section 2 about s_w and s_c. s_a is a weighted average savings ratio, $s_a = s_c + (1-\delta)s^w$, where δ is the share of capitalists' disposable income to total disposable income.

2) See chapter 3 in Atkinson and Stiglitz (1980).

3) One possible explanation is to appeal to the empirical fact that the savings ratio tends to be stable over a long time.

cy instruments. Moreover, we can also eliminate such an ad-hoc assumption regarding the relative magnitude of the savings ratios of workers and capitalists, because our model itself endogenously provides this information.

At first sight, it may seem that the framework we employ is quite similar to that developed recently by Turnovsky (1982), Judd (1985), and Becker (1985), that is, the *perfect foresight equilibrium* model with an infinity-lived representative agent. However, there are several substantial differences between our model and theirs. First, we are mainly concerned with tax incidence in a framework of *heterogeneous agents*. To make the analysis tractable, we shall employ a two-class setting in which the agents associated with different classes, capitalists and workers, have heterogeneous preferences, discount factors, and factor endowments. These ingredients may give rise to the diversity of agents in the sense that their savings or consumption behavior responds to tax changes in different ways. This simple two-class structure also enables us to investigate the effects of tax changes on income distribution among heterogeneous agents.

Unfortunately, the *generalized* perfect foresight (*or Ramsey*) equilibrium model with heterogeneous agents developed by Becker (1980) is not appropriate for our long-run analysis of capital taxation. As shown in Becker, if agents have *different* constant discount rates, in the long-run the most patient agent ends up with all of the capital stock and the other agents have none. This extreme long-run distribution result does not allow a usual analysis of tax incidence because, although the effect of changes in capital income taxes on the capital stocks held by agents is the most natural index for evaluating tax incidence in a growing economy [see Homma (1981)], it is not applicable.

To avoid such a difficulty, we formulate our model in the form of a differential game played by the two classes in which each class uses the savings ratio as its strategy. That is, the agents in each class behave collectively as a single decision unit, so that they have power to influence the market prices or wages through control over their capital holdings. Under such a circumstance they choose optimal savings plans, subject to their intertemporal budget constraints, given the actions of the other class's agents. This setting views a capitalist economy as a *noncooperative dynamic game* between workers and capitalists. Further justifications for this are that (i) similar models were adopted by Tokita (1971) and Lancaster (1973), who dealt with a situation of conflict between capitalists and workers in a dynamic macroeconomic setting, and (ii) it seems reasonable that certain elements of a game situation, such as labor unions and cartels of firms, do exist in a capitalist society. This is mainly because workers and capitalists both wish to collude within a class due to identity of interests, and agree to behave like a single agent.

The plan of the paper and its principal conclusions are as follows: In section 2 we introduce the basic model used in the subsequent sections. In section 3 we derive the optimal savings behavior of agents in each class. It is shown that in our two-class model there exists a unique steady-state equilibrium (*Pasinetti equilibrium*), which is globally asymptotically stable if the discount rate of workers is larger, but not much different, than that of capitalists. In section 4 we carry out a comparative static analysis to evaluate *differential tax incidence* on the distribution of private capital (wealth) and income between the two classes. This exercise reveals the *opposite* conclusion to that obtained in the earlier literature of dynamic tax incidence. That is, using either disposable income or private capital (or wealth) as a measure, capitalists will bear more than 100 percent of the tax burden resulting from the increase in capital income taxes, whereas workers will be better off, provided some mild assumptions are imposed on the production technology.

2. The Model

The technology of the economy can be described by the production function $Y=F(K, L)$ which relates a single homogeneous output Y, available for consumption or capital accumulation, to services of capital K, and labor L. F is also assumed to be three-times continuously differentiable and homogeneous of degree one, with positive marginal products and diminishing marginal returns with respect to each factor. This homogeneity assumption means that the production function may be written in intensive form:

$$y=f(k), \tag{1}$$

where $y=Y/L$, $k=K/L$, $f'(k)=\partial f/\partial k>0$, and $f''(k)=\partial^2 f/\partial k^2<0$. Furthermore, to ensure the existence of steady-state equilibria, we assume

$$f(0)=0,\quad f(\infty)=\infty,\quad \lim_{k\to 0} f'(k)=\infty,\quad \text{and}\quad \lim_{k\to\infty} f'(k)=0, \tag{2}$$

which are the so-called *Inada conditions*. It is moreover assumed that there is neither depreciation nor technical change.

Assuming that firms are price takers, profit maximization requires

$$r(t)=f'(k(t)), \quad (3)$$

and $$w(t)=f(k(t))-k(t)f'(k(t)), \quad (4)$$

where t denotes time, and $r(t)$ and $w(t)$ represent the interest and the wage rates which prevail in the capital and the labor markets, respectively, at each moment in time. Each agent's ownership of firms is proportional to the current level of its capital holdings, so that conditions (3) and (4) are sufficient to describe real activities of firms.

The government never plays any constructive role in this economy. The government is assumed to purchase real goods and services, $G(t)$, in the goods market, which it finances out of receipts of capital income taxes, $\tau_k r(t)K(t)$, and labor income taxes, $\tau_w w(t)L(t)$, where τ_k and τ_w represent the tax rates on capital income and labor income respectively. Its balanced budget constraint can be expressed by

$$G(t)=\tau_k r(t)K(t)+\tau_w w(t)L(t). \quad (5)$$

We do not allow public capital accumulation[4] or any forms of government debts, such as money or government bonds. As a result, policy variables available to the government are the tax rates imposed on capital income and labor income, along with government expenditure.[5] In order to focus on the incidence of taxation, we assume that the government always tries to keep the per capita level of government expenditure, g, constant. In other words, our main concern is the so-called *balanced growth incidence or differential in-*

4) Homma (1981) incorporated public investment into his model.

5) The utility function can be extended to include benefits from consuming government expenditure, i.e. $U_i(c_i, g)$, $i=c, w$. Since g is assumed to be constant over time, the element g in the utility function plays no role in our analysis.

cidence.[6] It follows from this restriction that (5) can be written in intensive form:

$$g=\tau_k r(t)k(t)+\tau_w w(t), \tag{6}$$

where $g=G(t)/L(t)$. For the sake of economic relevance, we shall impose[7] $0<\tau_k<1$ and $0<\tau_w<1$.

In this economy, society is divided into two classes, workers and capitalists. This division will be useful for studying an income distribution problem among heterogeneous agents and how the different propensities to save of agents affect capital accumulation. We here postulate that there are many infinitely-lived individuals, whose preferences and endowments are homogeneous (or identical) within the same class, while heterogeneous across the different classes. To avoid further complications, it is moreover assumed that the populations of both classes are initially equal and grow exogenously at the same rate, say n. Each class owns the capital stock, $K_i(t)$, $i=w, c$, and accumulates it as the only asset available for savings purposes in the economy. Notice that the sum of the capital stock owned by each class equals the total capital stock in the economy, that is, $k(t)=k_w(t)+k_c(t)$, where $k_w(t)=K_w(t)/L(t)$ and $k_c(t)=K_c(t)/L(t)$. Since we assume that capitalists do not supply any labor, they receive only after-tax capital income proportional to their capital holdings.

On the other hand, workers' income consists of after-tax labor income and after-tax capital income proportional to their capital

6) See chapter 8 in Atkinson and Stiglitz (1980).

7) This assumption limits the feasible range of government expenditure, that is, $0<(g-\tau_w w)/rk<1$ and $0<(g-\tau_k rk)/w$, in other words, g must satisfy $g<w$ and $g>rk$. Thus the assumption that $rk<w$ may therefore be implicitly required.

holdings. For simplicity, we assume that labor is supplied inelastically by workers only, which is normalized to be unity, and that workers' instantaneous utility functions are concave, non-negative, additively separable in time, and discounted by a positive constant rate of discount, ρ_w. Hence, given expectations regarding the time paths of prices and tax rates, the objective of workers is to choose their consumption plans over an infinite lifetime so as to maximize their intertemporal utility,

$$\text{Max} \qquad \int_0^\infty U_w(c_w(t))e^{-\rho_w t}dt,$$

subject to

$$c_w(t)+\dot{k}_w(t)+nk_w(t)=w(t)(1-\tau_w)+r(t)k_w(t)(1-\tau_k), \quad (7)$$

$$k_k(0)=k_k^0,$$

where $k_w(t)$, $\dot{k}_w(t)$, $c_w(t)$, and k_w^0, respectively, denote the workers' capital stock per capita, net capital accumulation per capita (or net savings per capita), consumption per capita, and initial capital stock at time 0. Eq.(7) implies that the instantaneous budget constraint must be satisfied at each moment in time.

To simplify the analysis we shall further assume that the marginal instantaneous utility is constant, that is, $U_w''(c)=0$[8]. By suitable choice of units of utility,

$$U_w(c_w(t))=c_w(t). \quad (8)$$

With this assumption, our utility maximization problem now simplifies to a consumption stream maximization problem. For comparison we shall use the savings ratio of workers, $s_w(t)$, as a con-

8) This assumption is not necessarily restrictive in the context of long-run tax incidence because the steady-state conditions, (22) and (32), are unaffected by the specification of utility functions. See also section 5.

trol variable for workers, instead of $c_w(t)$. Then, workers' disposable income per capita, consumption per capita, and net savings per capita can be expressed as:

$$y_w(t)=w(t)(1-\tau_w)+r(t)k_w(t)(1-\tau_k), \tag{9}$$

$$c_w(t)=(1-s_w(t))[w(t)(1-\tau_w)+r(t)k_w(t)(1-\tau_k)], \tag{10}$$

$$\dot{k}_w(t)=s_w(t)[w(t)(1-\tau_w)+r(t)k_w(t)(1-\tau_k)]-nk_w(t). \tag{11}$$

By substituting (3) and (4) into $r(t)$ and $w(t)$ in (9), (10), and (11) and then substituting these into (7) and (8), we have

$$\text{Max } W_w \equiv \int_0^{\infty}(1-s_w)[f(k)-k_c f'(k)(1-\tau_k)-g]e^{-\rho_w t}\,dt, \tag{12}$$

subject to

$$\dot{k}_w=s_w[f(k)-k_c f'(k)(1-\tau_k)-g]-nk_w,$$

$$k_w(0)=k_w^0.$$

For notational convenience, the time argument is hereafter omitted where it is not strictly necessary. In this formulation n, g, ρ_w, τ_k and k are all exogenous variables and only s_w is a choice variable for workers. It is also important to note that although the time paths of the wage and interest rates do not appear in problem (12), workers must anticipate these paths to solve their problems.

In the case of capitalists, we also assume, by the same reasoning, that their instantaneous utility functions display constant marginal utility, $U_c''(c)=0$. Owing to this specification, the formulation for the capitalists' problem can be simplified straightforwardly along the same lines sketched in the case of workers. Capitalists will choose their savings plans, s_c, over an infinite lifetime so as to maximize their intertemporal utility subject to their instantaneous budget constraints, taking the time paths of prices and tax rates as given,

$$\text{Max } W_c \equiv \int_0^{\infty} (1-s_c)(1-\tau_k)\, k_c f'(k) e^{-\rho_c t}\, dt, \tag{13}$$

subject to

$$\dot{k}_c = s_c(1-\tau_k)\, k_c f'(k) - nk_c,$$
$$k_c(0) = k_c^0,$$

where k_c, $\dot{k}_c$, s_c, and ρ_c denote the capitalists' capital stock per capita, net capital accumulation per capita, savings ratio, and discount rate, respectively. Capitalists also regard n, k_k^0, τ_k, and ρ_c as exogenous variables and only s_c as a choice variable.

3 . Cournot-Nash Equilibrium and SteadyState Equilibrium

In order to solve their consumption-savings problems, workers (or capitalists) must anticipate future prices. In other words, they must anticipate the time paths of future capital stocks satisfying (3) and (4), or equivalently, the capital stocks owned by the other class. As mentioned in the introduction, workers (or capitalists) anticipate the price paths which clear all the future markets, assuming that the consumption-savings decisions of the other class are insensitive both to their savings decisions and capital stocks. Thus, in solving problem (12) [(13)] workers (capitalists) regard the decisions of the other class as given at each moment in time.

Hence, our problem is reduced to a kind of noncooperative defferential game played by workers and capitalists in which the payoff for each class is their total discounted utility (or consumption streams), the strategy of each class is the savings ratio, and the equilibrium concept is an *open-loop Nash equilibrium*. Before solving the game, we briefly state the formal definitions:

Definition 1: *The space of open-loop strategies of each class, $i=c,w$, is*

$$S_i \equiv \{s_i(t) \mid s_i(t) \text{ is measurable on } [0,\infty) \text{ and } 0 \leqq s_i(t) \leqq 1 \quad t \in [0,\infty)\}.$$

Open-loop strategies are functions of time alone. They do not depend on the current state of the game. Playing open-loop strategies means that the agents in each class determine their actions *in advance* for all $t \in [0,\infty)$[9].

Denote the objective functions of workers and capitalists in problem (12) and (13) by $W_w(s_w(t),\ s_c(t))$ and $W_c(s_w(t),\ s_c(t))$, respectively.

Definition 2: *The strategy pair* $(\hat{s}_w(t),\ \hat{s}_c(t))$ *is said to constitute the open-loop Nash equilibrium of the differential game defined in problem* (12) *and* (13) *if and only if*

$$W_w(\hat{s}_w(t),\ \hat{s}_c(t)) \geqq W_w(s_w(t),\ \hat{s}_c(t)), \tag{14}$$
$$W_c(\hat{s}_w(t),\ \hat{s}_c(t)) \geqq W_c(\hat{s}_w(t),\ s_c(t)),$$

for all feasible strategy pairs $(s_w(t),\ s_c(t)) \in S_w \times S_c$.

This behavior hypothesis enables us to apply the standard maximum principal to solve problem (12) and (13). Thus the Hamiltonian function for problem (12) is

$$H_w = e^{-\rho_w t}\{(1-s_w)[f(k)-k_c f'(k)(1-\tau_k)-g] + q_w\{s_w[f(k)-k_c f'(k)(1-\tau_k)-g]-nk_w(t)\}\}, \tag{15}$$

9) The open-loop Nash equilibrium is not a satisfactory concept, because it does not satisfy subgame perfectness, that is, it excludes the possibilities of the learning process about the other class's behavior all over the horizon. However, more satisfactory treatment of a differential game is beyond the scope of this paper.

where q_w is the current shadow price of the capital stock owned by workers. Notice that q_w has the economic interpretation of the demand price of one unit of investment. Solving the first order necessary condition, $\partial H_w / \partial s_w = 0$, yields

$$q_w = 1. \tag{16}$$

This states that the savings ratio of workers is determined by the value of q_w. Furthermore, the dynamic behavior of workers will be completely governed by the above savings ratio. Taking into account (16), we can construct three possibilities of dynamic motion, depending on the value of s_w (or q_w) as shown below:

(1) The case when $q_w > 1$ and $s_w = 1$. The canonical equations of this dynamic motion are

$$\dot{k}_w = f(k) - k_c f'(k)(1-\tau_k) - g - nk_w, \tag{17}$$

$$\dot{q}_w / q = (\rho_w + n) - [f'(k) - k_c f''(k)(1-\tau_k)]. \tag{18}$$

Workers save all of their disposable income.

(2) The case when $q_w < 1$ and $s_w = 0$. The canonical equations are

$$\dot{k}_w = -nk_w, \tag{19}$$

$$\dot{q}_w = (\rho_w + n) q_w - [f'(k) - k_c f''(k)(1-\tau_k)]. \tag{20}$$

Workers consume all of disposable income, so their net savings are negative.

(3) The case when $q_w = 1$.

$$nk_w = s_w [f(k) - k_c f'(k)(1-\tau_k) - g], \tag{21}$$

$$\rho_w + n = f'(k) - k_c f''(k)(1-\tau_k). \tag{22}$$

Workers consume a constant per capita amount of their disposable income, so their net savings are zero.

Using the above results and taking k_c and τ_k as given, the dynamic behavior of capital accumulation by workers is illustrated in Figure 1. Close inspection of this diagram reveals that the steady-state equilibrium, $(\hat{k}_w, 1)$, displays a saddlepoint property. Moreover, optimality requires that the optimal path must satisfy the transversality condition:

$$\lim_{t\to\infty} e^{-\rho_w t} q_w k_w = 0. \tag{23}$$

Thus the optimal path of capital accumulation by workers lies exactly on the stable arm, AA-line, and asymptotically converges to the steady-state equilibrium denoted by $(\hat{k}_w, 1)$ in Figure 1 as t approaches ∞, given the fixed values of k_c and τ_k. If the value of either k_c or τ_k changes, then the system may have different stable arms and thus different steady-state equilibria. Due to the transversality condition, however, the optimal path always follows a stable arm. By further observation of the motions of k_w and q_w on the stable arm, AA-line, in Figure 1, we can find the following compact but important relationship between q_w and $\dot{k}_w$, namely,

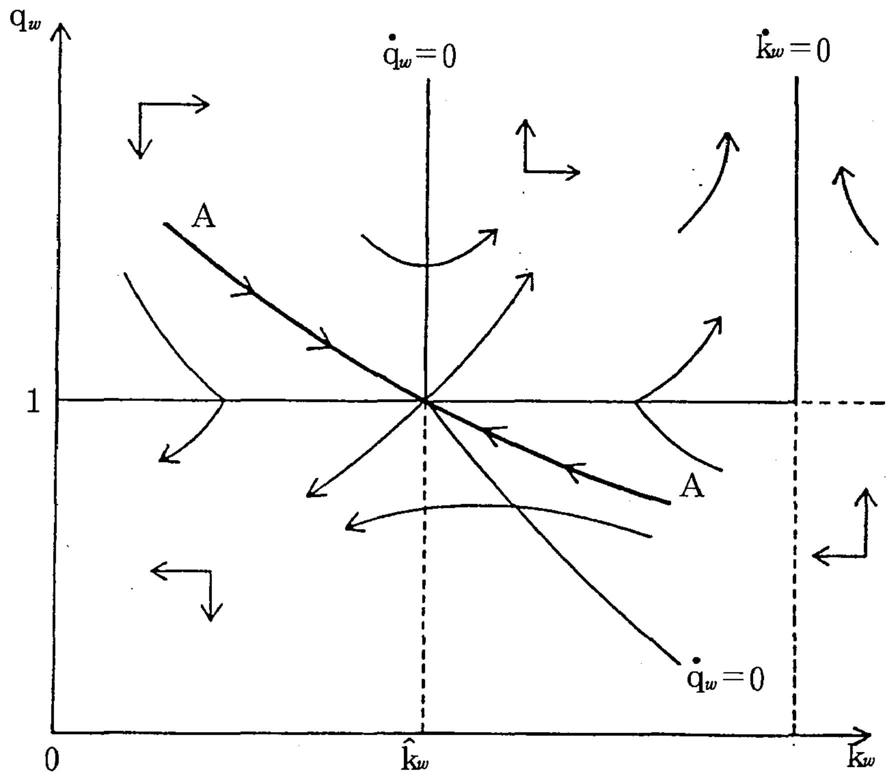

FIGURE 1 Optimal paths of (k_w, q_w).

$$\dot{k}_w \gtreqless 0 \quad \text{if and only if} \quad q_w \gtreqless 1. \tag{24}$$

On the other hand, capitalists are also assumed to maximize their intertemporal utility subject to their instantaneous budget constraints, taking the decisions of workers and the tax rates as given. The Hamiltonian function for problem (13) is

$$H_c = e^{-\rho_c t}\{(1-s_c)(1-\tau_k)\, k_c f'(k) - q_c[s_c k_c f'(k)(1-\tau_k) - nk_c]\}, \tag{25}$$

where q_c is the current shadow price of capital owned by capitalists. The first order necessary condition, $\partial H_c / \partial s_c = 0$, implies

$$q_c = 1. \tag{26}$$

That is, the solution is also of a *bang-bang* type. Therefore, three dynamic motions for capitalists' savings behavior emerge in the same manner indicated in the case of workers:

(1) The case when $q_c > 1$ and $s_c = 1$. The canonical equations of this dynamic motion are

$$\dot{k}_c = k_c f'(k)(1-\tau_k) - nk_c, \tag{27}$$

$$\dot{q}_c / q_c = \rho_c + n - [f'(k) + k_c f''(k)](1-\tau_k). \tag{28}$$

(2) The case when $q_c < 1$ and $s_c = 0$. The canonical equations are

$$\dot{k}_c = -nk_c, \tag{29}$$

$$\dot{q}_c = (\rho_c + n) q_c - [f'(k) + k_c f''(k)](1-\tau_k). \tag{30}$$

(3) The case when $q_c = 1$. We have[10]

$$nk_c = s_c k_c f'(k)(1-\tau_k), \tag{31}$$

$$\rho_c + n = [f'(k) + k_c f''(k)](1-\tau_k). \tag{32}$$

From these results, the dynamic behavior of capitalists' opti-

10) For simplicity, we assume uniqueness of k_c satisfying (32), given k_w.

mal savings can be illustrated in $(k_c,\ q_c)$-space as in Figure 1, given the fixed values of k_w and τ_k. The steady-state equilibrium therefore is a saddlepoint. Similarly, the transversality condition is given by

$$\lim_{t\to\infty} e^{-\rho_c t} q_c k_c = 0. \tag{33}$$

Thus the optimal path of capital accumulation by capitalists always approaches the steady-state equilibrium, $(\hat{k}_c, 1)$, along the stable arm, given the fixed values of k_w and τ_k. Consequently, observing the dynamic motions of k_c and q_c on the stable arm, we can also deduce the following analogous results between q_c and $\dot{k}_c$, namely,

$$\dot{k}_c \gtreqless 0 \quad \text{if and only if} \quad q_c \gtreqless 1. \tag{34}$$

To justify our comparative statics in the steady-state Nash equilibrium, we must ask under what conditions its existence, uniqueness, and stability are established. Since the detailed procedure can be found in Appendix (A I) and (A II), we shall summarize the results as follows:

Proposition 1: *Let us assume* $f''' > 0$[11]. *Given* ρ_c, *n, and* τ_k, *if* ρ_w *satisfies the following inequalities,*

$$f'(k_c^2) - k_c^2 f''(k_c^2)(1-\tau_k) > \rho_w + n > f'(k_w^2), \tag{35}$$

then there is a unique steady-state Nash equilibrium (Pasinetti

11) Alternatively, to prove only the uniqueness of the steady-state equilibrium, the condition $f''' > 0$ is not required. This is because the conditions of the global univalence theorem [Theorem 20.10 in Nikaido (1968)] are satisfied without it, for the Jacobian matrix [D] is one-signed. However, $f''' > 0$ plays a crucial role in ensuring stability of the steady-state Nash equilibrium. Indeed, if $f''' > 0$ is not assumed, then the slope of curve II cannot be determined, so curve I may intersect curve II from the below in Figure 2. Although this reflects the reverse coexistence condition, $\rho_c > \rho_w$, it is not a realistic assumption for a capitalist society [see footnote 12].

equilibrium).

Given ρ_w, *n, and* τ_k, *if* ρ_c *satisfies the inequalities,*

$$f'(k_w^1)(1-\tau_k)>\rho_c+n>[f'(k_c^1)+k_c^1 f''(k_c^1)](1-\tau_k), \tag{36}$$

then there is a unique steady-state Nash equilibrium (Pasinetti equilibrium), where k_w^1, k_w^2, k_c^1, *and* k_c^2, *respectively, are given by*

$$\rho_w+n=f'(k_w^1), \tag{37}$$

$$\rho_c+n=f'(k_w^2)(1-\tau_k), \tag{38}$$

$$\rho_w+n=f'(k_c^1)-k_c^1 f''(k_c^1)(1-\tau_k), \tag{39}$$

$$\rho_c+n=[f'(k_c^2)+k_c^2 f''(k_c^2)](1-\tau_k). \tag{40}$$

Proposition 2 : *If there exists a unique steady-state equilibrium (Pasinetti equilibrium), it is globally asymptotically stable under* $f'''>0$.

A few remarks should now be made. First, proposition 1 means that if the discount rate of workers is greater than that of capitalists and this difference is not very large [see Figures 4 and 5 in Appendix (AI)], then every agent has a positive amount of capital in the steady-state equilibrium[12], which is called *Pasinetti equilibrium*. Second, our analysis of tax incidence should be confined to such an equilibrium, because our main concern is the effect of tax changes on the long-run distribution of the capital stocks (or wealth) between the two classes, which will be used to evaluate

12) If the conditions of proposition 1 are not met, namely, either (i) if the discount rate of capitalists is too much smaller than that of workers, then the anti-Pasinetti equilibrium is realized, where only capitalists hold all of the capital stocks of the economy, or (ii) if the discount rate of capitalists is greater than that of workers, then another anti-Pasinetti equilibrium is realized, where only workers hold all of the capital stocks. These situations are similar to, but not identical, with the long-run consequences of the perfect foresight equilibrium model with heterogeneous consumers [Becker (1980)].

long-run tax incidence. Third, there is a sharp contrast between our coexistence condition and that of Homma (1981). Homma dealt with the neoclassical growth model with fixed savings ratios, so that his coexistence condition [i.e. inequality (15) in Homma] is determined by the three structural parameters of his model; the growth rate of population, the savings ratios of both classes, and the share of capitalists' disposable income to total disposable income. Our coexistence conditions, on the other hand, depend on more basic parameters: the growth rate of population, the discount rates of the two classes, and the *initial* rate of capital income taxes, in addition to the production technology.

4. The Effects of Changes in Capital Income Taxes

In this section we shall proceed to analyze the effects of changes in capital income taxes on the steady-state savings ratios and the capital stocks of workers and capitalists, keeping government expenditure constant. In so doing, totally differentiating (21), (22), (31), and (32), and then then rearranging yields the following simultaneous system of equations:[13)]

$$\begin{bmatrix} D \end{bmatrix} \begin{bmatrix} ds_c \\ ds_w \\ dk_c \\ dk_w \end{bmatrix} = \begin{bmatrix} s_c f' \\ -s_w k_c f' \\ -k_c f'' \\ f' + k_c f'' \end{bmatrix} d\tau_k, \tag{41}$$

13) Inspection of the system (41) reveals that the two variables dk_w and dk_c, are independently determined by the bottom two equations, and the other two variables, ds_w and ds_c, are sequentially determined by the top two variables. This is called the *block recursive* system.

where [D] =

$$\begin{bmatrix} f'(1-\tau_k) & 0 & s_c f''(1-\tau_k) & s_c f''(1-\tau_k) \\ 0 & f-k_c f'(1-\tau_k)-g & s_w[f'\tau_k-k_c f''(1-\tau_k)] & s_w[f'-k_c f''(1-\tau_k)]-n \\ 0 & 0 & f''\tau_k-k_c f'''(1-\tau_k) & f''-k_c f'''(1-\tau_k) \\ 0 & 0 & (2f''+k_c f''')(1-\tau_k) & (f''+k_c f''')(1-\tau_k) \end{bmatrix} \tag{42}$$

For symbolical convenience, let f', f'', and f''' represent $f'(k)$, $f''(k)$, and $f'''(k)$, respectively, and let $|D|$ denote a determinant of the matrix [D],

$$|D| = [f'(1-\tau_k)nk_w|J|]/s_w < 0, \tag{43}$$

where $$|J| = (1-\tau_k)(f'')^2(\tau_k-2) < 0, \tag{44}$$

which are all evaluated at the steady-state $(k_w^*,\ k_c^*) > 0$.

Applying Cramer's rule to eqs. (41), we obtain

$$\frac{dk_w}{d\tau_k} = \frac{2k_c(f'')^2 - k_c(f'')^2\tau_k + f'f''\tau_k - k_c f'f'''(1-\tau_k)}{|J|}, \tag{45}$$

$$\frac{dk_c}{d\tau_k} = \frac{-2k_c(f'')^2 + k_c(f'')^2\tau_k - f'f'' + k_c f'f'''(1-\tau_k)}{|J|}. \tag{46}$$

Summing (45) and (46) yields

$$\frac{dk}{d\tau_k} = \frac{dk_w}{d\tau_k} + \frac{dk_c}{d\tau_k} = \frac{f'}{f''(2-\tau_k)} < 0. \tag{47}$$

This means that an increase in capital income taxes unambiguously reduces the entire capital stock of the economy. This result qualitatively coincides with Homma's and Boyd's(1988)[14]. It should also be noted that Felstein (1976), Yeh (1979), and Homma (1981) all require the assumption $s_c > s_w$ to unambiguously determine the sign of its Jacobian and to guarantee local stability of a steady-state equilibrium, whereas in our model this ad-hoc assumption is not needed at all, for our model shows that this property is automatically fulfilled in the steady-state equilibrium[15]. However, the signs of (45) and (46) cannot be unambiguously determined, for their numerators are ambiguous. To get economically appealing results, more restrictive assumptions on the shape of the production function have to be made. Rearranging (45) and (46) yields

$$\frac{dk_w}{d\tau_k}=\frac{k_c(1-\tau_k)[(f'')^2-f'f''']+f''[f'\tau_k+k_cf'']}{|J|}\gtrless 0. \tag{48}$$

$$\frac{dk_c}{d\tau_k}=\frac{-k_c(1-\tau_k)[(f'')^2-f'f''']-f''[f'+k_cf'']}{|J|}< 0. \tag{49}$$

where the sign of (49) is determined by the following assumption:

$$(f'')^2\leqq f'f'''. \tag{50}$$

---

14) Another difference must be mentioned: In the perfect foresight equilibrium model with either homogeneous or heterogeneous agents, the following steady-state condition holds: $(1-\tau_k)f'=\rho+n$, where ρ is a discount factor of the most patient agent; see Becker (1980) and Boyd (1988). Differentiating this with respect to τ_k yields

$$\frac{dk}{d\tau_k}=\frac{f'}{f''(1-\tau_k)}< 0.$$

Comparing this with (47), we see that the effect of capital income taxes on the aggregate capital/labor ratio is *smaller in magnitude* in our model than in their perfect foresight equilibrium models. This is because in our model workers also have capital in the steady-state.

15) Our model *endogenously* explains the inequality $s_c > s_w$ in the steady-state equilibrium as follows. From (21) and (31), it turns out that

$$s_c=\frac{n}{f'(k)(1-\tau_k)} \text{ and } s_w=\frac{nk_w}{f(k)-k_cf'(k)(1-\tau_k)-g}$$

$$s_c-s_w=\frac{n[f(k)-k_cf'(k)(1-\tau_k)-g]}{[f(k)-k_cf'(k)(1-\tau_k)-g]\,f'(k)(1-\tau_k)}>0.$$

This assumption implies that the elasticity of the demand for capital is *nondecreasing* with respect to the capital/labor ratio; see Appendix(III). Loosely speaking, when the capital/labor ratio is higher, the demand for capital is sensitively responsive to a change in the interest rate. Note also that (50) ensures the stability condition, $f''' > 0$.

However, the sign of (48) is still ambiguous due to the ambiguity of the term $f'\tau_k + k_c f''$ in (48). Recalling that $f' + k_c f'' > 0$ from (32), if the *initial* tax rate τ_k lies within the range $[\tau_k^*, 1)$ where $\tau_k^* = (-k_c f'')/f' < 1$, then we have

$$\frac{dk_w}{d\tau_k} > 0. \tag{51}$$

Using (48) and (49), on the other hand, we get

$$\frac{ds_w}{d\tau_k} = \frac{nk_c(1-\tau_k)[(f'')^2 - f'f'''][f - k_c f'(1-\tau_k) - g]}{[f - k_c f'(1-\tau_k) - g]^2 |J|} + \frac{nf''[f'\tau_k + k_c f'']}{[f - k_c f'(1-\tau_k) - g]\ |J|} \gtreqless 0, \tag{52}$$

$$\frac{ds_c}{d\tau^k} = \frac{-s_c(f'')^2}{|J|} > 0. \tag{53}$$

Similarly, $ds_w/d\tau_k > 0$ at $\tau_k \in [\tau_k^*, 1)$, (54)

Thus, we can summarize these results as follows :

Proposition 3 : *Suppose that the government substitutes labor for capital income taxes, keeping government expenditure constant in the steady-state equilibrium (Pasinetti equilibrium). Then, the aggregate capital stock of the economy and the savings ratio of capitalists unambiguously fall. Furthermore, provided the elasticity of* *the demand for capital is nondecreasing with respect to the capital/labor ratio* [*i.e.* $(f'')^2 \leq f'f'''$], *capitalists bear the resulting burden in terms of the capital stock owned by themselves, and there exists a lower bound of capital income taxes,* τ_k^*, *such that if the initial value of capital income taxes lies within the range* $[\tau_k^*, 1)$ *where* $\tau_k^* = (-k_c f'')/f' < 1$, *then the tax burden of workers unambiguously falls and the savings ratio of workers rises, and thus capitalists bear more than* 100 *percent of the tax burden.*

Specifically, under the Cobb-Douglas function, $k^{\alpha}(\alpha<1)$, we can get the unambiguous results:

$$\frac{dk_w}{d\tau_k} = \frac{\alpha^2(\alpha-1)(k_w\tau_k + k_c\alpha)k^{2(\alpha-2)}}{|J|} > 0, \tag{55}$$

$$\frac{dk_c}{d\tau_k} = \frac{\alpha^2(\alpha-1)\{k_c(\tau_k-1) - k_w - k_c\alpha\}k^{2(\alpha-2)}}{|J|} < 0, \tag{56}$$

$$\frac{ds_w}{d\tau_k} > 0. \tag{57}$$

To intuitively understand proposition 3, we suppose that the economy is initially in the steady-state E in Figure 2. Consider an increase in capital income taxes. This increase causes the capitalists' net return on capital [i. e. the right hand side of (32)] to fall[16]. Since capitalists regard k_w as given, capitalists reduce k_c to keep the equality of (32). This is shown as an inward shift of curve II to the origin in Figure 2. On the other hand, although curve I also shifts

16) The right hand side of (22) can be interpreted as a long-run demand curve for capital which workers face, while that of (32) as a long-run demand curve for capital which capitalists face. Put differently, in our model these heterogeneous agents face the *different* long-run demand curves for capital [cf. Becker (1980) and Boyd (1988)]

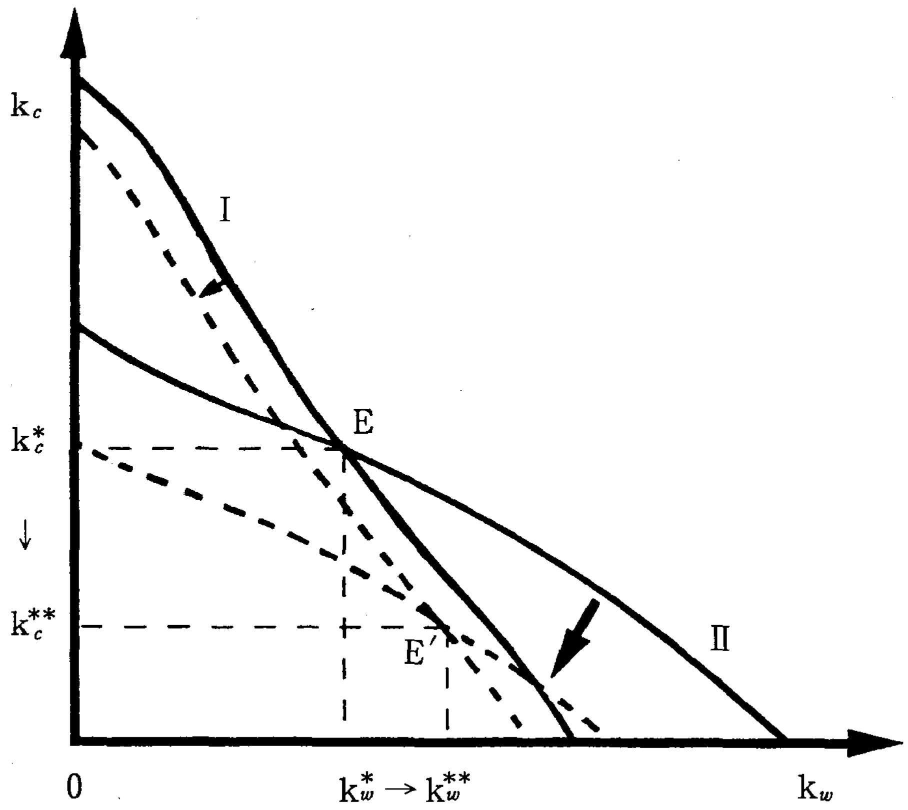

FIGURE 2 An increase in capital income taxes shifis both curve I and II inward to the origin ; Pasinetti equilibrium moves from point E to point E′.

inward to the origin, this shift is *smaller in magnitude* than that of curve II. This is because the workers'net return on capital [i. e. the right hand side of (22)] is less affected by the increase in τ_k than that of capitalists. More precisely, the induced decrease in k_c has two effects on the workers' net return: First, the aggregate capital stock falls, thereby driving up the gross return (i. e. f'), and second, the workers' tax base of capital income taxes (i. e. $-k_c f''$) may become smaller. Thus these two effects would bring about a rise in the workers' net return when $\tau_k \geqq \tau_k^*$, regardless of the increase in τ_k. This encourages workers to own more capital. Furthermore, the increase in k_w in turn raises the aggregate capital stock, thereby leading to a further reduction in k_c. The economy continuously repeats the above process until it reaches the new steady-state E′, where k_c and k are both lower, but k_w is higher than it would previously, as shown in Figure 2.

On the other hand, a rise in either ρ_c, n, or the initial tax rate τ_k increases the left hand side of (32). To restore the equality of (32), k_c must fall, thus leading to a decline in the lower bound τ_k^* [i.e. $(-k_c f'')/f' < 1$]. In other words, the capitalists' tax base of capital income taxes (i.e. $f' + k_c f''$) is larger, while the workers' tax base (i.e. $-k_c f''$) is smaller. Thus when τ_k increases, curve II shifts greatly, while curve I shifts less. Consequently, the higher is either ρ_c, n, or τ_k, the more likely it is that the increase in f′ will dominate a negative effect resulting from the increase in τ_k on the term $-k_c f''$ in (22). If this is the case, the workers' net return rises, thereby increasing k_w, as shown in Figure 2.

Proposition 4: *If either the initial level of capital income taxes, the discount rate of capitalists, or the rate of population becomes larger, then the lower bound of capital income taxes, τ_k^*, is smaller, that is, the range of the initial capital income tax, in which the tax burden of workers resulting from the increase in capital income taxes falls, is larger.*

Proposition 4 means that whether or not workers is better off depends on either the initial level of capital income taxes, the discount rate of capitalists, or the rate of population. Furthermore, Proposition 4 is contrary to the earlier results obtained by Yeh (1979) and Homma (1981) that the higher is the initial rate of capital income taxes, the more likely it is that the burden resulting from a rise in capital income taxes shifts from capitalists to workers [see(iii)in the introduction in this paper].

Here, we have used the per capita level of the capital stock owned by each class as a measure to evaluate tax incidence. Unfortunately, this is not necessarily a universally approved measure. There are several alternatives in the literature of tax incidence, such as disposable income, utility level, etc. For this reason, we should verify our results by applying these alternatives. Since in the steady-state equilibrium the savings ratio remains constant and the utility level is precisely measured by the consumption level, it suffices to verify our results in terms of disposable income per capita. Since workers' and capitalists' disposable incomes per capita are respectively

$$y_w = w(1-\tau_w) + rk_w(1-\tau_k) \text{ and } y_c = rk_c(1-\tau_k), \tag{58}$$

substituting (3), (4), and (6) into (58), and then differentiating with respect to τ_k yields

$$\frac{dy_w}{d\tau_k} = \frac{k_c f'(1-\tau_k)^2[(f'')^2 - f'f''']}{|J|} > 0, \tag{59}$$

$$\frac{dy_c}{d\tau_k} = \frac{-k_c f'(1-\tau_k)^2[(f'')^2 - f'f'''] - (1-\tau_k)(f')^2 f''}{|J|} > 0, \tag{60}$$

if $(f'')^2 \leqq f'f'''$. Summing (59) and (60) yields

$$\frac{dy}{d\tau_k} = \frac{dy_w}{d\tau_k} + \frac{dy_c}{d\tau_k} = \frac{(f')^2}{f''(2-\tau_k)} < 0. \tag{61}$$

Eq. (61) implies that total disposable income per capita of the economy unambiguously falls. When assumption (50) is imposed, furthermore, relatively stronger results obtain under the measure of disposable income per capita than under that of the capital stock per capita (or wealth) in the sense that the burden of workers unambiguously falls, regardless of the initial value of τ_k. Note further that capitalists bear more than 100 percent of the tax burden in terms of disposable income, as shown by (61)

This can be explained as follows. Capitalists receive the higher gross rate of return on capital due to a fall in the aggregate capital/labor ratio, but their steady-state capital stock becomes smaller. Capitalists' disposable income falls because under assumption (50) k_c must fall greatly, thereby canceling out a rise in the return on capital. In contrast, workers' disposable income would increase, because the workers' capital income rises due to increases in both k_w and the gross rate of return on capital. In addition, although the wage rate falls due to the fall in k, the tax substitution effect resulting from a reduction in labor income taxes is positive. Consequently, these effects contribute to improve workers' disposable income.

Comparing the results obtained so far with Feldstein (1974), Grieson (1975), Yeh (1979) as well as Homma (1981), we see remarkable differences. According to their results, a substantial part of the burden resulting from the increase in capital income taxes is borne by workers, whereas our results would indicate the *opposite*. The reason for this difference is that (i) our model is based on the intertemporal optimizing behavior of agents, while theirs are not, (ii) workers and capitalist behave *asymmetrically* in response to changes in capital income taxes because these agents face the different long-run demand curves for capital which are dependent each other, and (iii)Feldstein and the other have focused on the distribution effect on labor income, while in our Kaldorian two-class setting workers also have capital income, in addition to labor income. Accordingly, the rise in capital income taxes reduces the wage rate in the long-run, making the situation of workers worse off in their models, while in our model the induced increase in capital owned by workers contributes to improve their disposable income. Although Homma adopted the same Kaldorian two-class model, his results differ significantly from ours. This is because in most cases of his model the capital stock owned by workers falls, since the savings ratios of both classes are constant, and thus there is no interaction between workers and capitalists.

5. Concluding Remarks

In this section, brief comments and possible extensions of our analysis will be stated: (i)We have assumed a special form of utility functions, that is, a *linear* utility function. The dynamic behavior of the model relies crucially on this specification, while the steady-state conditions [i. e. (22) and (32)] are independent of a form of utility functions. This implies that our incidence results regarding either the capital stocks or disposable income continue to hold under more general utility functions, although the saving ratio and the consumption level of each class rely on it. (ii) We here adopt the open-loop Nash strategy. However, if more variants concerning the behavior hypothesis of agents are allowed, such as Stackelberg, closed-loop, and bargaining game strategies, one may obtain a variety of outcomes regarding both the dynamic behavior and long-run tax incidence, because the resulting steady-state equilibrium may be different. (iii) Our main concern is the long-run effect of tax changes, ignoring the adjustment process which governs the economy approaching the steady-state equilibrium. Apart from technical difficulties, comparative dynamics of changes in capital income taxation deserves further study in the present framework; see Boadway (1979). Furthermore, the comparative dynamic results depend heavily on both the specification of utility functions and the underlying behavior hypothesis.

Appendix

(AI) *Proof of Proposition* 1. The proofs given here are essentially based on Tokita's (1972), so we shall reconstruct his proofs with proper modifications. Let us rewrite eqs. (22) and (32). That is, for workers,

$$\rho_w + n = f'(k) - k_c f''(k)(1-\tau_k), \tag{22}$$

for capitalists

$$\rho_c + n = [f'(k) + k_c f''(k)](1-\tau_k). \tag{32}$$

First, in order to illustrate (22) and (32), we must find their slopes. In so doing, we totally differentiate (22) and then rearrange it as follows,

$$\frac{dk_c}{dk_w} = \frac{f''(k) - k_c f'''(k)(1-\tau_k)}{f''(k)\tau_k - k_c f'''(k)(1-\tau_k)} < 0, \tag{A1}$$

under assumption $f'''(k) > 0$. Furthermore, we find

$$\frac{dk_c}{dk_w} = -1 - \frac{f''(k)(1-\tau_k)}{f''(k)\tau_k - k_c f'''(k)(1-\tau_k)} < -1. \tag{A2}$$

Thus the slope of this curve (curve I) is smaller than -1, as illustrated in Figure 3.

On the other hand, the slope of the locus satisfying (32) (curve II) is so complex that we cannot uniquely characterize it. Alternatively, summing (22) and (32) yields

$$\rho_c + \rho_w + 2n = 2f'(k)[1 - (\tau_k / 2)]. \tag{A3}$$

By totally differentiatin (A3) and rearranging it, the slope of (A3) is

$$\frac{dk_c}{dk_w} = -1. \tag{A4}$$

Taking into account (A2) and (A4), we find that if curve I and line (A3) intersects in the positive orthant of (k_w, k_c)-space, it must be unique. Obviously, the same conclusion also holds for the case of the combination between the loci (22) and (32), that is, curve I intersects curve II from the above as illustrated in Figure 3.

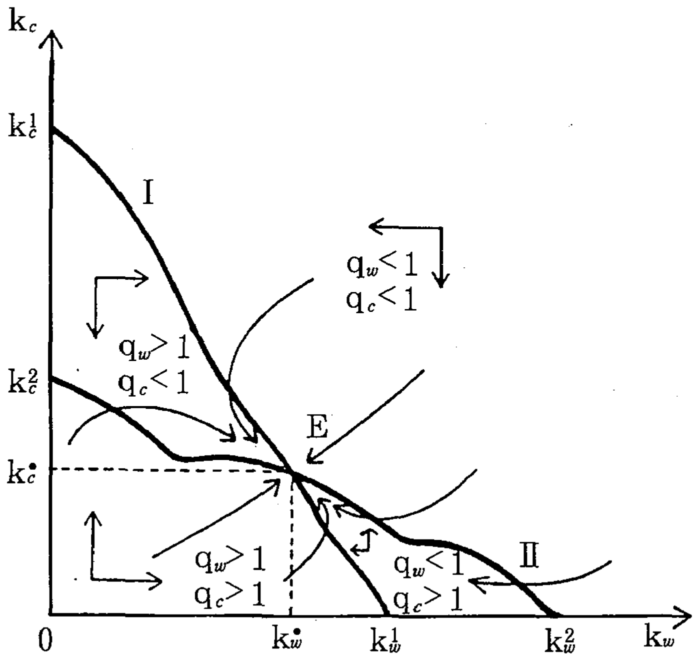

FIGURE 3 Optimal paths of (k_w, k_c).

To prove that there exists the Pasinetti equilibrium point, we must show that the intersection of the two curves always lies in the positive orthant of (k_w, k_c)-space. Suppose that curve I has the k_c^1 intercept on the k_c-axis and the k_w^1 intercept on the k_w-axis, while curve II has the k_w^2 intercept on the k_w-axis and the k_c^2 intercept on the k_c-axis; see Figure 3. Furthermore, since k_w^1 and k_c^1 satisfy (22) (i.e. curve I), substituting two intercept points $(k_w^1, 0)$ and $(0, k_c^1)$ into (k_w, k_c) in (22), respectively, yields

$$\rho_w + n = f'(k_w^1) \quad \text{and} \quad \rho_w + n = f'(k_c^1) - k_c^1 f''(k_c^1)(1 - \tau_k). \tag{A5}$$

Similarly, substituting two axis points $(k_w^2, 0)$ and $(0, k_c^2)$ into (k_w, k_c) in (32) (i.e. curve II), respectively, we find another relation:

$$\rho_c + n = f'(k_w^2)(1 - \tau_k) \quad \text{and} \quad \rho_c + n = [f'(k_c^2) + k_c^2 f''(k_c^2)](1 - \tau_k). \tag{A6}$$

Consulting Figure 3, furthermore, one sees that if

$$k_w^2 > k_w^1 \qquad \text{and} \qquad k_c^1 > k_c^2 \tag{A7}$$

are met on both the axes, then there is the intersection of curve I and curve II in the positive orthant of (k_w, k_c)-space. Hence we must find under what conditions (A7) occurs. First, we consider the feasible range of ρ_w to meet (A7) when ρ_c, n, and τ_k are given. Since $\rho_c + n$ and τ_k are exogenously fixed, k_c^2 and k_w^2 are obtained by (A6) as illustrated in Figure 4. Then, given these values of k_c^2 and k_w^2, we can choose the range of $\rho_w + n$ which determines the values of k_c^1 and k_w^1 by (A5) to satisfy (A7), as indicated in Figure 4. In other words, as long as $\rho + n$ lies in the interval indicated in Figure 4, both k_w^2 and k_c^2 satisfy (A7). Thus, $\rho_w + n$ must satisfy

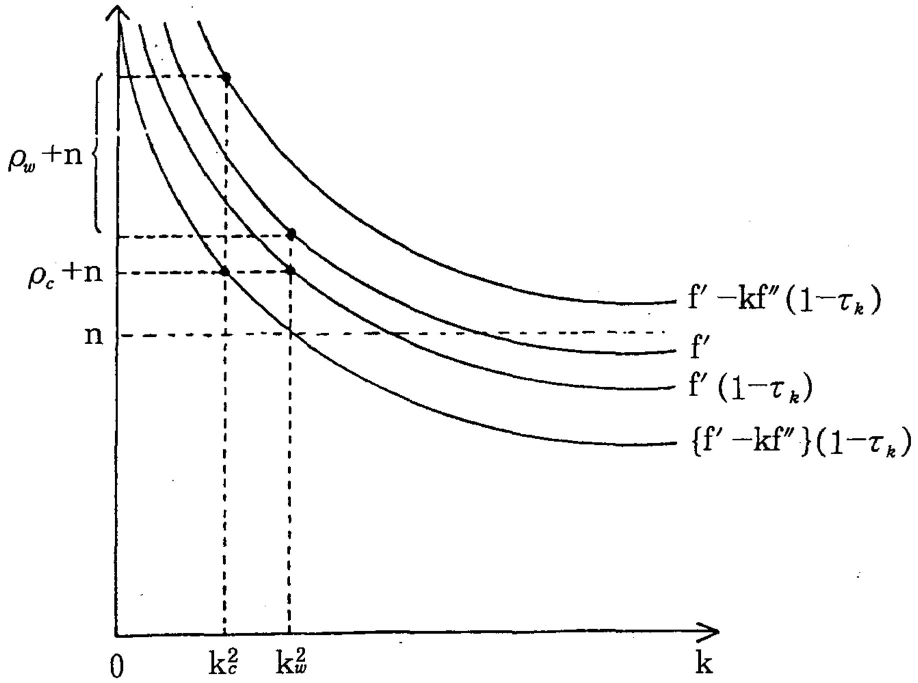

FIGURE 4 Given ρ_c, n, and τ_k, if the value of ρ_w lies in the indicated open interval, the Pasinetti equilibrium exists.

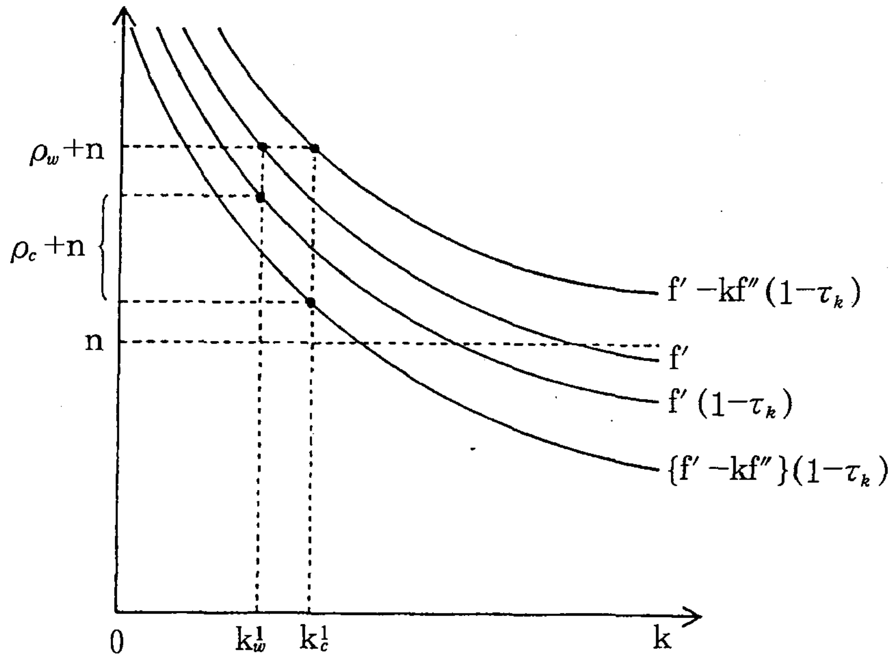

FIGURE 5 Given ρ_w, n, and τ_k, if the value of ρ_c lies in the indicated open interval, the Pasinetti equilibrium exists.

$$f'(k_c^2)-k_c^2 f''(k_c^2)(1-\tau_k)>\rho_w+n>f'(k_w^2). \qquad \text{(A8)}$$

On the other hand, taking ρ_w, n, and τ_k as given, it follows that ρ_c+n must lie in the interval which is displayed in Figure 5 so as to meet (A7). Mathematically speaking, ρ_c must satisfy

$$f'(k_c^1)(1-\tau_k)>\rho_c+n>[f'(k_c^1)+k_c^1 f''(k_c^1)](1-\tau_k). \qquad \text{(A9)}$$

Otherwise, we may have no intersection in the positive orthant of (k_w, k_c)-space; see footnote 12. Q. E. D.

(A II) *The Proof of Proposition* 2. In what follows, we shall assume that there exists the unique intersection between curve I and curve II in the positive orthant of (k_w, k_c)-space, which is denoted by point E in Figure 3. Note also that point E corresponds precisely to the Pasinetti equilibrium $(k_w^*, k_c^*)>0$. By virtue of (22) and (32), we can see that in each region enclosed by curve I and curve II, q_w(or q_c) may be greater or less than unity, as illustrated in Figure 3. Moreover, recalling the results obtained in section 3, that is, (24) and (34), the dynamic behavior of k_w and k_c will be solely determined by the values of q_w and q_c. Then, close observation of the motions of k_w and k_c indicated by the arrows in Figure 3 reveals that from any starting point (k_w^0, k_c^0) in the positive orthant of (k_w, k_c)-space, the system will always converge to the steady-state equilibrium (k_w^*, k_c^*), as shown in Figure 3. Furthermore, a limit cycle cannot exist because all the phase arrows that cross the boundary of the region enclosed curve I and II inward. Therefore, the system is globally asymptotically stable, provided curve I intersects curve II from the above at least once in the positive orthant of (k_w, k_c)-space. This completes the proof of Proposition 2. Q. E. D.

(AⅢ) *The Implications of Assumption* (50). We give three interpretations to understand assumption $f'f''' \geqq (f'')^2$

First, the elasticity of demand for capital (or savings) is defined as $\varepsilon = -f'/(kf'')$. By straightforward calculation we can see that if the elasticity of the demand for capital is *nondecreasing* with respect to k, i.e. $d\varepsilon/dk > 0$, then $f'f''' > (f'')^2$. However, the converse may not be true.

Second, define the following function: $R = -f''(k)/f'(k)$, (A10) which is another measure to evaluate the convexity of $f'(k)$. This measure is the same as the *Arrow-Pratt measure of absolute risk aversion* concerning the degree of the convexity of a utility function. Differentiating (A10) with respect to k yields

$$(dR/dk) = [f'f''' + (f'')^2]/(f')^2. \qquad \text{(A11)}$$

Thus it follows that $dR/dk \lesseqgtr 0$ if and only if $f'f''' \gtreqless (f'')^2$.

Finally, the third interpretation is given by the following classifications:

Definition 1: The demand function for capital, $f'(k)$, is a *negative-exponential* if and only if $f'''f' = (f'')^2$ for all $k > 0$,

Definition 2: The demand function for capital, $f'(k)$, is *less concave than a negative-exponential* if and only if $f'''f' < (f'')^2$ for all $k > 0$,

Definition 3: The demand function for capital, $f'(k)$, is *more convex than a negative-exponential* if and only if $f'''f' > (f'')^2$ for all $k > 0$,

where the negative-exponential demand function is defined by $f'(k) = ae^{-bk}$ with $a > 0$ and $b > 0$. Thus assumption (50) corresponds precisely to Definitions 1 and 3.

References

Atkinson, A. B., and J. E. Stiglitz, 1980, *Lectures on Public Economics*, McGraw-Hill, London.

Becker, R. A., 1980, On the long-run steady state in a simple dynamic model of equilibrium with heterogeneous households, *Quarterly Journal of Economics* 47, 375-382.

Becker, R. A., 1985, Capital income taxation and perfect foresight, *Journal of Public Economics* 26, 147-167.

Boadway, R., 1979, Long-run tax incidence ; a comparative dynamic approach, *Review of Economic Studies* 46, 505-511.

Boyd, J. H. Ⅲ, 1988, Dynamic tax incidence with heterogeneous households, *Working Paper*, University of Rochester, Rochester, New York.

Feldstein, M. S., 1974, Incidence of a capital income tax in a growing economy with variable savings rates, *Review of Economic Studies* 41, 505-513.

Grieson, R. S., 1975, The incidence of profits taxes in a neoclassical growth model, *Journal of Public Economics* 4, 75-85.

Harberger, A. C., 1962, The incidence of the corporation income tax, *Journal of Political Economy* 70, 215-240.

Homma, M., 1981, A dynamic analysis of the differential incidence of capital and labor taxes in a two-class economy, *Journal of Public Economics* 15, 363-373.

Judd, K, J., 1985, Redistribution taxation in a simple perfect foresight model, *Journal of Public Economics* 28, 59-83.

Lancaster, K., 1973, The dynamic inefficiency of capitalism, *Journal of Political Economy* 81, 1092-1109.

Nikaido, H., 1968, *Convex Structure and Economic Theory*, Academic Press, New York and London.

Tokita, T., 1972, Continual planning revision and stability of accumulation program, *The Economic Studies Quarterly* 23, 38-47.

Turnovsky, S. T., 1982, The incidence of taxes : A dynamic macroeconomic analysis, *Journal of Public Economics* 18, 161-194.

Yeh C. N., 1979, The incidence of the value added tax in a neoclassical growth model, *Public Finance* 34, 277-291.