A note on optimal taxation in the presence of externalities

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Abstract

In this paper, we reexamine the optimal tax problem with identical consumers in the presence of externalities by taking explicitly the interrelationship between externality and consumption into account. In setting out the optimal tax conditions in the presence of externalities, it is standard to follow Sandmo (1975) to employ the following assumptions: (i) separable externalities; and (ii) independent demands. We dispense with these restrictive assumptions and show the Sandmo's "*additivity property*" where externalities only affect the tax formula for an externality generating good and do not affect other tax formulas.

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1 Introduction

Addressing environmental issues typically involves taxes and other policy instruments. From economists' standpoint, environmental issues can be divided into two categories.

The first one consists of the issues associated with environmental tax reforms. A typical research strategy is to presume that there exist distortionary taxes already. Researchers then ask the following question: Can revenues from environmental taxes be used to finance cuts in distortionary taxes? If the answer is yes, then we say that the "double-dividend" hypothesis holds. Oates (1995) discussed the possibility of the hypothesis.[1]

The second category addresses optimal tax problems associated with externalities. This problem began with Sandmo (1975), who showed that externalities only affect the tax formula for an externality generating good and do not affect other tax formulas. This expresses the Sandmo's "*additivity property*". Cremer et al. (1998) incorporated self-selection constraints as taste differentiation in a model with n types consumers, and Sheshinski (2004) studied redistributional problems at the optimal personal taxes with non-identical consumers.

In setting out the optimal tax conditions in the presence of externalities, it is standard to follow Sandmo (1975) to employ the following assumptions: (i) separable externalities; and (ii) independent demands. These assumptions simplify an interrelationship between externality and consumption. Under these assumptions, Sandmo derived the well-known "*inverse elasticity rule*" where the optimal tax formula is inversely related to the demand elasticity.

In this paper, we reexamine the optimal tax problem with identical consumers in the presence of externalities by taking explicitly the interrelationship between the externality and the consumption into account. We do not

[1] Bovenberg and de Mooij (1994), Fullerton (1997) compared second-best tax formulas with Pigovian taxes in this context.

impose these restrictive assumptions.

The plan of the paper is as follows: The next section provides the basic model. Section 3 derives the optimal tax formulas.

2 The structure of the model

2.1 The basic model

The economy consists of n consumers, who have identical preferences and consume two goods, x_i, $i = 1, 2$. Denote by $X_i = nx_i$ the total amount of x_i. Let x_0 be a labor supply and let $1 - x_0$ be leisure. We consider a negative consumption externality which is created by the total consumption of x_2. For simplicity, production of x_i is assumed to be linear. A consumer price vector is denoted by $P = (P_0, P_1, P_2)$ and a producer price vector by $p = (p_0, p_1, p_2)$ as given. We choose labor as our *numéraire*, so that $P_0 = p_0 = 1$.

The utility function of the representative consumer is:

$$u = u\left(1 - x_0, x_1, x_2, X_2\right). \tag{1}$$

We assume that u is strictly concave and satisfies: $u_0 \equiv \partial u/\partial(1 - x_0) > 0$, $u_i \equiv \partial u/\partial x_i > 0$ for $i = 1, 2$ and $u_3 \equiv \partial u/\partial X_2 < 0$. Let S be the transfer payments to the representative consumer. The budget constraint is:

$$-x_0 + \sum_{i=1}^{2} P_i x_i = S. \tag{2}$$

The consumer maximizes (1) for a *given* externality subject to (2).

A transformation function in the production side is:

$$-X_0 + \sum_{i=1}^{2} p_i X_i = 0. \tag{3}$$

Let T be the government's revenue requirements and let $t_i = P_i - p_i$ be a unit tax. The government's budget constraint is:

$$\sum_{i=1}^{2} t_i X_i = n \sum_{i=1}^{2} (P_i - p_i) x_i = T. \tag{4}$$

By Walras' law, $T = nS$. [2]

2.2 The consistency condition

The representative consumer maximizes utility given prices P and externality X_2. We write the demand x_i as a function of P and X_2:

$$x_i = x_i(P, X_2), \quad i = 0, 1, 2. \tag{5}$$

We impose a consistency condition on the externality X_2. That is, we require that the externality X_2 arises from the optimizing behavior on the part of the consumers who take *the externality* X_2 as well as P. Formally, this consistency condition can be written as:

$$X_2 = n x_2(P, X_2). \tag{6}$$

3 The second-best problem

3.1 General form

We state the government's second-best optimization problem and do not impose the assumptions of separabilities and independent demands. Substituting (5) into (1), we have the representative consumer's indirect utility function:

$$v(P, X_2) \equiv u\left(1 - x_0(P, X_2), x_1(P, X_2), x_2(P, X_2), X_2\right). \tag{7}$$

[2]Subtract (3) and (4) from the sum of the budget constraint (2) and take $nx_i = X_i$ into account.

The government maximizes the sum of the indirect utility functions (7) with respect to consumer prices subject to the government's budget constraint (4) as well as the consistency condition (6). Let $\mathscr{L}$ be the Lagrangian expression, and let β and γ be the Lagrange multipliers associated with the government's budget constraint and the consistency condition, respectively:

$$\begin{aligned}\mathscr{L}(P, X_2, \beta, \gamma) =& nv(P, X_2) - \beta \left[n \textstyle\sum_{i=1}^{2}(P_i - p_i)x_i(P, X_2) - T\right] \\ & - \gamma \left[X_2 - nx_2(P, X_2)\right]. \end{aligned} \tag{8}$$

We have the first-order conditions for this problem:

$$\begin{aligned} & - nu_0 \frac{\partial x_0}{\partial P_k} + n \sum_{i=1}^{2} u_i \frac{\partial x_i}{\partial P_k} - \beta n \left[\sum_{i=1}^{2}(P_i - p_i)\frac{\partial x_i}{\partial P_k} + x_k(P, X_2)\right] \\ & - \gamma \left[-n \frac{\partial x_2}{\partial P_k}\right] = 0, \quad k = 1, 2, \end{aligned} \tag{9}$$

$$\begin{aligned} & - nu_0 \frac{\partial x_0}{\partial X_2} + n \sum_{i=1}^{2} u_i \frac{\partial x_i}{\partial X_2} + nu_{X2} - \beta n \left[\sum_{i=1}^{2}(P_i - p_i)\frac{\partial x_i}{\partial X_2})\right] \\ & - \gamma \left[1 - n \frac{\partial x_2}{\partial X_2}\right] = 0, \end{aligned} \tag{10}$$

where $u_0 \equiv \partial u / \partial(1 - x_0)$, $u_i \equiv \partial u / \partial x_i$ and $u_{X2} \equiv \partial u / \partial X_2$.

Differentiating (2) with respect to (P, X_2), and differentiating (6) with respect to P, [3] we have the optimal tax formulas by rearranging (9) and (10):

$$t_1 = (1 - \mu) \left[\frac{n^2(x_2 x_{21} - x_1 x_{22})}{|J|(1 - nx_{2X2})}\right], \tag{11}$$

$$\begin{aligned} t_2 =& (1 - \mu) \left[\frac{n^2(x_2 x_{21} - x_1 x_{22})(-nx_{1X2})}{|J|(1 - nx_{2X2})} + \frac{n^2(x_1 x_{12} - x_2 x_{11})}{|J|}\right] \\ & + \mu P_2 \left(-n \frac{u_{X2}}{u_2}\right), \end{aligned} \tag{12}$$

[3] We rewrite the consistency condition (6) and assume that the problem has a unique solution, which we call (P^*, X_2^*). In addition, we require that $n\frac{\partial x_2}{\partial X_2}(P^*, X_2^*) \neq 1$. By the implicit function theorem, therefore, these exists a differentiable function $X_2 = X_2(P)$ defined on a neighborhood u of P^* such that $X_2^* = X_2(P^*)$. (6) can be written as $X_2(P) = nx_2(P, X_2(P)), \forall P \in u$.

where $|J| = n^2(x_{11}x_{22} - x_{12}x_{21})(1 - nx_{2X2})^{-1}$ is the Jacobian determinant,and $u_2 = \lambda P_2$, $\mu = -(\lambda/\beta)$, $nu_{X2}/\beta = \mu P_2\,(-nu_{X2}/u_2)$, $x_{ik} = \partial x_i/\partial P_k$, $x_{iX2} = \partial x_i/\partial X_2$, and $X_{2k} = \partial X_2/\partial P_k$ for $i, k = 1, 2$, and λ is the Lagrange multiplier associated with the budget constraint for the representative consumer.

From these equations we can make the following observations. First, (11) and (12) still retain the famous "*additivity property*" identified by Sandmo where externalities only affect the tax formula for the externality generating good (or good 2) and do not affect the tax formula for good 1. Second, (11) is similar to Sandmo's: Externalities do not affect this tax formula. Third, (12) differs from Sandmo's. The explicit treatment of the consistency condition (6) requires a revision of the Sandmo's formula: The interrelationship between the externalities and the consumption of good 2 explicitly appears in this tax formula.

3.2 Special cases

We reexamine the properties of optimal tax formulas by imposing Sandmo's assumptions. First, considering the case of independent demands, so that $x_{ik} = 0$ for $i \neq k$ and $X_{21} = 0$. The tax formulas for goods 1 and 2 still retain the Sandmo's additivity property and the "*inverse elasticity formulas*". The explicit treatment of the consistency condition (6) requires a revision of the tax formula for good 2: The interrelationship between the externalities and the consumption of good 2 explicitly appears in this tax formula.

Second, considering the case of separable externalities, so that $x_{iX2} = 0$ for i. The tax formulas for goods 1 and 2 still retain the Sandmo's additivity property. Even though we explicitly take the consistency condition (6) into account, the interrelationship between the externalities and the consumption of good 2 vanishes in these tax formulas.

Third, considering the case of independent demands and separable externalities, so that $x_{ik} = 0$ for $i \neq k$, $x_{iX2} = 0$ and $X_{21} = 0$. The tax formulas for goods 1 and 2 coincide with the inverse elasticity formulas and retain the additivity property identified by Sandmo. Even though we explicitly take the consistency condition (6) into account, the interrelationship between the externalities and the consumption of good 2 vanishes in the tax formula for good 2.

References

[1] Bovenberg, A. L. and de Mooij, R. A. (1994) Environmental levies and distortionary taxation, *American Economic Review*, Vol. 84, pp. 1085-1089.

[2] Cremer, H., Gahvari, F. and Ladoux, H. (1998) Externalities and optimal taxation, *Journal of Public Economics* , Vol. 70, pp. 343-364.

[3] Fullerton, D. (1997) Environmental levies and distortionary taxation: Comment, *American Economic Review*, Vol. 87, pp. 245-251.

[4] Oates, W. E. (1995) Green taxes: Can we protect the environment and improve the tax system at the same time?, *Southern Economic Journal* , Vol. 61, pp. 915-922.

[5] Sandmo, A. (1975) Optimal taxation in the presence of externalities, *Swedish Journal of Economics*, Vol. 77, pp. 86-98.

[6] Sheshinski, E. (2004) On atmosphere externality and corrective taxes, *Journal of Public Economics* , Vol. 88, pp. 727-734.