On the Existence of Unemployment Equilibria under Wage Rigidity *

Jun Iritani and Ko Sumino

Abstract

This paper presents the existence proof of unemployment equilibria where the aggregate equilibrium is compatible with an individual worker's disequilibrium. The model is a simple two-sector model with unemployment and wage rigidity. In one sector, the wage is determined competitively, while in the other a higher wage is set according to some wage setting rule. Those workers who work in the second sector face with a risk of unemployment. We require in equilibrium that the expected wage in this sector is equal to the competitive wage.

JEL Classification Number : C62, D51

Keywords : Unemployment, General Equilibrium, Wage Rigidity

1 Introduction

One of the main themes in economic theory has been to find a mechanism generating unemployment. The key concepts in the study of unemployment have been price rigidity and rationing of demand and supply.[1)]

---

* Authors are indebted for valuable comments and helpful suggestions to Professors Hiroo Sasaki (Waseda University), Tomoichi Shinotsuka (Otaru University of Commerce), Kiyoshi Kuga (Osaka University), Kenji Yamamoto (Otaru University of Commerce), Hiroaki Nagatani (Osaka University), Ken Urai (Osaka University).

The price rigidity, which plays an important role in the present paper, can be traced back to the pioneering works, such as Benassy (1975) and Younes (1975). They have fixed prices and quantity constraints. Benassy considers the fact underlying unemployment as the one that demands do not necessarily satisfy consumers' budgets. He defines the concept of "effective demand for a commodity" and specifies a rationing scheme which transforms the vector of effective demands to a feasible allocation. Younes, on the other hand, develops a model where consumers' budget constraints are satisfied and their consumption plans are subject to quantity constraints. In his model, unemployment exists in a way that agents can not supply what they want. Malinvaud (1977) considers unemployment as discrepancies between planned and actual trades. His approach is similar to Benassy's in the treatment of unemployment.

In his seminal work, Drèze (1975) represented price rigidities as the restricted variation of prices in intervals of the upper and lower bounds of prices. In addition, he introduces a kind of quantity constraint called a rationing scheme, which is described by upper and lower bounds of net trades. The rationing scheme is generated endogenously in equilibrium, while the price rigidities are given exogenously.

Some contributions to equilibrium theory of unemployment have found new insights by reformulating price rigidities or quantity constraints in the Drèze model. Van Der Laan (1984) considers the case that supply sides only obey quantity constraints and that price bounds may depend on the general price level. Weddepohl (1987) studies bounds for prices and net

1) Recently, the concepts, however, are not much addressd as in the earlier works. The job search theory of the labor market and the theory of dual labor markets have thrown new lights on the theory of unemployment, e.g. Pissarides (1990), Davidson-Martin-Matusz (1988), and Bulow-Summers (1986).

trades as functions of rationing indexes. These models share one feature with that of Younes (1975): Planned activities of buying and selling coincide with actual ones in equilibrium. Their models are similar to Younes'.

Kurz (1982) and Wu (1988) offer another mechanism generating unemployment. Kurz (1982) incorporates a price rigidity and an exogenous price linkage into an exchange economy. Initial endowments may be also unemployed in his model according to some probability law. Wu (1988) works with a stochastic quantity rationing rule. Despite modeling differences, equilibria in both Kurz's and Wu's models have something in common with those of Benassy (1975) and Malinvaud (1977): Planned trades and actual ones may not coincide in equilibrium.

The purpose of this paper is to present a new approach to the theory of unemployment and to show that non-null unemployment is generated endogenously in a simple two sector model. Our equilibrium model contains part of Benassy (1975) and Malinvaud (1977) in the sense that planned trades and actual ones of individual agents may not coincide in equilibrium. Furthermore, our model shares one feature with Younes (1975) and Drèze (1975) in that the sum of planned trades is equal to that of actual ones in equilibrium.

In this paper, we assume a wage setting mechanism in one sector of our economy. The mechanism associates a list of prices and a rate of employment with a wage level of the sector. We call it a wage setting function. This function differs from the price indexation rules in the existing literature in that it incorporates the employment rate of the sector. Our formulation is of some intuitive appeal from the real world point of view.[2)]

2) For instance, the wage rate in the Japanese steel industry is determined by the economic state of the industry indicated by its employment rate and profit level, and so on, as well as the overall state of the economy summarized by the general price level, e.t.c..

Mathematically, our wage setting function is capable of generating price rigidity discussed by Drèze (1975), Van der Laan (1984), and Weddenpohl (1987). The function also allows us to handle the Keynesian relative wage hypothesis, argued by Summers (1988).

The essential structure for the generation of unemployment equilibria in this paper consists in the coexistence of two different wages. The wage set by the wage setting function is higher than the competitive wage. The wage difference caused by the function will generates workers' movements from the law wage sector to the high one. The workers who want to work in the high wage sector may not get jobs. Thus possible equilibria will require that the competitive wage is equal to the expected one. Harris and Todaro (1970) introduce such an equilibrium condition under a dynamic migration setting. The equality condition can describe an equilibrium in a static equilibrium model[3] and it will turn out to be very useful in the section 2. 4. That is, we can establish a relation like Walras law by the condition. By this, the sum of planned trades coincide with that of actual ones in equilibrium, while planned and actual trades of individual agents may be different. In this sense, our equilibrium concept may be regarded as a hybrid of two groups, i. e., Drèze-Younes and Benassy-Malinvaud.

In Section 2 below, we introduce notations and assumptions and explain our model. In Section 3, we establish the existence theorem.

2 The Model

In this section, we shall develop a simple two-sector model with an endogenous unemployment in labor.

---

3) See for example Miyagiwa (1988).

2.1 Assumptions

Assume that there are two commodities and that one sector produces one commodity from capital and labor. The production technology of each sector is linearly homogeneous. Each worker supplies one unit of labor. Let K_0 and L_0 denote the total amount of capital and the number of workers, respectively. K_0 and L_0 are positive constants. We assume that

(P) the production function $F_i : (K_i, L_i) \in R^2_+ \mapsto y_i \in R_+$ is continuous, concave and homogeneous of degree one, and $F_i\ (K_i, L_i) = 0$, if $K_i = 0$ or $L_i = 0$, $i = 1, 2$.[4)]

Every consumer in the economy has an identical utility function $u(x_1, x_2)$ defined on R^2_+. We assume that

(U) the utility function $u(x_1, x_2)$ is continuous, quasi-concave, homothetic and non-decreasing in R^2_+ and strictly quasi-concave and strictly increasing in R^2_{++}. Furthermore, it holds that

$$u(\hat{x}_1, \hat{x}_2) = \inf\{u(x_1, x_2) \mid (x_1, x_2) \in R^2_+\} \text{ if and only if } \hat{x}_1 = 0 \text{ or } \hat{x}_2 = 0.$$

This assumption may sound restrictive, but makes the situation much simple. By this we can obtain the result that the demand for the product gets arbitrary large when the product price goes to zero and when the consumer's income remains positive.

We assume that all the prices but the wage of the first sector are determined competitively. In particular, a competitive wage prevails in the second sector. We also assume a wage setting function that sets the wage

---

4) R_+ (resp. R_{++}) is the set of non-negative (resp. positive) reals. R^n_+ (resp. R^n_{++}) is the n times Cartesian product of R_+ (resp. R_{++}).

of the first sector, which is denoted by w_1, depending on commodity prices p_1, p_2, a competitive wage w_2 of the second sector, a rental of capital r, and an employment ratio ξ of the first sector. Let the wage setting function of the first sector is given by $\Psi(p_1, p_2, w_2, r, \xi)$, which is continuous and defined on $R^4_+ \times [0, 1]$. We do not discuss how a wage negotiation process, which underlies such a wage setting function, could be made and why the wage setting function should be formed in the first sector. We assume that

(WR) the function $\Psi(p_1, p_2, w_2, r, \xi)$ is bounded from below in the sense that $\Psi(p_1, p_2, w_2, r, \xi) \geqq \delta w_2$, for some constant $\delta(\geqq 1)$. Let Δ be a bounded subset of R^3_+ whose generic element is (p_2, w_2, r). There exists a sufficiently large number b depending on the set Δ such that $\Psi(p_1, p_2, w_2, r, \xi) \leqq b$ for any $(p_1, p_2, w_2, r, \xi) \in R_+ \times \Delta \times [0, 1]$.

This assumption implies that the wage of the first sector is bounded when the competitive sector's prices are bounded. The upper and lower bounds of the wage b and δw_2 correspond to the price rigidity discussed by Drèze (1975), Van Der Laan (1984), and Weddepohl (1987). The functional relation that the wage of the first sector depends on the other prices corresponds to the exogenous price linkage discussed by Kurz (1982). Further, we assume that

(H) the wage setting function $\Psi(p_1, p_2, w_2, r, \xi)$ is homogeneous of degree one with respect to (p_1, p_2, w_2, r) for each fixed ξ.

The homogeneity of $\Psi(\cdot)$ implies that the wage of the first sector is indexed to the whole prices. We have recourse to assumption **(H)** to obtain a demand and supply system that is homogeneous of degree zero with respect to prices.

We can find the simplest example of the wage setting function defined above such that

$$\Psi(p_1, p_2, w_2, r, \xi) = \delta w_2.$$

Another example is provided such that

$$\Psi(p_1, p_2, w_2, r, \xi) = (1 - \rho((\xi))w_2 + \rho(\xi)\pi w_2,$$

where $\rho(\xi)$ is a continuous function whose range is [0, 1] and $\pi(> 1)$ is a positive constant. A few comments will be in order on this example. Suppose that negotiations are made with respect to wage determination between the employer and the employee in the first sector and that the functions $\rho(\xi)$ and $1-\rho(\xi)$ represent the employee's and the employer's powers in the wage negotiations, respectively. The employee prefers the high wage πw_2, $\pi > 1$ to the law wage w_2, while the employer oppositely. It is reasonable that the value of $\rho(\xi)$ gets arbitrary small when ξ goes to the vicinity of zero. The power of workers rises as the number of workers increases, that is $\rho(\xi)$ increases as ξ increases as long as the number of employees is small. Finally, $\rho(\xi)$ begins to decrease when ξ exceeds some critical value because the high employment ratio implies that the labor market becomes competitive. And thus we can assume $\rho(\xi)$ is reversely U-shaped.

We can interpret the wage setting function $\Psi(\cdot)$ as a variant of the wage determination according to the relative wage hypothesis. We restrict the class of wage setting functions satisfying

(WR′) the function $\Psi(p_1, p_2, w_2, r, \xi)$ is bounded in the sense that

$$\underline{\delta} w_2 \leqq \Psi(p_1, p_2, w_2, r, \xi) \leqq \overline{\delta} w_2,$$

for some constants $\underline{\delta}$ and $\overline{\delta}$ ($\overline{\delta} \geqq \underline{\delta} \geqq 1$).

It is easy to see that the condition (**WR′**) implies (**WR**). By the condition (**WR′**) together with (**H**), we restrict the ratio of wages w_1/w_2 in a closed interval. And thus we can regard (**WR′**) as an expression of the Keynesian relative wage hypothesis when $\overline{\delta} = \underline{\delta}$ is the case (see Summers (1988)).

The role of government here is to give unemployment benefits to the unemployed. Suppose that the amount of unemployment benefits is indexed to the competitive wage such as αw_2 where α is a constant and $0 < \alpha < 1$. The expenditure is financed by a revenue raised by the capital and the labor taxes on producers. Let t_K and t_L denote the capital tax rate and the labor tax rate, respectively.

Let Q_i denote the set of all pairs of the capital and the labor inputs (k_i, λ_i) by the use of which the i-th sector can produce one unit of the i-th product. We assume that each sector minimizes its production cost. Let $k_i(T_L w_i, T_K r)$ and $\lambda_i(T_L w_i, T_K r)$ be capital and labor demand functions to produce one unit of the i-th product respectively, where w_i and r are factor prices and $T_L := 1 + t_L$ and $T_K := 1 + t_K$.[5] We assume Q_i has enough curvature so that the correspondences $k_i(T_L w_i, T_K r)$ and $\lambda_i(T_L w_i, T_K r)$ are single valued, $i = 1, 2$.

The product price of the i-th sector is defined as

$$p_i := T_K r k_i(T_L w_i, T_K r) + T_L w_i \lambda_i(T_L w_i, T_K r), \; i = 1, 2.$$

We assume that

(**B**) for any sequence $\{(w_2^m, r^m) \mid m = 1,2,\ldots\}$, whose limit is (w_2, r), the following conditions are satisfied

(i) if $w_2 > 0$ and $r = 0$, then $k_2(T_L w_2^m, T_K r^m) \to \infty$ as $m \to \infty$,

5) The symbol " := " implies that the left hand side is defined by the right hand side.

(ii) if $w_2 = 0$ and $r > 0$, then $\lambda_2(T_L w_2^m, T_K r^m) \to \infty$ as $m \to \infty$,

(iii) if $w_2 = 0$ or $r = 0$, then $p_2(T_L w_2^m, T_K r^m) \to 0$ as $m \to \infty$.

The assumption **(B)** implies that the unit isoquant Q_2 is unbounded and that Q_2 has two axes as asymptotes.

Let y_i denote the i-th sector output level. Let $K_i(T_L w_i, T_K r, y_i)$ and $L_i(T_L w_i, T_K r, y_i)$ be demands for factors of the i-th sector and be defined such that

$$K_i(T_L w_i, T_K r, y_i) := y_i k_i(T_L w_i, T_K r),\ i = 1, 2,$$
$$L_i(T_L w_i, T_K r, y_i) := y_i \lambda_i(T_L w_i, T_K r),\ i = 1, 2.$$

Let I denote the income of a consumer. Utility maximizing behavior of a consumer such as

$$\max u(x_1, x_2) \quad \text{subject to} \quad p_1 x_1 + p_2 x_2 \leqq I,$$

leads us to demands for commodities $x_i(p_1, p_2, I)$, $i = 1,2$.

2.2 Equilibrium Conditions

A worker chooses in which sector to work by comparing the expected utilities he would gain. Let prices and the first sector employment ratio be (p_1, p_2, w_1, w_2) and ξ, respectively. The worker who wants to work in the first sector faces the probability of employment ξ. The expected utility u^1 from working in the first sector is

$$u^1 = \xi u(x_1(p_1, p_2, w_1 + R), x_2(p_1, p_2, w_1 + R))$$
$$+ (1 - \xi) u(x_1(p_1, p_2, \alpha w_2 + R), x_2(x_1(p_1, p_2, \alpha w_2 + R)),$$

where R is the income from capital. The utility u^2 from working in the second sector is

$$u^2 = u(x_1(p_1, p_2, w_2 + R), x_2(p_1, p_2, w_2 + R))$$

Note that the utility function of every worker, here, is identical and thus all workers want to work in the first (resp. second) sector if the situation $u^1 > u^2$ (resp. $u^1 < u^2$). If every worker is indifferent to the choice of the sector for him to work, then there exists a possible equilibrium which satisfies the condition $u^1 = u^2$.

Instead of these expected utility formula, we assume for simplicity that the workers supply their labor to the sector that offers the higher expected wage. The equilibrium condition we use in this paper then is the following equality,

$$\xi w_1 + (1 - \xi)\alpha w_2 = w_2.$$

Moreover, one should also note that both two conditions $\xi w_1 + (1 - \xi)\alpha w_2 = w_2$ and $u^1 = u^2$ are equivalent when the utility function is homogenous of degree one. This implies that the workers are indifferent between the expected and competitive wages in the sense that their utility depends on their expected incomes. This may be a slightly different from the usual risk neutral presentation.

We shall discuss the boundedness of the price setting function after defining unemployment equilibria and give an example where no reasonable equilibria exist when $\Psi(\cdot)$ is not bounded from above.[6)]

The labor market equilibrium condition is stated as

6) Note that the equality does not hold if $w_1 (= \Psi(\cdot))$ is strictly less than w_2 for any (p_1, p_2, w_2, r, ξ), this relates to the boundedness of the wage setting function.

$$\frac{L_1(T_L w_1, T_K r, y_1)}{\xi} + L_2(T_L w_2, T_K r, y_2) = L_0.$$

The first term in the left hand side of the above condition is the sum of the employed and unemployed workers in the first sector. Note that there are no rationing schemes of labor in the equilibrium condition defined above. Therefore the questions of who are unemployed or employed among workers are not involved here. In this sense, we do not discuss the micro structure of labor market, but concentrate ourself to the aggregate labor balance. We can rewrite this into a stochastic equilibrium formula as follows

$$L_1(T_L w_1, T_K r, y_1) = \xi(L_0 - L_2(T_L w_2, T_K r, y_2)).$$

The right hand side of the equation is the expected supply of labor to the first sector and the left hand side is the labor demand. That is to say, this is a stochastic equilibrium, which is similar to Wu (1990).

Let the total income of workers be given by $I(w_1, w_2, r, \xi)$. We have

$$\begin{aligned} I(w_1, w_2, r, \xi) &:= rK_0 + w_2 L_0, \text{ if } \xi w_1 + (1-x)\alpha w_2 \leqq w_2, \\ &:= rK_0 + (\xi w_1 + (1-\xi)\alpha w_2) L_0, \\ &\quad \text{if } \xi w_1 + (1-\xi)\alpha w_2 > w_2. \end{aligned}$$

The assumption (U) of homotheticity of the utility function enables us to write total demand X_i for the i-th commodity as a function of the total income and two commodity prices, i. e.

$$X_i(p_1, p_2, w_1, w_2, r, \xi) := x_i(p_1, p_2, I(w_1, w_2, r, \xi)), \; i = 1,2.$$

Finally, we consider the government budget. A triplet (T_K, T_L, α) is called a *tax scheme*. A tax scheme (T_K, T_L, α) is called *feasible* if the government budget is balanced, i. e.,

$$\alpha w_2 L_1(T_L w_1, T_K r, y_1)\frac{1-\xi}{\xi} = t_K r(K_1(T_L w_1, T_K r, y_1)$$

$$+ K_2(T_L w_2, T_K r, y_2)) + t_L(w_1 L_1(T_L w_1, T_K r, y_1) + w_2 L_2(T_L w_2, T_K r, y_2)).$$

The right hand side is the tax revenue. The left hand side is the government transfer to the unemployed since the number of unemployed workers is $L_1(T_L w_1, T_K r, y_1)(1-\xi)/\xi$ when the labor market is balanced.

2.3 Definition of unemployment Equilibria

We are now fully equipped to define unemployment equilibria.

Definition A triplet of an allocation $(X_1^*, X_2^*, y_1^*, y_2^*, K_1^*, K_2^*, L_1^*, L_2^*)$,[7] prices $(w_1^*, w_2^*, r^*, p_1^*, p_2^*)$ and an employment ratio of the first sector ξ^* $(0 < \xi^* < 1)$ is an unemployment equilibrium relative to a wage setting function $\Psi(\cdot)$ and a tax scheme (T_L^*, T_K^*, α) when the following conditions are satisfied.

(a) **[market equilibrium]**

(1) $L_1(T_L^* w_1^*, T_K^* r^*, y_1^*)/\xi^* + L_2(T_L^* w_2^*, T_K^* r^*, y_2^*) = L_0,$

(2) $K_1(T_L^* w_1^*, T_K^* r^*, y_1^*) + K_2(T_L^* w_2^*, T_K^* r^*, y_2^*) = K_0,$

(3) $X_i(p_1^*, p_2^*, w_1^*, w_2^*, r^*, \xi^*) = y_i^*, i = 1,2,$

(b) **[price setting and wage expectation]**

(4) $w_1^* = \Psi(p_1^*, p_2^*, w_2^*, r^*, \xi^*),$

(5) $\xi^* w_1^* + (1-\xi^*)\alpha w_2^* = w_2^*,$

(c) **[government budget]**

(6) the tax scheme (T_L^*, T_K^*, α) is feasible.

---

7) The values of X_1^*, X_2^*, K_1^*, K_2^*, L_1^*, and L_2^* are those of corresponding functions evaluated at (y_1^*, y_2^*, ξ^*), $(w_1^*, w_2^*, r^*, p_1^*, p_2^*)$ and (T_K^*, T_L^*).

Equation (1) implies that employed and unemployed workers sum into L_0, which illustrates an equilibrium statement incorporating unemployment. The capital market clearing condition is (2). The equalities of demands and supplies for commodities are stated in (3). Equation (4) is the first sector wage setting. The equality between the competitive wage and the expected wage is described by (5). The statement (6) characterizes the government budget constraint.

One characteristic feature in the definition is that the employment ratio ξ^* is endogenously determined. That is, workers have a belief on the employment ratio in advance which coincides with the resulting ratio. There, we assume some belief forming mechanism similar to the one in the rational expectation theory.

2.4 Walras law

Walras law is one of the usual building blocks in the existence proof of general equilibrium. Our model so far developed describes a disequilibrium. In addition, there are two wages w_1 and w_2. And thus we can not expect Walras law holds. We can, however, establish a quasi-Walras law when we restrict wages appropriately. This makes the existence proof much easy.

Let $((y_1, y_2), (w_2, r), \xi)$ be any element of $R^2_{++} \times S^\circ \times (0, 1]$, where S° is the relative interior of the set $S := \{(w_2, r) \in R^2_+ \mid w_2 + r = 1\}$. Let $\tilde{w}_1$ be a function of w_2 and ξ, such that

$$\tilde{w}_1 := \frac{w_2}{\eta(\xi)}, \ \eta(\xi) := \frac{\xi}{1-\alpha(1-\xi)}.$$

Replace every w_1 with $\tilde{w}_1$ and we can confine ourselves to the state that the expected wage of the first sector, which is $\xi\tilde{w}_1 + (1-\xi)\alpha w_2$, coincides with competitive wage w_2. In this circumstance, we can regard the total income as a function of w_2 and r, such that

$$\tilde{I}(w_2, r) := w_2 L_0 + r K_0.$$

Let $\tilde{X}_i(p_1, p_2, w_2, r) := x_i(p_1, p_2, \tilde{I}(w_2, r))$, $i = 1,2$. Let E_K and E_L be the excess demands for capital and labor, respectively. We now

$$E_K := K_1(T_L w_2/\eta(\xi), T_K r, y_1) + K_2(T_L w_2, T_K r, y_2) - K_0,$$
$$E_L := L_1(T_L w_2/\eta(\xi), T_K r, y_1)/\xi + L_2(T_L w_2, T_K r, y_2) - L_0.$$

For simplicity, we write $\tilde{X}_i$, $\tilde{K}_1$, $\tilde{K}_2$, $\tilde{L}_1$ and $\tilde{L}_2$ instead of $\tilde{X}_i(p_1, p_2, w_2, r)$, $\tilde{K}_1(T_L w_2/\eta(\xi), T_K r, y_1)$, $\tilde{K}_2(T_L w_2, T_K r, y_2)$, $\tilde{L}_1(T_L w_2/\eta(\xi), T_K r, y_1)$, and $\tilde{L}_2(T_L w_2, T_K r, y_2)$. By consumers' budget constraints and the linear homogeneity of production functions, we have

$$\begin{aligned} p_1(\tilde{X}_1 - y_1) &+ p_2(\tilde{X}_2 - y_2) + w_2 E_L + r E_K \\ &= p_1\tilde{X}_1 + p_2\tilde{X}_2 - w_2 L_0 - r K_0 \\ &\quad - (p_1 y_1 - T_L w_2 \tilde{L}_1/\eta(\xi) - T_K r \tilde{K}_1) \\ &\quad - (p_2 y_2 - T_L w_2 \tilde{L}_2 - T_K r \tilde{K}_2) \\ &\quad + w_2\tilde{L}_1/\xi - T_L w_2 \tilde{L}_1/\eta(\xi) + w_2\tilde{L}_2 - T_L w_2 \tilde{L}_2 - t_K r(\tilde{K}_1 + \tilde{K}_2) \\ &= \alpha w_2 \tilde{L}_1 (1-\xi)/\xi - t_L(\tilde{w}_1\tilde{L}_1 + w_2\tilde{L}_2) - t_K r(\tilde{K}_1 + \tilde{K}_2). \end{aligned}$$

Finally, let E_G be the net revenue of the government defined as

$$E_G := t_L(\tilde{w}_1\tilde{L}_1 + w_2\tilde{L}_2) + t_K r(\tilde{K}_1 + \tilde{K}_2) - \alpha w_2 \tilde{L}_1(1-\xi)/\xi .$$

We have then,

$$p_1(\tilde{X}_1 - y_1) + p_2(\tilde{X}_2 - y_2) + w_2 E_L + r E_K + E_G = 0.$$

This is our Walras law. Note that the relation holds when $w_1 = \tilde{w}_1$.

2.5 Uniform Boundedness of the Price Setting Function

We discuss, here, analytically the reason why the price setting function must be uniformly bounded from above. We present an example in which there are no reasonable equilibria under a price setting function that is not uniformly bounded from above. Let the wage setting function be

$$\Psi(p_1, p_2, w_2, r, \xi) := vw_2/\xi,$$

where v is a positive constant which is strictly greater than unity. The function is not bounded from above and does not satisfy the condition **(WR)**.

Substituting w_1 in (5) with this $\Psi(\cdot)$, we have

$$vw_2 + (1 - \xi)\alpha w_2 = w_2.$$

Assuming w_2 is positive in reasonable equilibria, we obtain

$$\xi = \frac{v-1+\alpha}{\alpha} > 1.$$

This is, however, impossible. And thus we need the uniform boundedness of the wage setting function unless the competitive wage is nil.

3 Existence Proof

Our main result is stated as follows:

Theorem Suppose that the wage setting function $\Psi(\cdot)$ satisfies **(H)** and **(WR)** with $\delta > 1$. If the assumptions **(P)**, **(U)**, and **(B)** hold, then there exist the unemployment equilibria relative to a wage setting function and a tax scheme.

We can immediately obtain the following lemma.

Lemma Under the assumption **(U)**, for any (p_1, p_2, I) it holds that

(PD) if $I > 0$ and $p_i > 0$, $i = 1, 2$, then $X_i(p_1, p_2, I) > 0$, $i = 1,2$.

For any sequence $\{(p_1^m, p_2^m, I^m \mid m = 1, 2,...\}$ in R^3_{++} whose limit is (p_1, p_2, I), we have

(BD) if $p_i = 0$ and $I > 0$, then $X_i(p_1^m, p_2^m, I^m) \to \infty$ as $m \to \infty$.[8)]

Note that all the tax rates can not be determined exogenously, since the rate α is a given constant and tax rates are determined in (6). We introduce a relation such as $t_K = tt_L$, where t is a non-negative constant. This is not a restrictive assumption at all but is a convention to determine two tax rates by a single variable. Below, we use an auxiliary endogenous variable τ in an interval (0, 1], and define $T_L := 1/\tau$ and $t_L := T_L - 1$.

It is easy to see that for any given $\alpha (1 > \alpha \geqq 0)$, there exists a sufficiently small positive number e (<1) such that $(1-\alpha)/\tau - 1 > 0$ for any τ satisfying $e \geqq \tau > 0$. By this property we can restrict possible tax rates to

8) (PD) is obvious. To prove (BD), first we consider a case that both prices, p_1 and p_2 are zero. By consumers' budget constraints, we have

$$p_1^m X_1^m + p_2^m X_2^m = I^m.$$

It is easy to see that at least one of X_1^m and X_2^m goes to infinity. Second, we consider a case that either p_1 or p_2 is zero. Without loss of generality, we suppose that $p_1^m \to 0$. Soppose that the sequence (X_1^m, X_2^m) have a upper bound. We can assume that a subsequence of $\{(X_1^m, X_2^m)\}$ converges to a point (X_1, X_2). We choose a point (X_1+1, X_2) such that $u(X_1, X_2) < u(X_1+1, X_2)$. Given sufficiently small $\varepsilon > 0$, we have

$$p_1^m(X_1+1-\varepsilon) + p_2^m(X_2-\varepsilon) = p_1^m X_1 + p_2^m X_2 + p_1^m(1-\varepsilon) - p_2^m \varepsilon.$$

The value of the right hand side is less than I^m for m sufficiently large. For every 2ε neighborhood of $(X_1 + 1, X_2)$, there is a point $(X_1+1-\varepsilon, X_2-\varepsilon)$ that is in the budget budget set for sufficiently large m. Since ε can be taken arbitrarily small and since $u(\cdot)$ is continuous, it follows that $u(X_1^m, X_2^m) < u(X_1+1-\varepsilon, X_2-\varepsilon)$. The consumption bundle $(X_1+1-\varepsilon, X_2-\varepsilon)$ is in the budget set for (p_1^m, p_2^m, I^m). This is a contradiction.

a compact set :

$$\{(t_L, t_K) \in R^2_+ \mid t_K = tt_L, t_L = 1/\tau - 1, \tau \in [e, 1]\}$$

when the wage in the fist sector is $\tilde{w}_1$. In fact, let τ satisfy $0 < \tau \leqq e$. Define $t_L = 1/\tau - 1$, $t_K = tt_L$. Suppose $w_1 = \tilde{w}_1$. Then we can see that for any $((y_1, y_2), (w_2, r), \xi)$ in $R^2_{++} \times S^\circ \times (0, 1]$,

$$\begin{aligned}
\text{tax revenue} - \text{government expenditure} \\
&\geqq t_L \tilde{w}_1 \tilde{L}_1 - \alpha w_2 \tilde{L}_1 (1 - \xi)/\xi \\
&= \{t_L(1 - \alpha(1 - \xi)) - \alpha(1 - \xi)\} w_2 \tilde{L}_1/\xi \\
&> \{t_L(1 - \alpha) - \alpha\} w_2 \tilde{L}_1/\xi \\
&= \{(1 - \alpha)/\tau - 1\} w_2 \tilde{L}_1/\xi > 0.
\end{aligned}$$

Thus the tax revenue always exceeds the government expenditure as long as $w_1 = \tilde{w}_1$. This together with the fact that the tax revenue is nil when $t_K = t_L = 0$ (i. e., $\tau = 1$) enables us to put the inverse of $1 + t_L$ in the closed segment [e, 1].

Proof of the existence

Let Γ be the attainable set :

$$\begin{aligned}
\Gamma := \{(y_1, y_2) \in R^2_+ \mid \quad & \text{there exist } (K_1, L_1) \text{ and } (K_2, L_2) \text{ satisfying} \\
& y_1 \leqq F_1(K_1, L_1),\ y_2 \leqq F_2(K_2, L_2), \\
& K_1 + K_2 \leqq K_0 \text{ and } L_1 + L_2 \leqq L_0\}.
\end{aligned}$$

It is clear that Γ is a convex compact set. We can find a sufficiently large number M such that if (K_i, L_i) is a pair of factors satisfying $F_i(K_i, L_i) \geqq M$, then it holds that $L_i > L_0$ or $K_i > K_0$, $i = 1, 2$. It is obvious that Γ is a subset of $Z := [0, M] \times [0, M]$.

For any element $((y_1, y_2), (w_2, r), \xi, \tau) \in Z \times S^\circ \times (0, 1) \times [e, 1]$, let us define

$t_L := 1/\tau - 1$, $t_K := tt_L$, $\tilde{w}_1 := w_2/\eta(\xi)$, $p_1 := T_K r k_1(T_L\tilde{w}_1, T_K r) + T_L\tilde{w}_1\lambda(T_L\tilde{w}_1, T_K r)$ and $p_2 := T_K r k_2(T_L w_2, T_K r) + T_L w_2 \lambda_2(T_L w_2, T_K r)$.

The set $\Delta \overset{\text{def}}{=} \{(p_2, w_2, r) \mid (w_2, r) \in S\}$ is bounded. In fact, for some fixed $(\bar{k}_2, \bar{\lambda}_2) \in Q_2$, we can see that $p_2 \leqq T_K r\bar{k}_2 + T_L w_2\bar{\lambda}_2 \leqq ((1+t)\bar{k}_2 + \bar{\lambda}_2)/e$ since $T_K < (1 + t)T_L$. By **(WR)**, we can see that $\Psi(p_1, p_2, w_2, r, \xi)$ is bounded for any $(p_1, p_2, w_2, r, \xi) \in R_+ \times \Delta \times [0, 1]$. And thus $|\Psi(p_1, p_2, w_2, r, \xi) - w_2|$ is also bounded. We can assume that $M > 1$ and $M > |\Psi(p_1, p_2, w_2, r, \xi) - w_2|$ for any $(p_1, p_2, w_2, r, \xi) \in R_+ \times \Delta \times [0, 1]$.

And thus the following mapping

(7) $\tilde{y}_1 := \min(\tilde{X}_1, M)$,

(8) $\tilde{y}_2 := \min(\tilde{X}_2, M)$,

(9) $\tilde{w}_2 := (1/\beta)\min(w_2 + \max(p_1(\tilde{X}_1 - y_1)/w_2 + E_L, 0), M)$,

(10) $\tilde{r} := (1/\beta)\min(r + \max(p_2(\tilde{X}_2 - y_2)/r + E_K, 0), M)$,

(11) $$\tilde{\xi} := \frac{\xi + \min(\max(-\Psi(p_1, p_2, w_2, r, \xi) + w_2/\eta(\xi), 0), M)}{1 + \min(|\Psi(p_1, p_2, w_2, r, \xi) - w_2/\eta(\xi)|, M)},$$

(12) $$\tilde{\tau} := \min\left\{\max\left(\frac{1 + \max(E_G, 0)}{1 + \max(-E_G, 0)}, e\right), 1\right\},$$

(13) $$\beta := \min(w_2 + \max(p_1(\tilde{X}_1 - y_1)/w_2 + E_L, 0), M) + \min(r + \max(p_2(\tilde{X}_2 - y_2)/r + E_K, 0), M),$$

leads us to

$$((\tilde{y}_1, \tilde{y}_2), (\tilde{w}_2, \tilde{r}), \tilde{\xi}, \tilde{\tau}) \in Z \times S^\circ \times [0, 1] \times [e, 1].$$

This procedure defines a continuous function f defined on the set $Z \times S^\circ \times (0, 1] \times [e, 1]$ into itself which is a dense subset of the compact convex set $Z \times S \times [0, 1] \times [e, 1]$. That is,

$$f : ((y_1, y_2), (w_2, r), \xi, \tau) \to ((\tilde{y}_1, \tilde{y}_2), (\tilde{w}_2, \tilde{r}), \tilde{\xi}, \tilde{\tau})$$

By the extension of mappings we can obtain the following correspondence

$\bar{f} : Z \times S \times [0, 1] \times [e, 1] \to Z \times S \times [0, 1] \times [e, 1]$ and

$\bar{f}$ is an extension of f.

In this event, we extend the function f to $\bar{f}$ so that the graph of $\bar{f}$ may be the closure of that of f in $(Z \times S \times [0, 1] \times [e, 1]) \times (Z \times S \times [0, 1] \times [e, 1])$ and be upper semi-continuous and closed convex valued (see Theorem 4.7 and Corollary 2 to Theorem 4.8 in Nikaido (1968, pp. 72-73)). By Kakutani's fixed point theorem (Kakutani (1941)), there exists a fixed point $q^* := ((y_1^*, y_2^*), (w_2^*, r^*), \xi^*, \tau^*)$, where the variables with asterisks are those evaluated at the fixed point. If the fixed point is not in $Z \times S^\circ \times (0, 1] \times [e, 1]$, then there exists a sequence

$$q^m := ((y_1^m, y_2^m), (w_2^m, r^m), \xi^m, \tau^m) \in Z \times S^\circ \times [0, 1] \times [e, 1],\ m = 1,2,...$$

satisfying

(14) $f(q^m) \to q^*$ and $q^m \to q^*$ m as $m \to \infty$,

since the graph of $\bar{f}$ is the closure of the graph of f.

(Step 1) The first problem to be solved is the boundedness of p_1^*. Suppose that p_1^* can not be defined or simply that $p_1^* = \infty$. By the definition of the first sector product price, this is the case only when q^* is not in the set $Z \times S^\circ \times (0, 1] \times [e, 1]$. Let $q^m := ((y_1^m, y_2^m), (w_2^m, r^m), x^m, \tau^m)$, $m = 1, 2,...$ be the sequence in $Z \times S^\circ \times (0, 1] \times [e, 1]$ which satisfies (14) and the property that $p_1^m \to \infty$, (as $m \to \infty$). Note that the equality

$$p_1^m = T_K^m r^m k_1(T_L^m \tilde{w}_1^m, T_K^m r^m) + T_L^m \tilde{w}_1^m \lambda_1(T_L^m \tilde{w}_1^m, T_K^m r^m)$$

holds for every m. This together with the fact that T_K^m and T_L^m are uniformly less than $(1+t)/e$ and $1/e$, enables us to know that $\tilde{w}_1^m = w_2^m/\eta(\xi^m) \to \infty$, (as $m \to \infty$). The necessary condition for this to be true is that $\xi^m \to 0$, (as $m \to \infty$). Then by (11) and **(WR)**, we have $0 = M/(1+M) > 0$. This is a contradiction. Thus p_1^* is bounded.

(Step 2) Either $w_2^* > 0$ or $r^* > 0$ is true, so that $\tilde{I}^* > 0$. By (7), (8) and Step 1, we have

$$0 < y_1^* \leqq \tilde{X}_1^* \text{ and } 0 < y_2^* \leqq \tilde{X}_2^*.$$

(Step 3) Suppose that $w_2^* = 0$. There exists some sequence $q^m \in Z \times S^0 \times (0, 1] \times [e, 1]$, $m = 1,2,\ldots$ satisfying (14). By the fact that $y_2^* > 0$ and by the assumption **(B)** (ii), we must have $\tilde{L}_2^m = y_2^m \lambda_2 (T_L^m w_2^m, T_K^m r^m) \to \infty$, (as $m \to \infty$). This implies that $E_L^m \to \infty$ (as $m \to \infty$). This leads us to $w_2^* > 0$ by (9). This is a contradiction.

(Step 4) Suppose that $r^* = 0$. There exists some sequence $q^m \in Z \times S^0 \times (0, 1] \times [e, 1]$, $m = 1,2,\ldots$ By the same reasoning as in Step 3, we have $\tilde{K}_2^m = y_2^m k_2(T_L^m w_2^m, T_K^m r^m) \to \infty$, (as $m \to \infty$). This implies that $E_K^m \to \infty$, (as $m \to \infty$). This leads us to $r^* > 0$ by (10). This is a contradiction. Thus we have

$$w_2^* > 0 \text{ and } r^* > 0.$$

(Step 5) Suppose that $\xi^* = 0$. There exists some sequence $q^m \in Z \times S^0 \times (0, 1] \times [e, 1]$, $m = 1,2,\ldots$ satisfying (14). The fact that $\xi^m \to 0$, (as $m \to \infty$) together with $w_2^* > 0$ leads us to $w_2^m/\eta(\xi^m) \to \infty$, (as $m \to \infty$). Since p_2^* is finite, the set $\{(p_2^m, w_2^m, r^m) \mid m = 1, 2,\ldots\}$ is bounded. By the assumption **(WR)**, $\Psi(p_1^m, p_2^m, w_1^m, w_2^m, \xi^m)$ is uniformly bounded. Then by (11) we have $0 = M/(1+M) > 0$. This is a contradiction. We obtain $\xi^* > 0$. This together with $w_2^* > 0$ and $r^* > 0$ deduces that $p_1^* > 0$ and $p_2^* > 0$.[9)]

(Step 6) Furthermore, suppose that $\xi^* = 1$ were true. Define $s := \Psi(p_1^*, p_2^*, w_2^*, r^*, \xi^*) - w_2^*/\eta(\xi^*) = \Psi(p_1^*, p_2^*, w_2^*, r^*, \xi^*) - w_2^*$. It must hold that $s > 0$ by **(WR)** with $\delta > 1$. And thus we get $1 = 1/(1+s) < 1$. This is a contradiction. Here we obtain

(15) $1 > \xi^* > 0$.

And thus, it holds that form (11)

$$\xi^* \min(|\Psi(p_1^*, p_2^*, w_2^*, r^*, \xi^*) - w_2^*/\eta(\xi^*)|, M)$$
$$= \min(\max(-\Psi(p_1^*, p_2^*, w_2^*, r^*, \xi^*) + w_2^*/\eta(\xi^*), 0), M).$$

The relation above leads us to

$$\Psi(p_1^*, p_2^*, w_2^*, r^*, \xi^*) - w_2^*/\eta(\xi^*) = 0.$$

(Step 7) By (12) we see

$$(12') \quad \tau^* = \min \left| \max\left(\tau^* \frac{1+\max(E_G^*, 0)}{1+\max(-E_G^*, 0)}, e \right), 1 \right|$$

Suppose that $\tau^* = e$. By the fact that the output of the first sector is positive and $0 < \xi^* < 1$, the discussion made immediately after Lemma enables us to see that $E_G^* > 0$. This implies that right hand side of (12′) is strictly greater than e. This is a contradiction.

Let us study the case of $0 < \alpha$. In this case we know that $E_G^* = 0$ and $e < \tau^* < 1$ must hold. The other case is $\alpha = 0$, that is, the government expenditure is nil. Thus we have $\tau^* = 1$ since positive tax rates imply posi-

9) Professor T. Shinotsuka pointed out that we should have shown the positivity of p_1^* in the earlier draft.

tive government revenue. We summarize these as

(16) $0 < \alpha < 1 \Rightarrow (e < \tau^* < 1 \text{ and } E_G^* = 0)$ or
$\alpha = 0 \Rightarrow (\tau^* = 1 \text{ and } E_G^* = 0)$.

(Step 8) By quasi-Walras law and (16), we have

$$w_2^* \{p_1^*(\tilde{X}_1 - y_1^*)/w_2^* + E_L^*\} + r^*\{p_2^*(\tilde{X}_2 - y_2^*)/r^* + E_K^*\} = 0.$$

Then either the first or the second term must be less than or equal to zero. If the first term is less than or equal to zero then $w_2^* = w_1^*/\beta^*$. By this together with $w_2^* > 0$, we have

(17) $\beta^* = 1$.

If the second term is less than or equal to zero, then we have (17) by the same discussion. $\beta^* = 1$, it holds that

(18) $p_1^*(\tilde{X}_1 - y_1^*) + w_2^* E_L^* = 0$ and $p_2^*(\tilde{X}_2 - y_2^*) + r^* E_K^* = 0$.

Suppose that $y_1^* \geqq M$. Then by the definition of M, It follows that

$\tilde{K}_i^* > K_0$ or $\tilde{L}_i^* > L_0$.

This contradicts the equalities in (18). And thus $y_i^* < M$, $i = 1, 2$. Finally we can establish :

(19) $E_K^* = 0$, $E_L^* = 0$, $y_i^* = \tilde{X}_i^*$, $i = 1, 2$

A pair of the allocation $(\tilde{X}_1^*, \tilde{X}_2^*, y_1^*, y_2^*, \tilde{K}_1^*, \tilde{K}_2^*, \tilde{L}_1^*, \tilde{L}_2^*)$ with $0 < \xi^* < 1$ and the price vector $(w_1^*, w_2^*, r^*, p_1^*, p_2^*)$ is an unemployment equilibrium relative to the wage setting function $\Psi(\cdot)$ and a feasible tax scheme (T_K^*, T_L^*, α). Q. E. D.

References

Benassy, J. P. (1986) *Macroeconomics : Non-Walrasion Approach*, New York : Academic Press.

Benassy, J. P. (1975), "Neo-Keynesian Disequilibrium Theory in a Monetary Economy," *Review of Economic Studies*, Vol. 42, 503-523

Bulow, J. I., and L. H. Summers (1986) "A Theory of Dual Labor Markets with Application to Industrial Policy, Discrimination, and Keynesian Unemployment," *Journal of Labor Economics*, Vol. 4, 376-414.

Davidson, D., Martin L., and S. Matusz (1988), "The Structure of Simple General Equilibrium Models with Frictional Unemployment," *Journal of Political Economy*, Vol. 96, 1267-1293.

Drèze, J. (1975), "Existence of exchange equilibrium under price rigidities and quantity rationing," *International Economic Review*, Vol. 16, 301-320.

Harris, J. R. and M. P. Todaro (1970), "Migration unemployment and development : A two-sector analysis," *American Economic Review*, Vol. 60, 126-142.

Kakutani, S. (1941), "A generalization of Brouwer's fixed point theorem," *Duke Mathematical Journal*, Vol. 8, 457-459.

Kurz, M. (1982), "Unemployment Equilibria in an Economy with Linked Prices," *Journal of Economic Theory*, Vol. 26, 100-123.

Laan, G. van der (1984), "Supply-Constrained Fixed Price Equilibria in Monetary Economies," *Journal of Mathematical Economics*, Vol. 13, 171-187.

Malinvaud, E. (1977), *The theory of unemployment reconsidered*, Oxford : Basil Blackwell.

Miyagiwa, K. (1988), "Corporate income tax incidence in the presence of sector-specific unemployment," *Journal of Public Economics*, Vol. 37, 103-112.

Nikaido, H. (1968), *Convex structures and economic theory*, New York : Academic Press.

Pissarides, C. A. (1990) *Equilibrium Unemployment Theory*, Oxford : Basil Blackwell.

Summers, L. H. (1988), "Relative Wage Hypothesis, and Keynesian Unemployment," *American Economic Review*, Vol. 78, 383-388.

Weddepohl, C. (1987), "Supply-Constrained Equilibria in Economies with Indexed Prices," *Journal of Economic Theory*, Vol. 43, 203-222.

Wu, H. (1988), "Unemployment Equilibrium in a Random Economy," *Journal of Mathematical Economics*, Vol. 17, 385-400.

Younes Y. (1975), "On the Role of Money in the Process of Exchange and the Existence of a Non-Walrasian Equilibrium," *Review of Economic Studies*, Vol. 42, 489-501.