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STOCHASTIC STRUCTURE OF BROKERED FOREIGN EXCHANGE AUCTIONS

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ABSTRACT

This paper models stochastic order flow generations in a FX market. Like other financial asset markets, in order for a trader to have perspective on price, he must take other traders' perspectives into consideration. In our model, prices are formed through the interactions between heterogeneous price perspectives and the order flow from macro economy. The auctions are continuous. So we do not have truistic point of time to define liquidation value. Also applying demand and supply on their asynchronous transactions is difficult. We use alternative benchmark value and arrival intensities of order flow. We derive determinants of volume and volatility.

KEY WORDS

Foreign Exchange, Volume/Volatility, Continuous Auction

1 Introduction

Order flow is transaction volume that is signed. Lyons [1] Chapter 10 summaries significant influences of order flow on foreign exchange rates. He does not, however, present a model of order flow generation itself. Our theme is the structure of such order flow generations. This structure also determines volatility. Using our approach, we sort out determinants' effects on volume and volatility. It is possible to explain intra-day variability of volume / volatility correlation.

Inter-bank FX transactions have two channels. One is through brokers, and the other without broker. We model the first type of the auction channel. Price quotes go through brokers. Dealers are not obliged to keep quoting both prices. Only when they want, they submit limit orders to a broker. We assume the environment of the market as follows. There is a broker. The auction is continuous and double sided. Buyers and sellers compete in their own sides. Among submitted limit orders, the broker keeps announcing the best buying and selling prices. Spot foreign exchange is the commodity to trade. The market is geographically local. It has trading hours. There are many dealers. Let's take a representative dealer. He is risk neutral. His objective is daily profit maximization. He is allowed to have open position up to one transaction unit. Deliveries and settlements are scheduled for another day. So no interest cost incurs in order to have intra-day open position. He trades with retail customers any time they want during business hours. Their arrivals are asynchronous and random.

Analyses of continuous auction in financial market need extra approach. Transactions take place asynchronously. To handle this, we use arrival intensity of buyers and sellers, instead of demand and supply. The arrival intensity is expected number of arrivals per unit time. Use of the arrival intensity is in line with Garman [2] and Amihud, Yakov and Mendelson [3]. The next issue poses non trivial question. At a microstructure level, distinction between equilibrium and deviation is not truism. You should take advantage of errors, if many believe it. In such a environment, adverse selection problem is not major issue. Mere perspectives change outcome. So judging dispersion of heterogeneous price perspectives becomes important to make decisions. We face the following question: How do you form price expectation rational way while you know traders' perspectives collectively influence the actual outcome? Our answer is as follows: Dealers as a whole absorb unbalanced order flow from macro fundamentals. Meanwhile, dealers submit heterogeneous limit order prices. As these limit orders, one by one, absorb the unbalanced flow, transaction prices change. Our dealer perceives this mechanism. So he tries to fathom a distribution of reservation prices among dealers. Also he tries to foresee time path of the order flow arrivals. Then he expects configuration of price's time path. Dealers may agree on such a price determination mechanism. However, still heterogeneous perspectives persist. It is because information is limited. The distribution of reservation prices is not observable. Also dealers know only a fraction of order flow from macro economy as their own transactions.

In the next section, we present our model. In the last section, we summarize effects of model's parameters on volume and volatility. And we discuss empirical applicability and the theoretical issue with financial asset market.

2 Model

2.1 Switching Process of Perspectives

Currencies are traded 24 hours on the globe. They are done so consecutively by geographically local markets. We consider such a local market. It opens in the morning and ends in the evening. FX dealers trade in the local market. Closing hours overlaps with the opening hours of the next market. It is possible for dealers to have transaction with overseas counterparts then. This implies that the market need not be cleared at the closing time. There are n_d of dealers. They have transactions with their retail customers. Meanwhile dealers may have wholesale transactions.

Not all of dealers are quoting their prices concurrently. If they are confident enough in their expectations, then they take positions. Their expectation switch between two states. Let I_j be an index function about dealer's state of expectation. Let I_j for $j = 1, 2, \ldots n_d$ be random variables such that

$$I_j = \begin{cases} 0 & \text{if in state 0} \\ 1 & \text{if in state 1} \end{cases} \tag{1}$$

If $I_j = 0$, then the *j*th dealer does not assume open position. If he has retail transaction, he pass it to the wholesale market. If he is in state 1, he has reservation price and, based on it, he is ready to have open position. While in state 1, he must be quoting buying, selling or both prices. The quoted price is equal to his reservation price. There is a unique broker in the market. Quoting price means submitting limit orders to this broker. While in state 0, dealers do not quote prices. If they have retail transaction, they hit someone else's limit order immediately. The number of limit orders coincides with the number of dealers who are in state 1. For a case such that one dealer quotes bid and ask, we neglect their spread. Let N_1 be the number of those in state 1;

$$N_1 = \sum_{j=1}^{n_d} I_j$$

Random variable I_j takes value of 1 or 0 according to two exponential distributions. The switching process of I_j is a continuous time Markov chain consisting of two states. Sojourn time to be in state 1 follows exponential distribution with parameter θ_1. Sojourn time in state 0 is also random variable which follows exponential distribution with parameter θ_0. Let $P_{00}(t)$ be probability of being 0 at time t, starting from state 0 . This probability is obtained, for example by the method given by Ross [4] p.320.

$$P_{00}(t) = \frac{\theta_0}{\theta_0 + \theta_1} \exp^{-(\theta_0+\theta_1)t} + \frac{\theta_1}{\theta_0 + \theta_1} \tag{2}$$

By equation (2), $E[N_1] = \frac{\theta_1}{\theta_0+\theta_1}$ for t large.

2.2 Who is quoting market bid rate ?

When transition from state 0 to state 1 occurs, the *j*th dealer picks his reservation price C_j. This random variable has finite support. Rather than using C_j, we use its quantile value X_j. In other words,

$$X_j \doteqdot \mathrm{P}\ (C_j \le c) \text{ where } c \text{ is a point in } C_j\text{'s support}$$

The random variables X_j's are *i.i.d* with uniform distribution over the unit interval. Different values of X_j result in long or short positions in our auction process. Transaction price coincides with the median of X_j's in a simple situation such that dealers trade based only on different price perspectives. Suppose that dealers arrive one by one at the market. They have reservation prices but no initial inventory. Capital gain is only motivation. Deliveries and payments are scheduled later after the auction. During the auction, long and short positions must match. Hence aggregate net position is zero. We assume that each dealer takes one transaction unit of open position. Dealers' employer exogenously impose this restriction. With such identical open positions, the number of dealers with long position matches that of short positions. In our auction process, those who have open positions submit limit orders. The prices are equal to their reservation prices. For a given time, the largest reservation price on the short side coincides with the highest buying price. Let Z^- and Z^+ be the number of dealers who have short and long positions at time t. Net aggregate position is zero. We have following accounting relationships.

$$Z^- + Z^+ + Z^0 = N_1 \tag{3}$$

$$Z^- - Z^+ = 0 \tag{4}$$

where

$$Z^0 = \begin{cases} 1 & \text{if } N_1 \text{ is odd} \\ 0 & \text{if } N_1 \text{ is even} \end{cases} \tag{5}$$

$Z^0 = 1$ means that one dealer has squire position, which means zero inventory in FX market, although he is ready to have open position. He is quoting both buying and selling prices. This dealer's reservation price is the median of all the X_j's.

The highest buying price is called market bid rate. We identify the market bid as the Z^b*th* smallest X_j. Z^b coincides with Z^- if N_1 is even. Otherwise, $Z^b = Z^- + 1$. Hence,

$$Z^b = Z^- + Z^0 \tag{6}$$

Solving equation (3) and (4), $Z^b = \frac{1}{2}(N_1 + Z^0)$. By investigating characteristics of Z^b, we can analyze volatility of the market's bid rate and hence can approximate volatility of transaction price.

Next we introduce demand and supply from the economy's fundamentals. They take a form of dealer's retail transactions. As dealers have retail transactions asynchronously, dealers have reverse transactions in the wholesale market. Let $R_d(t)$ and $R_s(t)$ be retail demand and supply from the fundamentals. They are aggregated across the dealers. They are accumulative from the morning until time t. Let $R(t)$ be excess demand defined as

$$R(t) = R_d(t) - R_s(t) \tag{7}$$

The dealers as a whole absorb this excess demand. At any given time, dealers' net position becomes equal to excess demand; $Z^- - Z^+ = R$, where we suppressed time t. Accounting equations are now as follows.

$$Z^- + Z^+ + Z^0 = N_1 \tag{8}$$
$$Z^- - Z^+ = R \tag{9}$$

Then the index for the market bid is now;

$$Z^b = \frac{1}{2}(N_1 + R + Z^0) \tag{10}$$

2.3 Arrival Process of Order Flow

2.3.1 Revisions of Expectations

Order flow takes a form of hitting the market bid or ask. In addition, we also include the following case as order flow arrival although it does not hit market rate: "Dealer enters state 1, picks reservation price. It turns out to be between the current bid and ask. He quotes bid and ask." We assume that bid/ask spread is negligible, if quoted by the same dealer. This kind of arrival of order flow is necessarily the case such that Z^0 changes from 0 to 1.

The order flow is generated by two sources. The first source is dealers' revisions of expectations. The second source is retail transactions. We describe the process of revision of expectations as a combination of a process of switching between two states and a process of choosing reservation price. The *j*th dealer has a random variable $I_j(t)$ of equation (1). It start with $I_j(0) = 0$ for all of dealers. The sojourn time s in state *i* has an exponentially distribution. Its parameter is θ_i. When I_j changes value from 0 to 1, *i.e.* entering state 1, he picks up his reservation price, X_j. This is a random variable. It is uniformly distributed over unit interval. When he leaves state 1, *i.e.* when I_j switching to 0, he abandons X_j. When he picks X_j, he compares it with market bid and ask. If $X_j > ask$ then he hits ask. He buys spot FX at ask. He comes to have long position. Then he submits his reservation price as his selling price. He expects transaction price will reach his selling price. He waits with this limit order. If $X_j < bid$ when entering state 1,then he hits bid. He has short position and waits with his limit order. His buying price is equal to his reservation price. If $bid < X_j < ask$, then he wouldn't hit neither bid or ask. Instead he quotes his reservation price as both of bid and ask. When he leaves state 1, he abandons his X_j and corresponding position. If he has long or short position then, he hits market bid or ask to close position. Thus switching between states give rise to the generation of ordered flow. The switching process of I_j generates arrivals and X_j process sorts it out between buying and selling.

2.3.2 Retail Transactions

Retail demand and supply $R_d(t)$ and $R_s(t)$ are accumulative quantities retail customers bought from and sold to dealers until time t. They are sums over all n_d of dealers. They have Poisson distributions with parameter λ_d and λ_s. Constructions of $R_d(t)$ and $R_s(t)$ are as follows.

Dealer's objective is daily profit maximization. He has retail customers. No marketing effort is made. He trades with them anytime they want during business hours. His profit is constant per transaction. Their arrivals constitute two Poisson processes; one for customer's buying and the other selling. Quantity of a retail arrivals is one transaction unit. These retail arrivals immediately change into Poisson arrivals at the market. Such conversion is due to the following assumption (p1) to (p3). (p1)" Dealer is risk neutral." (p2)" There is a restriction on the maximum size of position; one transaction unit." (p3)"Exceeding the restriction on the position due to retail transaction is allowed but only for a moment." If he is in state 1, he must have constructed open position. Because of (p1), the open position must be at its maximum. So (p2) must have been binding. Meanwhile anytime he may have a retail transaction. Had this occurred, he counterbalances the retail transaction by hitting the market bid or ask. This is by (p3). The retail arrivals become Poisson arrivals in the wholesale market by the immediate counterbalancing. If dealer is in state 0, he does not want to assume open position. By this reason, he counterbalances retail transaction. Poisson arrivals pass through to the market. Aggregated across dealers, Poisson arrivals are also Poisson. Thus R_d and R_d are Poisson variables.

Excess demand R is defined as equation(7). Dealers as a whole absorb this excess demand. We introduce an assumption on the value of R. Namely, R cannot be bigger than dealers' aggregate open position; for a given N_1,

$$-N_1 \leq R \leq N_1 \tag{11}$$

If $R = N_1$, then it means that all the dealers in state 1 have short positions.

2.4 Transitions between Market States

2.4.1 Transition Intensities

Combinations of N_1 and R constitute a set of states of the market. Let $\omega(n, r)$ be a state of the market such that $N_1 = n$ and $R = r$. For each N_1, R takes value such that $-N_1 \leq R \leq N_1$. And $N_1 = 0, 1, \ldots, n_d$. Hence, there are $n_d(n_d + 1)$ of ω's. Processes of N_1 and R determine transition intensities between ω's. Let Q be infinitesimal operator defined on them. It is a $n_d(n_d + 1) \times n_d(n_d + 1)$ matrix. It lists transition of intensities. We construct Q matrix as follows. For a given $N_1 = n$, we consider $(2n + 1) \times (2n + 1)$ matrix. Let A_n be this matrix. Its element $A_n(j, k)$ corresponds to $\omega(n, k - n - 1)$, for

$j = 1, 2, \ldots, 2n + 1$. Its value is transition intensity from and to $\omega(n, k - n - 1)$. For a given *j*th row, for $1 < j < 2n + 1$,

$$A_n(j, j-1) = \lambda_s \tag{12}$$
$$A_n(j, j) = -\{(n_d - n)\theta_0 + n\theta_1 + \lambda_d + \lambda_s\} \tag{13}$$
$$A_n(j, j+1) = \lambda_d \tag{14}$$

And because of inequality (11), $A_n(1, 1) = -\lambda_d$ and $A_n(2n + 1, 2n + 1) = -\lambda_s$. The diagonal elements of A_n are negative. Negative value means intensity to exit that state. We put A_n as sub-matrices of Q so that

$$Q = \begin{pmatrix} A_0 & & \ldots & 0 \\ 0 & A_1 & \ldots & 0 \\ & & \ldots & \\ 0 & & \ldots & A_{n_d} \end{pmatrix} \tag{15}$$

Matrix Q's sum of the row is 0. The *j*th row of A_n has positive entries outside of A_n in matrix Q.

2.4.2 Transition Probabilities

For a given infinitesimal operator Q, we can obtain transition probability matrix $P(t)$ by solving Kolmogrov's backward equation:

$$P'(t) = QP(t) \tag{16}$$

The solution is given by

$$P(t) = e^{Qt} \tag{17}$$

Exponential notation of matrix implies that $e^{Qt} = I + Qt + \frac{1}{2}Q^2t^2 + \frac{1}{3!}Q^3t^3 + \ldots$. Equation (17) converges to stationary probabilities as $t \to \infty$.

2.5 Expected Change in a Unit Time Interval

2.5.1 Distributions of Market Bid Rates

We have three finite intervals to define reservation prices. Reservation price X_j is uniformly distributed over unit interval. From X_j we construct Y_j and C_j as follows.

$$Y_j = J(X_j) \text{ such that } J'(x) > 0, \tag{18}$$
$$J(0) = 0, \text{ and } J(1) = 1$$
$$C_j = c_0 + c_1 Y_j \text{ where } c_0, c_1 > 0 \tag{19}$$

In other words, an inverse function of $J(x)$ is Y_j's (accumulative) distribution function. We denote it as $H(y)$. Random variables X_j where $j = 1, 2 \ldots n_d$ are *i.i.d.*.So are Y_j and C_j. We call their supports Q, S and C intervals; Q for quantile, S for standardized, C for currency denomination. Let X be bid rate on Q interval. Similarly, Y and C be bid rates on S and C intervals. Notations are summarized in table 2.5.1. Random variable C is what we observe empirically. Its stochastic properties boil down to those of Y's. They are what we investigate.

Table 1. Bid Rates on Three Intervals

reservation price	bid rate	support
X_j	X	Q interval; $[0, 1]$
Y_j	Y	S interval; $[0, 1]$
C_j	C	positive finite C interval; $[c_0, c_0 + C_1]$

The bid rate is Z^b*th* reservation price from the smallest. If $N_1 + R = 2\kappa + 1$, for positive integer κ, then $Z^b = \kappa$. The random variable X has Beta distribution. Its density is given by

$$f(x) = \frac{\Gamma(n_d + 1)}{\Gamma(n_d - \kappa + 1)\Gamma(\kappa)} x^{\kappa - 1}(1 - x)^{(n_d - \kappa + 1) - 1} \tag{20}$$

This is Beta density$(\kappa, n_d - \kappa + 1)$. If $N_1 + R = 2\kappa$, then density of bid rate is also given by equation (20).

Next, we calculate moments of Y. We express the density for Y_j as a sum of two uniform distributions;

$$h(y) = \alpha h_1(y) + (1 - \alpha)h_2(y) \tag{21}$$

where $\alpha > 0$. The terms on RHS are given by

$$h_1(y) = 1 \qquad \text{for } y \in [0, 1] \tag{22}$$
$$h_2(y) = \frac{1}{\beta_2 - \beta_1} \qquad \text{for } y \in [\beta_1, \beta_2] \tag{23}$$

where $1 > \beta_2 > \frac{1}{2} > \beta_1 > 0$. Then we obtain distribution function for Y_j; $H(y)$.

$$\mathrm{H}(y) = \begin{cases} \alpha y & \text{for } y \in [0, \beta_1] \\ \alpha y + (1 - \alpha)\frac{y - \beta_1}{\beta_2 - \beta_1} & \text{for } y \in [\beta_1, \beta_2] \\ \alpha y + 1 - \alpha & \text{for } y \in [\beta_2, 1] \end{cases} \tag{24}$$

Let $J(x)$ be an inverse of $\mathrm{H}(y)$ such that, for $0 \le x \le 1$,

$$J(x) = \alpha J_1(x) + (1 - \alpha)J_2(x), \tag{25}$$

where terms on RHS are given by $J_1(x) = \frac{1}{\alpha}x$ for $x \in [0, 1]$ and

$$J_2(x) = \begin{cases} \frac{1 - \alpha}{(\beta_2 - \beta_1)\alpha + 1 - \alpha}(-\frac{1}{\alpha}x + \beta_1) \\ \quad \text{for } x \in [\alpha\beta_1, \alpha\beta_2 + 1 - \alpha] \\ 0 \quad \text{otherwise} \end{cases} \tag{26}$$

Then we can calculate $E[J(X)]$. Since X has Beta distribution (20), $E[J_1(X)] = \frac{\kappa}{\alpha(n_d + 1)}$. And $E[J_2(X)]$ is obtained as a sum of incomplete beta integrals.

$$E[J_2(X)] = \int_{\alpha\beta_1}^{\alpha\beta_2 + 1 - \alpha} J_2(x)f(x)dx \tag{27}$$

2.5.2 Expected Price Change by Transition

We consider price change during unit time interval. Let τ be this interval. We like to measure volatility by the following;

$$\mathrm{Var}\,[J(X(\tau)) - J(X(0))]\,. \tag{28}$$

However, $X(\tau)$'s density depends on a market state at τ. So does $X(0)$. It is not tractable. Instead we calculate an average of $\left(E[J(X(\tau))|\omega(m,l)] - E[J(X(0))|\omega(n,r)]\right)^2$. We use it to approximate equation (28). Then we derive attributable factors' effects on volatility.

Let $\Pr(m,l\,|n,r)$ be transition probability from $\omega(n,r)$ into $\omega(m,l)$ during τ. Each $\omega(n,r)$ has a density for the market bid and hence, $\mathrm{E}[J(X)|\omega(n,r)]$. Let $\mu(n,r)$ be this expected value. We calculate $\mu(m,l)-\mu(n,r)$. We approximate precise expected price change by this difference of expected values. Let $\Delta_1(n,r)$ and $\Delta_2(n,r)$ be as follows;

$$\Delta_1(n,r) \equiv \sum_{m,l} \Pr(m,l\,|n,r)\,(\mu(m,l)-\mu(n,r)) \tag{29}$$

$$\Delta_2(n,r) \equiv \sum_{m,l} \Pr(m,l\,|n,r)\,(\mu(m,l)-\mu(n,r))^2 \tag{30}$$

Market state $\omega(n,r)$ has the stationary probability, which is the limit of equation (17). It is probability to be there on average for t large. Let $\pi(n,r)$ be that stationary probability. We approximate variance of interval's price change by the following quantity.

$$\Delta_v \equiv \sum_{n,r} \pi(n,r)\left\{\Delta_2(n,r) - (\Delta_1(n,r))^2\right\} \tag{31}$$

2.5.3 Variability of Volatility

In order to obtain qualitative conclusions about Δ_v, we assume the following three conditions.

(a1) $\lambda_d = \lambda_s$: $E[R]=0$
(a2) $\beta_2 = 1-\beta 1$ where $0<\beta_1<0.5$:
$h(y)$ of equation(21), is symmetric around 0.5
(a3) $\alpha\beta_1 = \epsilon$, where $0<\epsilon<0.5$

Random variable R is difference of two Poisson variables as defined by equation (7). By having (a1), $\omega(n,r)$'s come to have positive stationary probabilities. If $r \neq 0$, then $\pi(n,r) = \pi(n,-r)$ holds. By having (a2), for $r \neq 0$, density functions for X corresponding to $\omega(n,r)$ and $\omega(n,-r)$ become symmetric around 0.5. Then $\mu(m,l)-\mu(n,r) = -\{\mu(m,-l)-\mu(n,r)\}$ holds. Then equation (29) becomes zero. Hence, equation (31) becomes simpler; $\Delta_v = \sum_{n,r}\pi(n,r)\Delta_2(n,r)$. By having (a3), besides (a2), for any value of α, kinks of $J(x)$ takes place at $x=\epsilon$ and $1-\epsilon$. And α is only variable in equation (27).

For small enough ϵ, we have $\alpha \approx 0$. In this case, $h(y)$ resembles to a spike on 0.5 on S-interval. In most values of $E[J(X)|\omega(n,r)]$ is very close to 0.5. Hence, $\Delta_2(n,r)$ is calculated within this concentrated range. The other extreme is large α for small ϵ. In this case $h(y)$ looks u-shaped. For a given N_1, as R changes sign, $E[J(X)|\omega(n,r)]$ jumps almost between two ends of S-interval. As α increases from $+0$, the value of equation (31) increases. Higher α means more heterogeneous expectations. Hence, we can conclude that, as reservation prices become more heterogeneous, bid rate volatility on S-interval increases.

Random Variable Y is the bid rate on S-interval. This is mapped onto C-interval as defined by equation (19). The bid rate comes to have larger volatility as c_1 increases. Larger c_1 also implies that reservation prices are more heterogeneous.

Random variable R is difference of two Poisson variables. By the central limit theorem, the difference of two Poisson variables converges to a normal distribution (Johnson, Kotz & Kemp [5]). Our R has both tails truncated; condition of equation (11); $-N_1 \le R \le N_1$. Still, the approximation works for our purpose. For a given state of the market $\omega(n,r)$, as λ_d and λ_s increase, transition probabilities from $R=r$ to more different values l of $R=l$ increases. As $|l-r|$ increases, so does $(\mu(m,l)-\mu(n,r))^2$ of Δ_2 . So does Δ_v.

Expectation revision cycle also changes volatility. As the cycle becomes smaller, transitions between $\omega(n,r)$'s with different value of $N_1 = n$ increase. For a given unit time interval, probabilities to reach different values of n increases. Random number N_1 comes to have a larger variance. Similarly to R, N_1's larger variance increases volatility.

2.6 Consistency of Model with Regard to Expectation Formation

Our model presupposes the heterogeneous reservation prices. Its price determination process allows the heterogeneity persist. The representative *j*th dealer estimates the distribution of C_j's. Also he estimates time path of $R(t)$. He may know specific $\Delta R(t)$; for example, multinational corporation's substantial transactions. Such changes in the arrivals means a peak or trough on $E[R(t)]$. By taking advantage of expected extrema as many as possible, his profits increases. So his present choice of position depends on the first local extremum of $E[R(t)]$. We assume he is risk neutral. He expects capital gain by reversing position at such a first local extremum. He substitutes model's bid rate for expected transaction price. Thus the first local extremum becomes his reservation price. Meanwhile, he observes only his own retail transactions. From them he estimates aggregate value $R(t)$. Also he estimates C_j's distribution. This is not directly observable. Because of these limitations, dealers have heterogeneous estimates on them. The heterogeneous reservation prices persist.

Table 2. **Shorter Expectation Revision Cycle**

θ_0 and $\theta_1 \uparrow$	
volume	volatility
+	+
cycle effect	volume effect

Table 3. **Smaller** $E[N_1]$

$\frac{\theta_0}{\theta_0+\theta_1} \downarrow$	
volume	volatility
depending on $\frac{1}{\theta_0}+\frac{1}{\theta_1}$	+ thickness effect

Table 4. **Larger Retail Transactions**

λ_d and $\lambda_s \uparrow$	
volume	volatility
+	+
	volume effect

Table 5. **More Heterogeneous Expectations**

$\alpha \uparrow 2$ and/or $c_1 \uparrow$	
volume	volatility
no change	+
	heterogeneity effect

3 Conclusion

Changes in model's parameter values give rise to expected trading volume and volatility. They are summarized in Table 2 to 5. We can explain empirical observations as combined results of these effects. As is explained in the following, volume /volatility correlation seems non-stationary. It is difficult for existing approaches like in Lyons[1] (p.146) to handle such variations.

Wada [6] reports statistically significant effects of volume on volatility, using spot USD/JPY tick data. The data cover from June '95 to April '96 in Tokyo. For every five minute interval, he regresses volatility on volume. Except for lunch time, the regression coefficients are all positive. They are also statistically significant except for a few in the afternoon. Lunch time period is different; very low volume and higher volatility. Thus volume and volatility tend to have positive correlation. However, it varies with an intraday pattern.

Our explanation: Demand and supply from macro economy R_d and R_s come faster in the morning. Also due to more uncertainty with regard to daily totals of R_d and R_s, reservation prices spread wider in the morning. Volume and heterogeneity effects work together. As for lunch time, R_d and R_s are smaller and revision cycle is longer. Volume is reduced. Meanwhile thickness effect of smaller N_1 makes volatility higher. The latter effect more than cancels the former. As a result, higher volatility can accompany small volume for lunch time.

The existing microstructure literature has not been satisfactory with regard to handling of the lack of apparent benchmark values in case of continuous auctions of financial assets. Planning horizon can be any length to seek capital gains. Defining liquidation value is not truism. This theoretical issue is coupled with self prophecy aspect of price perspectives. Since feedback from macro economy is weak and hence price perspectives are very influential, a trader has to consider perspective's dispersion among others to have his own. The lack of apparent benchmark value becomes nontrivial to handle. Our model provides alternative approach. We do no rely on the exogenously given liquidity value. Our benchmark value is the first local extremum on the expected time path. It becomes reservation price. Its distribution among dealers determines price and its volatility. Meanwhile exogenous forces of retail transactions also enter the price determination process. Our approach looks promising.

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