THEORY OF EXPORT CREDIT INSURANCE

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in
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by

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Theory of Export Credit Insurance  

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The present study conducts positive analyses on various aspects of export credit insurance. It is shown that export credit insurance is a useful device to protect domestic exporting firms against various political risks and default risk in the foreign market. Under a certain type of the reimbursement method, export credit insurance can make the exporting firm's production decision independent of the risk and the attitude toward risk.  

The government insurance agency can increase the level of exports by setting a lower premium rate. This export promotion effect through export credit insurance is larger when the exporting firm is more risk averse. If the premium rate per coverage is set at a fair rate, a risk averse firm will export as much as a risk neutral firm does in the absence of insurance.  

The presence of a domestic market restricts the effectiveness of export credit insurance. If the domestic price is higher than a net foreign price (the foreign price which is discounted by a premium rate), the firm does not export at all even though the insurance is available at a fair rate.  

The exporting firms will purchase full coverage of export credit insurance if the premium rate is set at a fair or a more than fair rate.
When they do purchase full coverage, they do not spend any money on the self-protecting activities which reduce the export credit risk. Therefore, there exists a problem of moral hazard.
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CHAPTER I

INTRODUCTION AND REVIEW OF THE LITERATURE

1.1 Introduction

There are always various types of risks in international trade. Some of them are inevitably beyond the control of a private company. For example, many companies lost their sales and investments during the 1980-81 Iranian revolution. As some developing countries have fallen into serious financial trouble in the course of the worldwide recession of the early 1980's, the fear of non-payment by foreign buyers has grown among companies that export to those markets. Even in the markets of the developed countries, looming protectionism creates uncertain circumstances for exporters. According to a well established result in the theory of the firm under uncertainty, the presence of uncertainty makes a firm produce less output than under certainty. Therefore a gloomy picture of the world community causes a shrinkage of world trade, and further deepens the recession.

Today, most governments in industrialized countries have certain programs designed to protect domestic firms from the risks in foreign trade. A vital part of these programs is export credit insurance. According to OECD (1982), the export credit insurance programs in most member countries are directly or indirectly organized by a government agency. Recently such insurance schemes have caught the eyes of economic
planners in developing countries. A number of these countries have now started government-backed export credit insurance programs in order to promote or diversity their exports. These programs can be particularly effective in giving a strong stimulus to less experienced exporters who want to ship more nontraditional goods such as manufactured products. In this connection, we should mention the report published by UNCTAD in 1976 which is titled "Export credit insurance as a means of expanding and diversifying exports of manufactures from the developing countries."

In the case of the United States, the Foreign Credit Insurance Association (FCIA) was formed in 1961 to insure U.S. exporting companies against unforeseen defaults by foreign buyers of political risks such as war, coup d'etat, currency blockage, and import restriction in foreign countries. According to FCIA (1982), the FCIA, which was an association of some 50 of the leading insurance companies, operated in cooperation with the Export-Import Bank of the United States (Eximbank), an independent government agency. The private insurance companies underwrote the commercial credit risks; Eximbank covered the political risks and reinsured certain excess commercial risks. According to Rodriguez (1983), however, the private insurance companies subsequently withdrew from the operation, and Eximbank started to operate the insurance program directly.

One of the prime motivations behind the present study stems from the recent controversy over the export incentive program in the United States. Strong free-trade advocates in the current administration, who ideologically oppose export credit subsidization of any kind, would like eventually to phase out the Eximbank. Supporters in Congress argue that
without Eximbank, exporters in the United States are forced into an unfavorable position in the world market in which exporters in other countries are supported by similar institutions. In November 1983, Congress barely passed the legislation that would extend the Eximbank's charter for at least another two years. The compromise between the two camps seemed to be made on two points: 1. Eximbank negotiates an agreement with other major exporting nations that reduces the amount of subsidy in export credit terms; 2. Eximbank starts to devote its main effort in short term credit insurance. Although several books on the role of Eximbank recently appeared, including Baron (1983), Feinberg (1982), Hartland-Thumberg and Gauford (1982), Hillman (1982), and Schneider (1974), the role of export credit insurance has not been formally discussed in the literature. The purpose of this study is to fill this gap in the literature by analyzing the role of export credit insurance in the theoretical framework.

We will attempt to answer the following questions in this dissertation:

1. How does export credit insurance promote exports?
2. What kind of conditions determine the effectiveness of the program?
3. Can export credit insurance be really regarded as a subsidy to exports?
4. What argument justifies the government's direct operation of the insurance program?
5. What problems might arise in the operation of the program?

This study can also be regarded as a positive analysis of the demand for insurance by corporations. As Mayers and Smith (1982)
pointed out, this topic has not been studied in either economics or finance despite its importance. They quoted the fact that "business insurance accounted for approximately 54.2 percent of the $79,032,923,000 in direct property and liability insurance premiums written in the United States in 1978." Although we deal with a special type of insurance, our theoretical framework could be applicable to other forms of insurance.

It might have been difficult to analyze this kind of topic theoretically and to derive meaningful conclusions before the recent development of the theory of the firm under uncertainty which uses the expected utility maximization approach. Since this approach has been also introduced into the insurance literature, it is fairly straightforward to set up the model for optimal insurance purchase by the firm.

In the rest of this chapter, Section 1.2 further clarifies the institutional aspects of export credit insurance; Section 1.3 reviews the theory of the firm under uncertainty; Section 1.4 provides a review of optimal insurance purchase; and finally, the organization of this dissertation is provided in Section 1.5.

1.2 Export Credit Insurance

The role of export credit insurance has not been formally discussed in the literature except in papers by Greene (1965) and Huszaph and Greene (1982). The first paper by Greene discussed the role and various aspects of a newly introduced export credit insurance scheme in the United States. The second paper by Greene, with Huszaph as coauthor, critically reviewed the twenty-year operation of FCIA and concluded that "export credit insurance must receive a thorough internal review and
streamlining of administrative and underwriting procedures before its use in exporting has a chance of being materially increased."

The purpose of this section is to summarize the main features of the export credit insurance program in the United States. Using the materials provided by the FCIA regional office in Houston, we state the basic scheme, the purpose, the benefits, types of policies, and premium rates and payment procedure.

The Scheme

According to UNCTAD (1982), there are five basic patterns of export credit insurance schemes in various countries, depending on the degree of government involvement.

(a) Through a government department or government agency (Japan, New Zealand, and the United Kingdom);
(b) Through an insurance company or a banking institution which acts as commissioned agent in the name and for the account of the government (Austria, the Federal Republic of Germany, and Ireland);
(c) Through a public corporation or a public fund which is autonomous but wholly government-owned (Australia, Belgium and Canada);
(d) Through a specialized institution jointly owned by the government and private insurance and banking enterprises (Switzerland, France, Portugal, and Spain);
(e) In cooperation with private insurance institutions which assume part of the risks for their own account (the Netherlands, the United States (before 1982)).

As stated in the previous section, private insurance companies recently withdrew from the operation of FCIA. Eximbank has changed its program mix, and 90 percent of its budget was spent on guarantees and
insurance in 1983. Therefore, the United States joined category (a) which is the most direct form of government-backed export credit insurance.

The Purpose

The FCIA was created in 1961 to assist the United States exporters to protect and finance their foreign receivables and inventories, and help in their effort to be competitive in overseas markets. Two types of risks are insured; political risk and commercial risk. Political risk is defined to be the unforeseen loss resulting from a sudden change in the worldwide political environment. Since risks such as war, civil strife, expropriation, and nationalization are usually beyond the control of private companies, FCIA offers 100 percent coverage for political risk. On the other hand, commercial risk is defined as the loss resulting from default or insolvency of the importer. Although this type of risk is manageable through a more careful selection of importer or market research, its difficulties in the foreign market are more severe than in the domestic market because of differences in bankruptcy or business laws. FCIA provides 90 percent coverage for commercial risk.

The Benefits

Purchasing FCIA insurance coverage has several advantages besides protecting foreign sales from the above mentioned risks.
1. The exporters are able to offer more competitive terms of payment to foreign buyers.
2. It provides a means of penetrating into higher-risk foreign markets.
3. It increases the financial stability of the exporter and it induces the commercial bank to discount its loan rate.
4. FCIA premium payments are tax deductible business expenses, and the modest self insurance provisions of FCIA policy can generally be used to create tax-exempt reserves.

**Types of Policies**

FCIA insurance policies are basically divided into two types: the Master Policy and the Medium-Term Policy.

The Master Policy is written for shipments during a one-year period and insures all or a portion of an exporter's eligible sales. Both short-term sales, with repayment terms generally up to 180 days, and medium-term sales, with repayment stretching out to five years, can be insured under the Master Policy.

The Medium-Term Policy covers capital and quasi-capital goods (primarily of U.S. manufacture sold in international trade on terms from six months to five years, and occasionally longer). The policy is written on a case-by-case basis. There is no requirement that the exporter insure all his medium-term transactions. And he may insure either a single or repetitive sale to an overseas buyer.

Besides the above two policies, there are the Combination Policy, which is a combination of short- and medium-term insurance, and the Small Business Policy for companies just beginning to export, or with only limited value.

The Master Policy also has a deductible feature similar to that of major medical or other forms of insurance. This deductible applies only to commercial coverage on an annual cumulative basis. The deductible does not apply to political coverage since, in FCIA's view, political
risks are normally of the unforeseen type, and losses in this area should be fully covered.

Premium Rates and Payment Procedure

FCIA determines its premium charges for the Master Policy primarily by the terms of sale being offered by the exporter, although the total volume to be insured and the exporter's past experience are also considered. On application, the exporter furnishes a history of the past year's exports, and a single composite rate is then computed for all shipments insured under the policy.

Payment of the premium is required monthly on the actual shipments made during the shipment period. A premium must be paid on all shipments, except those made to buyers in ineligible countries, or those specifically excluded by policy endorsement.

1.3 Theory of the Firm Under Uncertainty

Recent developments in the theory of the firm under uncertainty have been remarkable. Since the appearance of the seminal work by Sandmo (1971), who studied the optimal behavior of the competitive firm under price uncertainty, a number of important researches have been conducted in this area. The most significant contribution by Sandmo is the introduction of risk aversion. Using the expected utility maximization approach developed by von Neumann and Morgenstern, he showed, among other things, that in the presence of uncertainty, the optimal output of the risk averse firm is less than that of the risk neutral firm.

The source of uncertainty in the Sandmo model comes from the fact that the firm must produce before price is known, where the price is a
random variable with a known probability distribution. Since there is no inventory allowed, and the firm must sell all of its product in a single period, a change in production is the only way to respond to the price uncertainty.

Theoretical works by those who followed the Sandmo approach (in a sense that they all use the maximization of expected utility from profit) can be divided into three broad categories: 1) analyses of the different types of firms such as monopoly, the labor managed firm, the cooperative firm, the price discriminating firm, the multinational firm; 2) analyses of different sources of uncertainty such as production uncertainty, the fluctuation of input price, stochastic input delivery; 3) introduction of alternative ways of responding to uncertainty such as the existence of a futures market, self-protection behavior by the firm, and the ex-post adjustment of input employment.

The theoretical analysis of this dissertation can be regarded as an extension of the Sandmo model in all three of these respects. We investigate the behaviors of both a price taker and a price setter. The source of uncertainty is essentially a default risk in the foreign market which none of the previous works has investigated. Finally, export credit insurance is a solid alternative for responding to uncertainty other than output adjustment.

Since the main purpose of this dissertation is to clarify the role of export credit insurance against uncertainty, it is useful to briefly review some of the papers in the third category.

Holthausen (1979) and Feder, Just, and Schmitz (1980) introduced a futures market into the Sandmo model, and showed that the futures
market has the effect of separating the output decision from the risk and the firm's attitude toward risk. The level of output is determined according to the current spot price of the futures contract, and the subjective probability distribution of the future spot price affects only the firm's involvement in futures trading. Among other things, as in the certainty case, a change in fixed cost does not affect the total output. Katz and Paroush (1979), and Batra, Donnenfeld, and Hadar (1982), derived similar results in models of an exporting firm and the multinational firms, respectively.

The other important extension, which is related to this dissertation, is the incorporation of an activity designed to mitigate the degree of uncertainty. When such behavior results in the reduction of the probability of undesirable outcomes, it is called "self-protection." Recently, two papers which dealt with self-protection in the model of a competitive firm under uncertainty appeared. One is the paper by Paroush (1981) which treats market research as self-protection. The other is one by Hiebert (1983) which deals with production uncertainty. The former is more relevant to our topic because risk is involved on the revenue side of the profit function.

Paroush assumes that information gathering by the firm can reduce the variance of the subjective distribution of market price. At a constant cost of search behavior, the firm's ex ante decision problem is to choose the levels of output and search for information. He showed that the decisions on output and on the amount of market research are interdependent and have to be made simultaneously. Furthermore, conditions were specified under which the activity of self-protection in
the form of market research has the same consequences as the activity of self-insurance in the form of trading in the future market.

1.4 The Economics Literature on Insurance

In the framework of the expected utility approach, various aspects of optimal insurance purchase were originally studied by Mossin (1968), V. Smith (1968), and Gould (1969) at almost the same time. They analyzed the problem from the point of view of an individual who faces a specific risk. He is offered an insurance policy which specifies the payment to be received from an insurance company. One section in Mossin's paper assumed that the individual can choose the fraction of the total risk which is to be insured, while V. Smith considered the case in which the insured can choose the level of the maximum amount of coverage. Both assumed that insurance is provided at a constant premium rate. Another section in the papers by Mossin and Gould studied an insurance policy with a deductible feature. They assumed that the insured can choose an optimal amount of deductible, and the premium payment is a decreasing function of the level of deductible. It is common in these models that insurance policies are exogenously determined by the insurance agency. A recent paper by Raviv (1979) made a major contribution in the economic literature on insurance by studying a Pareto optimal insurance contract. The insurance policies were endogenously determined in his model. Using optimal control theory, he showed that an optimal insurance contract must include a deductible feature or a coinsurance feature. Borch (1983) also studied the optimal insurance contract in a competitive market, using a similar approach.
Two very important topics in the optimal purchase of insurance which have been extensively studied by economists are "moral hazard" and "adverse selection." The term "moral hazard" is used to describe the negative effect on incentives for self-protection by the insured. For example, in the case of medical insurance, the presence of medical insurance may make the insured person careless about health, so that the probability of becoming ill might increase. In the economics literature, the concept of moral hazard has been discussed mostly in the context of market equilibrium. Several authors, including Arrow (1971), Pauly (1968), and Helpman and Laffont (1975), argued that a competitive equilibrium may or may not exist in the presence of moral hazard, and might be Pareto inefficient. Marshall (1976) also studied insurance contracts which prevent a misallocation of resources resulting from moral hazard. More closely to the line of discussion on the demand for insurance, Ehrlich and Becker (1972) analyzed moral hazard by formally introducing the concept of self-protection which is defined as the human effort to reduce the probabilities of hazardous events. They showed that market insurance and self-protection can be complements; market insurance may lead to a reduction in the probabilities of hazardous events under certain conditions.

It has been pointed out by several authors, including Arrow (1963), Akerlof (1970), and Rothchild and Stiglitz (1976), that the competitive market mechanism may not lead to a desirable state when the insured agent has more information about the probability of a loss than the insurer. This is the so-called "adverse selection" problem. In the case of medical insurance, if the insurance agency cannot distinguish
a high-risk individual from a low-risk individual, a fair premium for a low-risk individual is not profitable for the insurance company because a high-risk individual also purchases full coverage. If the market premium rate is increased as a result of the exit of some insurance companies, the rate will become unfair for a low-risk individual. Hence, a low-risk individual purchases less than full coverage, and selling insurance coverage becomes less profitable. This process may continue until all the low-risk individuals are driven out of the market, and the market premium rate becomes exactly fair for a high-risk individual. Therefore, insurance service may not be provided for a low-risk individual at all through the competitive market. The above argument is used by several authors, such as Dahlby (1981), Johnson (1977, 1978), Pauly (1974), as a justification for some kinds of government intervention (example: regulation, compulsory insurance) into the insurance market. The same argument may well hold for the export credit insurance program. Indeed, the adverse selection problem may be very relevant to our model because it will be extremely costly for a small competitive insurance company to gather all the necessary information about various foreign markets.

Finally, we should mention the studies on nonlinear pricing in insurance contracts. It is not always optimal for a monopolist to charge a single price for its product, regardless of the quantity demanded. If there is enough information about the demand conditions, a nonlinear price schedule can provide a means for increasing profits. Practical examples of nonlinear prices are found in quantity discounts at the supermarket, telephone and rent-a-car rates. Nonlinear pricing in the
commodity market has been studied by Ramsey and Hadar (1974), Spence (1977), and Mirman and Sibley (1980). An application of nonlinear pricing for the insurance contract was considered by Stiglitz (1977) and Schlesinger (1983). Since the insurer can offer various menus of insurance contracts which as specified by the relation between the premium rate and the level of coverage, it could discriminate among individuals with different risks through a nonlinear pricing in an ex ante sense. Hence it could serve as a device for solving the adverse selection problem.

1.5 Dissertation Organization

This dissertation is organized as follows. Chapter II presents the basic models for export credit insurance where the firm is a price taker and exports its entire product to the foreign market. The demand for insurance is studied under several payment methods. Chapter III extends the analysis to the case of a price setter, i.e., monopoly, and assumes that there exists a riskless domestic market. Comparison is made with respect to the demand for export credit insurance between the models of Chapters II and III. Chapter IV studies a moral hazard problem by assuming that the probability distribution of export credit risk is affected by self-protecting behavior on the part of the firm, such as spending money for market research. Finally, Chapter V suggests further extensions of this study and states concluding remarks.
CHAPTER II

OPTIMAL COVERAGE:
MODELS WITHOUT A DOMESTIC MARKET

2.1 Introduction

In this chapter we present some basic models that are suitable for the analysis of export credit insurance. Using a simple model of the theory of the firm under uncertainty, we examine the fundamental role of the insurance and its impact on the firm's decision making. In order to focus on the analysis of the different features of insurance contracts, we assume that the firm is a price taker, and exports all of its product to a foreign market. After explaining the nature of the risk in Section 1, we analyze two different types of insurance contracts in Sections 2 and 3. Section 4 studies an insurance contract which includes a deductible feature. Finally, a summary of the results is provided in Section 5.

2.2 A Model Without Insurance

The model in this chapter is similar to the one studied by Sandmo (1971) except for the nature of the risk. We assume that the price of the product in the foreign market is known with certainty to the firm before the production decision is made. Furthermore, it is assumed that there is no exchange rate risk (i.e., sales are billed in units of the domestic currency). However, the firm may not be able to receive
the full payment for its product from the foreign buyers because of unforeseen defaults or political risks in the foreign country. In such a case, it is convenient to express random total sales in the multiplicative form.

The random profit of the firm, $\pi$, is written as follows:

$$\pi = (1 - \alpha)px - f(x) - b,$$

where $p$ is the price of the product, $x$ is the level of export, $f(x)$ is the total variable cost function, $b$ is fixed costs, and $\alpha$ is a random variable which is distributed between zero and one.

We make the following assumptions:

A.1 The objective of the firm is to maximize the expected utility of profits.

A.2 The utility function satisfies the von Neumann-Morgenstern axioms of preference.

A.3 The utility function $u(\pi)$ is twice differentiable and has derivatives with the following signs: $u' > 0$, $u'' \leq 0$.

A.4 The variable cost function is also twice differentiable and has derivatives with the following signs: $f' > 0$, $f'' > 0$.

Now the firm's decision problem is described as follows:

$$\text{Max } E u(\pi) = \int_{0}^{1} u(\pi)g(\alpha)d\alpha,$$

where $E$ is the expectation operator, and $g(\alpha)$ is the probability density function of $\alpha$.

Assuming that the objective function possesses an interior maximum, we have the necessary condition
\[ Eu' \cdot [(1 - \alpha)p - f'] = 0. \]  

(2.1)

The sufficient condition for this problem is satisfied under assumptions A.3 and A.4 as follows:

\[ Eu'' \cdot [(1 - \alpha)p - f']^2 - f'' Eu' < 0. \]

In the above model, the variable \( \alpha \) represents the proportion of total sales that is lost due to various risks in the foreign market. In the case of \( \alpha = 0 \), which corresponds to the riskless case, the firm receives the full amount of its sales. In the Sandmo model, the expected price is regarded as the certainty price, and the optimal level of the risk neutral firm is referred to as the certainty output. He proved the main result that under price uncertainty output is smaller than certainty output. This proposition must be slightly modified in our model since even the output level of the risk neutral firm is not free from the effect of the risk.

Proposition 2.1
(a) Under uncertainty in the foreign market, the risk neutral firm exports less than in the no-loss case, and the risk averse firm exports less than the risk neutral firm does.
(b) As the firm becomes more risk averse, its level of exports decreases.

The Proof of Proposition 2.1(a)
Rearranging (2.1), we have

\[
\left(1 - \frac{Eu'}{Eu'} \right) p = f'.
\]  

(2.1')
Let $x^c$, $x^n$, $x^a$ denote exports in the no-loss case (certainty), exports of the risk neutral firm, and exports of the risk averse firm, respectively. We know the following fact:

$$ Eu'\alpha = \bar{\alpha}Eu' + \text{Cov}(u',\alpha), $$

where $\bar{\alpha}$ denotes the expected value of $\alpha$.

In the case of a risk neutral firm ($u'' = 0$), $\text{cov}(u',\alpha)$ is zero, and in the case of a risk averse firm ($u'' < 0$), $\text{cov}(u',\alpha)$ is positive because $u'$ and $\alpha$ are increasing functions of $\alpha$. In the three cases (no loss, risk neutrality, risk aversion), the expression in (2.1') takes the following specific forms, respectively:

$$ p = f'(x^c) \quad \text{(no loss),} \quad (2.2) $$

$$ (1 - \alpha)p = f'(x^n) \quad \text{(risk neutrality),} \quad (2.3) $$

$$ \left( 1 - \bar{\alpha} - \frac{\text{cov}(u',\alpha)}{Eu'} \right) p = f'(x^a) \quad \text{(risk aversion).} \quad (2.4) $$

Since $\left( 1 - \bar{\alpha} - \frac{\text{cov}(u',\alpha)}{Eu'} \right) p < (1 - \bar{\alpha})p < p$, and $f'$ is an increasing function of $x$, we obtain $x^a < x^n < x^c$, which completes the proof.

Part (b) of Proposition 2.1 also holds in Sandmo's model, although he did not address this question. In order to prove it, we use the following approach. First of all, we characterize increased risk aversion by a concave transformation of the original utility function, which is one of the equivalent characterizations of increased risk aversion introduced by Pratt (1964). Then the next step is to compare the optimal level of export under the original utility function with
that under the transformed utility. This will be done by evaluating the first derivative under the transformed utility at the optimal level under the original utility function. Since the expected utility function is concave, if the sign of the derivative is negative, the proof is complete.

The Proof of Proposition 2.1(b)

Let $\psi[u(\pi)]$ denote the new utility function whose derivatives satisfy $\psi' > 0, \psi'' < 0$. The new objective function is

$$V(x) = E\psi[u(\pi)].$$

Assuming the existence of an interior maximum, the necessary condition under the transformed utility function is

$$V_x = E\psi' \cdot u' \cdot [(1 - \alpha)p - f'] = 0. \quad (2.5)$$

The sufficient condition is shown to be satisfied as follows:

$$V_{xx} = E\psi'' \cdot \{u' \cdot [(1 - \alpha)p - f']\}^2 + E\psi' \cdot u'' \cdot [(1 - \alpha)p - f']^2 - f'' \cdot E\psi' \cdot u' < 0. \quad (2.6)$$

Let $x_1, x_2$ denote the optimal solutions to (2.1) and (2.5), respectively. Our task is to show $x_2 < x_1$. We evaluate $V_x$ at $x_1$.

$$V_x |_{x=x_1} = \int_0^c \psi' \cdot u' \cdot [(1 - \alpha)p - f'] g(\alpha) d\alpha$$

$$+ \int_c^1 \psi' \cdot u' \cdot [(1 - \alpha)p - f'] g(\alpha) d\alpha, \quad (2.7)$$

where $c = \frac{p-f'}{p}$. Clearly, the first integral is positive, and the
second integral is negative. Applying the mean value theorem for integrals, we can write (2.7) as

\[ V_{x_1} = \psi'(\alpha_1) \int_0^c u' \cdot [(1 - \alpha)p - f'] g(\alpha) d\alpha \]

\[ + \psi'(\alpha_2) \int_0^1 u' \cdot [(1 - \alpha)p - f'] g(\alpha) d\alpha, \]

where \(0 < \alpha_1 < c\) and \(c < \alpha_2 < 1\). Since \(\psi\) is a concave function, the following inequality must hold:

\[ V_{x_1} < \psi'(c) \int_0^c u' \cdot [(1 - \alpha)p - f'] g(\alpha) d\alpha \]

\[ + \psi'(c) \int_0^1 u' \cdot [(1 - \alpha)p - f'] g(\alpha) d\alpha. \quad (2.7') \]

The right-hand-side of (2.7') is combined to yield

\[ \psi'(c) \int_0^1 u' \cdot [(1 - \alpha)p - f'] g(\alpha) d\alpha, \] which is zero from (2.1). Therefore, we have established \(V_{x_1} < 0\). Hence (2.6) implies that \(x_2 < x_1\).

### 2.3 Insurance Contracts With Proportional Reimbursement

In this section we study the case in which export credit insurance is available with the type of reimbursement method according to which the reimbursement payment is proportional to the coverage. This type of payment method was used by Mossin (1968) in his well-known paper. We will refer to it as the proportional method. The firm's decision problem is to choose the amount of insurance coverage and the level of export. The premium rate of the insurance is assumed to be exogenously set by a government agency. The firm can choose the amount of coverage up to the
amount of total sales. If the firm chooses the maximum amount of coverage, namely, the coverage is equal to total sales, we call it full coverage. If the firm selects a coverage which is less than the value of total sales, it is called partial coverage.

The proportional method specifies that the compensation paid by the insurance agency in case of loss is in proportion to the purchased amount of coverage. Suppose, for example, that 80 percent of the total sales is lost by an unforeseen default of the importer. Then the exporting firm will receive an amount equal to 80 percent of the coverage purchased from the export credit insurance agency. Under this specification, if the firm purchases a partial coverage, it must incur some loss, no matter how small the damage might be. According to this compensation method, the compensation, \( m \), is given by

\[
m = ay \quad \text{for all } a, \quad 0 < a < 1,
\]

where \( y \) is the amount of coverage.

Now the profit function is written as follows:

\[
\pi = (1 - a)px + m - f(x) - b - qy,
\]

where \( q \) is the premium rate per dollar of coverage.

The firm chooses optimal values of \( x \) and \( y \) in order to maximize its expected utility of profit subject to the condition \( 0 \leq y \leq px \).

The decision problem is described as

\[
\text{Max } V(x,y) = \text{Eu}(\pi) = \int_0^1 u[(1 - a)px + ay - f(x) - b - qy]g(\alpha)d\alpha
\]

subject to \( 0 \leq y \leq px \).
First we examine the boundary conditions for \( y \). If the firm does not export at all, there is no need for insurance. In order to avoid this trivial case, we hereafter assume that the firm exports some positive amount. Now the first two partial derivatives with respect to \( y \) are

\[
\begin{align*}
V_y &= Eu' \cdot (\alpha - q), \\
V_{yy} &= Eu'' \cdot (\alpha - q)^2 < 0.
\end{align*}
\] (2.8)

In order to facilitate the discussion, we introduce the following definition.

Definition 2.1

The premium rate \( q \) is called unfair, fair, or more than fair according to whether it is larger, equal to, or smaller than the expected value of \( \alpha \).

In the case of risk neutrality, the second derivative is zero, and there is no interior solution to the problem; the solution is either no coverage (\( y = 0 \)) or full coverage (\( y = px \)), depending on whether the first derivative is negative or positive. It is easy to see from (2.8) and Definition 2.1 that a risk neutral firm chooses no coverage if the premium rate is unfair, and full coverage if it is fair.

In the case of risk aversion, the second derivative is strictly negative. A necessary and sufficient condition for the firm to take full coverage, that is, to choose \( y = px \), is clearly that the first derivative at this point is nonnegative, that is,

\[
V_y|_{y=px} = u'[(1 - q)px - f(x) - b] \cdot (\alpha - q) \geq 0.
\]
Since $u' > 0$, a fair or more than fair premium is a necessary and sufficient condition for full coverage. Similarly, a necessary and sufficient condition for no coverage is that the first derivative at $y = 0$ is nonpositive, that is,

$$V_y|_{y=0} = Eu'[ (1 - \alpha)p_x - f(x) - b] \cdot (\alpha - q) \leq 0. \quad (2.9)$$

In this case, we obtain only a necessary condition. We can rewrite the right-hand-side of (2.9) as follows:

$$V_y|_{y=0} = Eu'[ (1 - \alpha)p_x - f(x) - b] \cdot (\alpha - q) + \text{cov}[u',\alpha - q] \leq 0. \quad (2.9')$$

It can be shown that $\text{cov}[u',\alpha - q]$ is positive because both $u'$ and $\alpha - q$ are increasing functions of $\alpha$. Thus in order for the inequality in (2.9) to hold, the first term of the right-hand-side in (2.9') must be negative. Hence, we see that an unfair premium is necessary for no coverage. It is important to note that an interior solution is possible for a risk averse firm, but this can happen only if the insurance premium is unfair.

We have established the following proposition.

Proposition 2.2

Under the proportional reimbursement method, the following statements are true:

(a) A risk neutral firm chooses no coverage (full coverage) if and only if the premium rate per dollar of insurance is unfair (fair or more than fair) and never chooses a partial coverage.
(b) A risk averse firm chooses full coverage if and only if the premium rate is fair or more than fair, and chooses no coverage only if it is unfair. If the premium rate is unfair, it may choose partial coverage.

Now we turn our interest to the case of an interior solution. We assume that the firm is risk averse, and that the premium rate is unfair. If the objective function possesses an interior maximum, then necessary conditions for the optimization problem are

\[ V_x = Eu' \cdot [(1 - \alpha) p - f'] = 0 \quad \text{(2.10)} \]
\[ V_y = Eu' \cdot (\alpha - q) = 0 \quad \text{(2.11)} \]

The sufficient conditions are

\[ \begin{cases} 
V_{xx} = Eu'' \cdot [(1 - \alpha) p - f']^2 - f''Eu' < 0, \\
V_{yy} = Eu'' \cdot (\alpha - q)^2 < 0, \\
V_{xx} \cdot V_{yy} - V_{xy} \cdot V_{yx} > 0.
\end{cases} \quad \text{(2.12)} \]

Conditions in (2.12) are always satisfied under the assumption of risk aversion and assumption A.4 in Section 2.1.

**Proposition 2.3**

If export credit insurance is available with the proportional reimbursement method, the optimal level of export is independent of the attitude toward risk and the distribution of the random variable.
Proof

From (2.11), we have \( \text{Eu}' \alpha = q \text{Eu}' \). Substituting this into (2.10) after writing it as \( p \text{Eu}' - p \text{Eu}' \alpha - f' \text{Eu}' = 0 \), we get

\[
p(1 - q) = f'.
\]

(2.13)

Since the expression in (2.13) contains neither the utility function nor the random variable, we have proved the proposition.

This result is similar to the role of a futures market as can be seen in the articles by Holthausen (1979) and Feder, Just, and Schmitz (1980). Differences in risk aversion or in the evaluation of foreign credit risks do not affect the production decision, although they affect the optimal purchase of coverage. A surprising and important part of this result is that the production decision is separated from the risk even if the firm purchases only a partial coverage, which means that the firm must incur a portion of the loss in total sales. In this case, export credit insurance is indeed a useful device for shielding the exporting firms from exposure to foreign credit risks. However, it will be shown that the result is sensitive to the reimbursement method used by the insurance agency.

Proposition 2.4

Under the proportional reimbursement method, the following statements are true:

(a) If the premium rate is fair, a risk averse firm exports as much as a risk-neutral firm does in the absence of insurance.

(b) If the premium rate is more than fair, a risk averse firm exports more than a risk neutral firm in the absence of insurance, and in
the extreme case in which the premium rate is zero, a risk averse firm exports as much as in the no-loss case.

Proof

Substituting \( q = \tilde{a} \) into (2.13) we have \( p(1 - \tilde{a}) = f' \). This equation is the same as (2.3), the condition for an optimal level of exports of a risk-neutral firm in the absence of export credit insurance. Therefore, the optimal level of exports is equal to \( x^n \) when \( q = \tilde{a} \). Since \( f' \) is an increasing function of \( x \), it follows that \( x^n < x \) when \( q < \tilde{a} \). In the extreme case when \( q = 0 \), condition (2.13) converges to (2.2) which is the condition for \( x^C \), exports in the no-loss case.

Several comments should be made about these results. As stated in Proposition 2.4, the insurance agency can increase exports by lowering the premium rate. Suppose firms in two different countries, which have the same production conditions, export to a common market in a risky third country. If the government of one country sets the premium rate lower than the other country, it can give the domestic firms a competitive advantage over firms in the other country. In this regard, one might say that the government intervenes in free trade by means of an export credit insurance program. However, so long as the premium rate is set at a fair rate, it is not the government agency that pays for the cost of risk, rather it is the private companies that pay for the risk in the form of premium payments. In this case the role of the government is solely to make the cost of the risk explicit. On the other hand, if the premium rate is intentionally set at a more than fair rate, which results in a higher export level than that of a risk-neutral firm in the absence of insurance, and a deficit is incurred in the
account of the insurance agency, one can argue that the government subsidizes exporting firms through the export credit insurance program.

Finally, we can infer from Propositions 2.1 and 2.4 that the effectiveness of export credit insurance as an export incentive program depends on the degree of risk aversion of the exporting firm. If the firm is very risk averse, the introduction of insurance will increase the level of export considerably. On the other hand, if the firm is risk neutral, the presence of insurance does not affect the level of exports at all as long as the program operates "cleanly" in the sense that the premium rate is set at a fair rate.

We must remember that some of the results obtained so far are sensitive to the reimbursement method employed in this section. In the next section we analyze a model which uses a different type of insurance payment method, and examine how the propositions derived in this section must be changed.

2.4 Insurance Contracts with Non-Proportional Reimbursement

In this section we study the model of export credit insurance which uses a payment method used by Smith (1968). Under this method, any amount of loss is compensated by the insurance agency as long as it is within the amount of the purchased coverage. In case of partial coverage, the exporting firm has to incur any loss in excess of the coverage. We will refer to it as the non-proportional method. This type of payment method is found in various types of casualty loss insurance. The reimbursement variable $m$ is formally defined as
Retaining the same notation and assumptions as in the previous section, we may write the firm's decision problem as follows:

$$\begin{align*}
\max V(x,y) &= \text{Eu}(\pi) = \int_0^{\frac{y}{px}} u[px - f(x) - b - qy]g(\alpha)d\alpha \\
&\quad + \int_{\frac{y}{px}}^1 u[(1 - \alpha)px + (1 - q)y - f(x) - b]g(\alpha)d\alpha ,
\end{align*}$$

subject to $0 \leq y \leq px$.

We assume once again that the firm produces some positive output. The first two partial derivatives with respect to $y$ are

$$\begin{align*}
V_y &= -q \cdot u'(\pi_1) \cdot G\left(\frac{y}{px}\right) + (1 - q) \int_{\frac{y}{px}}^1 u'(\pi_2)g(\alpha)d\alpha , \\
V_{yy} &= q^2 \cdot u''(\pi_1) \cdot G\left(\frac{y}{px}\right) + (1 - q)^2 \int_{\frac{y}{px}}^1 u''(\pi_2)g(\alpha)d\alpha - \frac{1}{px} u''(\pi_1)g\left(\frac{y}{px}\right) ,
\end{align*}$$

where $\pi_1 = px - f(x) - b - qy$, $\pi_2 = (1 - \alpha)px + (1 - q)y - f(x) - b$, and $G$ is a cumulative distribution of $\alpha$.

The second partial derivative is strictly negative for a risk averter ($u'' < 0$) and a risk-neutral firm ($u' = 0$), and may well be so even for some risk lover ($u'' > 0$). One should not be surprised at the fact that a risk-neutral or a risk-loving firm may purchase some insurance. As we have already discussed in Section 2.2, the nature of
risk in the insurance problem is a pure loss. Unlike in the problem of optimal portfolio investment, there is no possible gain in the case of default risk. The expected value of total sales in our model is always less than that in the no-loss case. Thus there is an incentive for buying insurance even among risk-neutral firms and some risk-loving firms.

We now investigate the boundary conditions for \( y \) by evaluating the first partial derivative at \( y = 0 \) and \( y = px \);

\[
V_y \big|_{y=0} = (1 - q) \int_0^1 u'(\pi_2) g(\alpha) d\alpha > 0, \tag{2.15}
\]

\[
V_y \big|_{y=px} = -qu'(\pi_1) \int_0^1 g(\alpha) d\alpha = -q \cdot u'(\pi_1) < 0. \tag{2.16}
\]

Since the objective function is strictly concave in \( y \), the inequalities in (2.15) and (2.16) imply that the solution for \( y \) is always in the interior if the premium rate is positive. We have established the following proposition.

**Proposition 2.5**

Under the non-proportional type of reimbursement method, a risk-neutral or risk-averse firm always chooses a partial coverage except when the premium rate is zero.

This result is in sharp contrast to Proposition 2.2. A risk neutral firm never purchases a partial coverage under the proportional type of reimbursement while it always purchases a partial coverage in the present case. The risk averse firm also purchases only a partial coverage irrespective of the level of the premium rate, while under the
proportional method it purchases a partial coverage only if the premium rate is unfair.

Under the non-proportional method, the objective function takes two different functional forms according to the domain of integration. This feature with the assumption of risk aversion gives us a considerable amount of algebraic complexities which prevent us from exploring further new insights in the problem. Therefore, we elect to concentrate on the risk neutral case in the rest of this section. In this way, we can still obtain interesting results without losing the essence of the model with the non-proportional reimbursement method.

Now the firm's objective is to maximize the expected profit:

$$\max_{x,y} V(x,y) = E\pi = \int_0^y \left[ px - f(x) - b - qy \right] g(\alpha) d\alpha$$

$$+ \int_{y}^{1} \left[ (1 - \alpha)px + y - f(x) - b - qy \right] g(\alpha) d\alpha .$$

Assuming the existence of an interior maximum, necessary conditions take the following forms:

$$V_x = \left[ 1 - \int_y^{1} g(\alpha) d\alpha \right] p - f' = 0 , \quad (2.17)$$

$$V_y = \int_y^{1} g(\alpha) d\alpha - q = 0 . \quad (2.18)$$

Sufficient conditions are shown to be satisfied as follows:
\[
\begin{align*}
V_{xx} &= -f'' - \frac{(py)^2}{(px)^3} g\left(\frac{y}{px}\right) < 0 , \\
V_{yy} &= -\frac{1}{px} g\left(\frac{y}{px}\right) < 0 , \\
V_{xx} V_{yy} - V_{xy} V_{yx} &= \frac{1}{px} f'' g\left(\frac{y}{px}\right) > 0 ,
\end{align*}
\]

(2.19)

where \( V_{xy} = V_{yx} = \frac{py}{(px)^2} g\left(\frac{y}{px}\right) \).

Let \( G(a) \) denote the cumulative distribution of \( a \). We can rewrite the first order conditions as follows:

\[
\left[ \frac{y}{px} G\left(\frac{y}{px}\right) + \int_{\frac{y}{px}}^{1} G(a) da \right] p - f' = 0 ,
\]

(2.17')

\[
1 - q - G\left(\frac{y}{px}\right) = 0 .
\]

(2.18')

We are ready to state the next proposition.

**Proposition 2.6**

Under the non-proportional type of reimbursement, a risk neutral firm exports more than under the proportional type of reimbursement.

**Proof**

From (2.18'), we can determine the proportion of total sales that the firm wants to insure, \( \frac{y}{px} \), at a given premium rate. The optimal proportion is obtained at a point of intersection between a horizontal line \( 1 - q \) and the cumulative distribution \( G(a) \) as depicted in Figure 2.1. Since the integral \( \int_{\frac{y}{px}}^{1} G(a) da \) represents the area shaded in Figure 2.1,
Figure 2.1. Non-proportional reimbursement
the following inequality must hold:
\[
\int_{V}^{px} G(a) da > (1 - q)(1 - \frac{V}{px}).
\]

Rearranging the terms and multiplying both sides by \( p \), we have
\[
[\frac{V}{px} (1 - q) + \int_{V}^{px} G(a) da]p > (1 - q)p.
\]

According to (2.13), the right-hand side of (2.20) is the marginal revenue with respect to output under the proportional method when the firm purchases insurance. In view of (2.17') and (2.18'), we know that the left-hand side in (2.20) is the marginal revenue under the non-proportional method. Since by assumption the marginal cost \( f' \) is an increasing function of \( x \), we have proved the proposition.

Under the proportional reimbursement method, a risk neutral firm purchases insurance (full coverage) if and only if the premium rate is more than fair, which implies that there is a deficit in the insurance agency's account. An increase in exports is attained at the expense of a deficit. Since Proposition 2.6 asserts that the level of export is higher under the non-proportional method, we ought to investigate what is happening in the insurance agency's account. The result is a bit surprising.

**Proposition 2.7**

Under the non-proportional type of reimbursement method, the insurance agency's account is always in a deficit when a risk neutral firm purchases insurance.
Proof

From (2.14), the expected indemnity payment by the insurance agency is given by

\[ E_m = y - px \int_0^y G(\alpha) d\alpha. \] (2.21)

From (2.18'), and in view of Figure 2.1, the following inequality must hold for any premium rate \( q \):

\[ \int_0^y G(\alpha) d\alpha < \frac{y}{px} (1 - q). \]

Rearranging the terms and multiplying both sides by \( y \), we have

\[ qy < y - px \int_0^y G(\alpha) d\alpha. \] (2.22)

Since the left-hand side of (2.22) is the premium revenue for the insurance agency, and the right-hand side is the expected indemnity from (2.21), we have established the proposition.

The implication of this proposition is significant. Of course, whether the firm is risk neutral or risk averse is an empirical question. But if it were risk neutral, the export credit insurance program would not be self-supporting. The government agency must subsidize risk-neutral exporting firms.

Next we investigate whether the optimal level of export is independent of the distribution of the random variable \( \alpha \) as was shown in Proposition 2.3. One way to investigate this problem is to introduce a shift parameter into the probability distribution, and examine the effect of a small change in the parameter. If the optimal level of
export is independent of the risk, a shift in the probability distribution will not have any effect on export.

We introduce a shift parameter $k$ into the cumulative distribution of $\alpha$. We write $G(\alpha,k)$ in place of $G(\alpha)$ in (2.17') and (2.18'). Let $G_k(\alpha,k)$ denote the partial derivative with respect to $k$. We assume that $G_k$ is not zero. Differentiating the first-order conditions with respect to $k$, we obtain the following system of equations:

$$
\begin{bmatrix}
V_{xx} & V_{xy} \\
V_{yx} & V_{yy}
\end{bmatrix}
\begin{bmatrix}
\frac{3x}{\partial k} \\
\frac{3y}{\partial k}
\end{bmatrix}
= 
\begin{bmatrix}
- \left\{ \frac{v}{px} G_k(\frac{v}{px},k) + \int \frac{1}{px} G_k(\alpha,k)d\alpha \right\} p \\
G_k(\frac{v}{px},k)
\end{bmatrix}.
$$

Solving the above for $\frac{3x}{\partial k}$, we have

$$
\frac{3x}{\partial k} = \frac{p}{\eta} \int \frac{1}{px} G_k(\alpha,k)d\alpha.
$$

Clearly, $\frac{3x}{\partial k}$ is not zero. Therefore we have established the following proposition.

**Proposition 2.8**

Under the non-proportional reimbursement method, the optimal level of export is **not** independent of the risk.

It is clear that, unlike under the proportional reimbursement method, the production decisions are not free from considerations of risk. Thus, the mere availability of insurance does not necessarily guarantee the separation of production decisions from the risk as one
might think. The separation property is sensitive to the type of reimbursement method used by the insurance agency.

Finally, we study the effect of a small change in the premium rate and the price of the exported commodity on the level of exports and the optimal amount of coverage. Differentiating the first order conditions (2.17) and (2.18) implicitly with respect to \( q \), we have the following system of the equations:

\[
\begin{bmatrix}
V_{xx} & V_{xy} \\
V_{yx} & V_{yy}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial x}{\partial q} \\
\frac{\partial y}{\partial q}
\end{bmatrix}
= \begin{bmatrix}
0 \\
1
\end{bmatrix}.
\]

Solving the above for \( \frac{\partial x}{\partial q} \) and \( \frac{\partial y}{\partial q} \), we have

\[
\frac{\partial x}{\partial q} = -\frac{1}{\text{ii}} \cdot \frac{y}{x} < 0, \tag{2.23}
\]

\[
\frac{\partial y}{\partial q} = -\frac{px}{q g(\frac{y}{px})} + \frac{y}{x} \cdot \frac{\partial x}{\partial q} < 0. \tag{2.24}
\]

**Proposition 2.9**

Under the non-proportional type of reimbursement method, as the premium rate becomes lower, a risk neutral firm increases the level of exports and the amount of insurance coverage.

As under the proportional method, the insurance agency can increase exports by lowering the premium rate. There are two effects working on the optimal amount of coverage. One is a direct effect which is always negative, as we can see in the first term of the right-hand side in (2.24). The other is an indirect effect operating through a change in
the level of production. Since a lower premium rate induces more export, there is an increased need for export credit insurance. Both effects are working in the same direction, so that a reduced premium rate unambiguously increases the demand for insurance.

We can analyze the effects of a change in the price of the product on the amounts of exports and coverage in the same manner. Differentiating the first-order conditions with respect to $p$, we have

\[
\begin{bmatrix}
V_{xx} & V_{xy} \\
V_{xy} & V_{yy}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial x}{\partial p} \\
\frac{\partial y}{\partial p}
\end{bmatrix}
= 
\begin{bmatrix}
-(1-\theta) + \frac{y^2}{(px)^2} g\left(\frac{y}{px}\right) \\
- \frac{xy}{(px)^2} g\left(\frac{y}{px}\right)
\end{bmatrix}
\]

where $\theta = \int_{px}^{1} \frac{1}{\alpha} g(\alpha) d\alpha$.

Solving the above for $\frac{\partial x}{\partial p}$ and $\frac{\partial y}{\partial p}$, we have

\[
\frac{\partial x}{\partial p} = \frac{1-\theta}{f^n} > 0,
\]

\[
\frac{\partial y}{\partial p} = \frac{y}{p} + \frac{y}{x} \frac{\partial x}{\partial p} > 0.
\]

Proposition 2.10

Under the non-proportional type of reimbursement method, as the price of the commodity becomes higher, a risk neutral firm increases the level of exports and the amount of insurance coverage.
2.5 Insurance Contracts With a Deductible Provision

In this section we consider an insurance contract which includes a deductible feature. According to the Master Policy which is provided by FCIA in the U.S., the exporting firm is not allowed to buy partial coverage, but it can choose a certain amount of deductible, \( z \). Under this provision, the firm covers any loss less than or equal to \( z \), and the insurance agency covers any excess over \( z \). The reimbursement variable, \( m \), is formally specified as

\[
m = \begin{cases} 
0 & \text{if } 0 < \alpha \leq \frac{z}{p_x} \\
\alpha p_x - z & \text{if } \frac{z}{p_x} < \alpha < 1
\end{cases}
\]

The premium payment, \( Q \), is assumed to be a decreasing and convex function of \( z \);

\[
Q = \phi(z), \quad \phi' < 0, \quad \phi'' > 0.
\]  

(2.25)

The function should also satisfy the restriction

\[
\phi(p_x) = 0,
\]  

(2.26)

because \( z = p_x \) implies that the firm does not purchase any insurance.

The exporting firm's profit function is

\[
\pi = (1 - \alpha)p_x + m - f(x) - b - Q.
\]

The exporting firm chooses the optimal amounts of \( z \) and \( x \) in order to maximize the expected utility of profit. The decision problem is described as follows:
Max \( V(x,z) = E(u) = \int_0^{\frac{z}{px}} u[(1- \alpha)px - f(x) - b - \phi(z)]g(\alpha)d\alpha \)

\[ + \int_{\frac{z}{px}}^1 u[px - z - f(x) - b - \phi(z)]g(\alpha)d\alpha \]

subject to \( 0 \leq z \leq px \).

Let \( \pi_1 \) and \( \pi_2 \) denote the arguments of the utility function in the first and second terms, respectively. The first two partial derivatives with respect to \( z \) are

\[ V_z = - \phi' \left[ \int_0^{\frac{z}{px}} u'(\pi_1)g(\alpha)d\alpha - (1 + \phi') \int_{\frac{z}{px}}^1 u'(\pi_2)g(\alpha)d\alpha, \right. \]

\[ V_{zz} = - \phi'' \cdot \left[ \int_0^{\frac{z}{px}} u'(\pi_1)g(\alpha)d\alpha + \int_{\frac{z}{px}}^1 u'(\pi_2)g(\alpha)d\alpha \right] \]

\[ + (\phi')^2 \int_0^{\frac{z}{px}} u''(\pi_1)g(\alpha)d\alpha + (1 + \phi')^2 \int_{\frac{z}{px}}^1 u''(\pi_2)g(\alpha)d\alpha \]

\[ + \frac{1}{px} u'(\pi_1)g\left(\frac{z}{px}\right). \]

The second derivative is not necessarily negative. Using a similar model to ours, Gould (1969) mistakenly claimed that the condition \( \phi'' > 0 \) is a sufficient condition for guaranteeing the negativity of \( V_{zz} \). This mistake was recently pointed out by Schlesinger (1983). Because of the presence of the last term, \( V_{zz} \) can be positive. This implies that we have to compare the expected utility of an interior maximum with those at the boundary points.
If the premium rate at \( z = 0 \) is fair or more than fair, then the expected utility at \( z = 0 \) is always greater than that at \( z = px \), as shown below. Since the utility function is concave, the following inequality must hold from Jensen's inequality:

\[
Eu[(1 - \alpha)px - f(x) - b] < u[px - f(x) - b - \bar{\alpha}px],
\]

where \( \bar{\alpha} \) is the expected value of \( \alpha \). Since \( u' > 0 \), the following inequality must also hold if the premium rate at \( z = 0 \) is fair or more than fair \( \left( \frac{\phi(0)}{px} \leq \bar{\alpha} \right) \):

\[
u[px - f(x) - b - \bar{\alpha}px] \leq u[px - f(x) - b - \phi(0)].
\]

Therefore, \( V|_{z=px} < V|_{z=0} \) if the premium rate is fair or more than fair.

We further investigate the boundary points by evaluating the first partial derivative at \( z = 0 \) and \( z = px \):

\[
V_{z}|_{z=0} = -(1 + \phi'(0))u'[px - f(x) - b - \phi(0)] \leq 0, \quad (2.27)
\]

\[
V_{z}|_{z=px} = -\phi'(px)Eu'[(1 - \alpha)px - f(x) - b] > 0. \quad (2.28)
\]

The sign of \( V_{z}|_{z=0} \) depends on whether the absolute value of the slope of premium function at \( z = 0 \) (\( |\phi'(0)| \)) is larger or smaller than one. Two cases are illustrated in Figure 2.2. The slope of the dotted line between \( \phi(0) \) and \( px \) represents the premium rate in the case of no deductible. The assumption of convexity implies that all the premium functions must pass through the inside of the triangle \( \phi(0) - 0 - px \). The 45-degree line which is drawn from the point of \( \phi(0) \) to the horizontal
Figure 2.2. Deductible function
axis must also lie in the triangle because the premium rate cannot exceed one. Case 1 and Case 2 represent the premium functions for which the absolute value of the slope at \( z = 0 \) is less or more than one, respectively. At a given amount of deductible \( z_0 \), as we can see from Figure 2.2, a reduction in the premium payment is considerably larger in Case 2 \( (\phi^2(z_0) < \phi^1(z_0)) \). The premium function in Case 2 is more favorable for exporting firms than those in Case 1. From (2.27), \(|\phi'(0)| > 1\) is a necessary and sufficient condition for the positivity of \( V_z \). Furthermore, from (2.28), we know that \( V_z \big|_{z=0} \) is always positive. Therefore, if the level of utility at \( z = 0 \) is higher than the expected utility at \( z = px \) as it is depicted in Figure 3.3, then \(|\phi'(0)| > 1\) must imply that an interior maximum is a global maximum. \( z^* \) in Figure 2.3 illustrates such a maximum. We have established the following proposition.

**Proposition 2.11**

If the premium rate with full coverage (zero deductible) is fair or more than fair, and the slope of the premium function at the origin is larger than one in magnitude, then the firm always chooses some amount of deductible.

Using a similar model, Mossin (1968) concluded that some positive amount of deductible is always optimal. However, his conclusion was derived under a specific type of premium function; namely,

\[
Q = \phi(z) = (1+k)\text{Em} = (1+k) \int_{\frac{z}{px}}^{\frac{1}{a}} (apx - z)g(\alpha)d\alpha, \quad (2.29)
\]

where \( k \) is a positive number which represents the loading factor. We
Figure 2.3. Expected utility function under a deductible provision.
can show that (2.29) satisfies the conditions which are imposed on the premium function in (2.25) and (2.26);

\[ \phi' = -(1 + k)[1 - G(\frac{z}{px})] < 0 , \]

\[ \phi'' = (1 + k) \frac{1}{px} g(\frac{z}{px}) > 0 , \]

\[ \phi(px) = (1 + k) \int_{\frac{px}{px}}^{1} (\alpha px - px)g(\alpha)d\alpha = 0 . \]

Therefore, Mossin's specification is a special case of ours. Evaluating the first derivative of the premium function at \( z = 0 \), we obtain

\[ \phi'(0) = -(1 + k). \]

Since \( k > 0 \), we have \( |\phi'(0)| > 1 \). This clearly shows that Mossin's special premium function is an example of our Case 2 illustrated above, in which the firm will take some positive deductible.

2.6 Summary

In this chapter, we have studied some basic models of export credit insurance in the framework of the theory of the firm under uncertainty. Using two different types of reimbursement methods, we analyzed and discussed some interesting features of export credit insurance. Unlike in models of futures markets, the presence of this insurance program does not necessarily separate the exporting firm's production decision from risk management, although it does so under a certain type of insurance payment.

It is commonly asserted that government-supported insurance should be priced so as to be self-supporting; namely, the premium rate should
be set at an actuarially fair rate so that the insurance agent's account is in balance on the average. However, we have shown that in order to increase exports effectively, the premium rate must be set as low as possible. The government can utilize export credit insurance aggressively to promote exports by intentionally setting a more-than-favorable premium rate. If this is done, the government is subsidizing exporting firms through export credit insurance.

The firm's attitude toward risk is also critical to the effectiveness of the program. Risk neutral firms never purchase insurance unless the premium rate is more than fair in one case, and the insurance agency's account is always in a deficit when risk neutral firms purchase insurance in the other case. Therefore the success of export credit insurance under self-supporting constraint hinges on the presence of a fairly large number of risk averse firms.

Finally, we have studied the insurance contract which has a deductible feature. It was shown that the exporting firms may not choose any amount of deductible unless a reduction in premium payment is substantial. The introduction of the deductible provision could be redundant.

Results in this chapter were derived under somewhat stringent conditions, i.e., we assumed that the firm is a price taker and exports all its products to the foreign market. These assumptions will be relaxed in the next chapter.
CHAPTER III

OPTIMAL COVERAGE:
MODELS WITH A DOMESTIC MARKET

3.1 Introduction

In the previous chapter it was assumed that the firm exports all its product to a foreign market. This assumption is relaxed in this chapter by introducing a domestic market; it will be assumed that the firm has an opportunity to sell its product in a riskless domestic market. We are interested in investigating how some of the results in Chapter II are affected by this modification of the model. We also study the case in which the firm has monopolistic power in either the domestic market or in both markets, and it is allowed to practice price discrimination between the two markets. The behavior of this type of firm has been traditionally used as an explanation for "dumping" by students of international trade. (In this regard, one can refer to Caves and Jones (1973, pp. 212-215), Davies and McGuinness (1982), and Ethier (1982).) The behavior of a price discriminating firm under uncertainty was recently studied by Katz, Paroush, and Kahana (1982). It will be shown that the policy implications discussed in their paper must be changed in the presence of export credit insurance.
In order to avoid unnecessary algebraic complexities, we assume that export credit insurance is provided with the proportional reimbursement method.

3.2 Competitive Firm

In the world with certainty, when the firm is a price taker in both domestic and foreign markets, the problem of the optimal allocation of its product between the two markets is rather trivial. If we assume that there is no transportation cost, the firm either exports all its product to the foreign market or sells only in the domestic market, depending on which of the two markets has the higher price. However, if we introduce uncertainty in the foreign market, the firm may engage in sales in both markets in order to hedge against the foreign risk.

We first analyze the case in which export credit insurance is not available. Retaining assumptions A.1 - A.4 and the notation in Section 2.1, we can write the profit function as follows:

\[ \pi = p_1(x - x_2) + (1-a)p_2x_2 - f(x) - b, \]

where \( x \) is the total output, \( x_2 \) is the level of exports, and \( p_i \) is the price in the \( i \)th market (\( i = 1 \) denotes the domestic market and \( i = 2 \) the foreign market).

The firm's decision problem is the maximization of expected utility from profit which is described as

\[
\max_{x,x_2} V(x,x_2) = \text{Eu}(\pi).
\]
First, we investigate the boundary conditions for export. The first two partial derivatives with respect to \( x_2 \) are
\[
V_{x_2} = \text{Eu}' \cdot [(1 - \alpha)p_2 - p_1],
\]
\[
V_{x_2 x_2} = \text{Eu}'' \cdot [(1 - \alpha)p_2 - p_1]^2 < 0.
\]

Since the second derivative is always negative in the case of risk aversion, a necessary and sufficient condition for the firm not to export is that the first derivative at \( x_2 = 0 \) is non-positive, that is,
\[
V_{x_2} \bigg|_{x_2=0} = \text{u}'[p_1 x - f(x) - b] \cdot [(1 - \alpha)p_2 - p_1] \leq 0.
\]

Since \( \text{u}' > 0 \), \( x_2 = 0 \) if and only if \( (1 - \alpha)p_2 \leq p_1 \), \( x_2 > 0 \) if and only if \( (1 - \alpha)p_2 > p_1 \). The firm exports at least some amount of its product to the foreign market when the expected marginal revenue in the foreign market is greater than the marginal revenue in the domestic market.

**Proposition 3.1**

In the presence of a domestic market, the inequality \( (1 - \alpha)p_2 > p_1 \) is a necessary and sufficient condition for the competitive firm to export some of its output to the foreign market.

On the other hand, the firm exports all its product to the foreign market if and only if the first derivative evaluated at \( x_2 = x \) is positive, that is
\[
V_{x_2} \bigg|_{x_2=x} = \text{Eu}'[(1 - \alpha)p_2 x - f(x) - b] \cdot [(1 - \alpha)p_2 - p_1] > 0.
\]
In this case, we obtain only a necessary condition. We can rewrite the right-hand side of (3.4) as follows:

\[ V_{x_2}^{x_2} \big|_{x_2 = x} = Eu'[(1 - \alpha)p_2x - f(x) - b] \cdot [(1 - \tilde{\alpha})p_2 - p_1] \]

\[ + \text{cov}[u',{(1 - \alpha)p_2 - p_1}]. \]  

(3.4')

It can be shown that \( \text{cov}[u',{(1 - \alpha)p_2 - p_1}] < 0 \) because \( u' \) is an increasing function of \( \alpha \), and \( {(1 - \alpha)p_2 - p_1} \) is a decreasing function of \( \alpha \). Thus in order for the inequality in (3.4) to hold, the first term of the right-hand side in (3.4') must be positive. Hence, we see that \( (1 - \tilde{\alpha})p_2 > p_1 \) is a necessary condition for \( x_2 = x \). The fact that this condition is only necessary implies that even if this inequality holds, some risk averse firms will not sell all of its product in the foreign market.

Next, we consider the case in which the solution is interior. Assuming the existence of an interior maximum, we have the following first order conditions:

\[ V_x = (p_1 - f') \cdot Eu' = 0, \quad \text{or} \quad p_1 = f', \]  

(3.5)

\[ V_{x_2} = Eu' \cdot [(1 - \alpha)p_2 - p_1] = 0. \]  

(3.6)

Second order conditions are

\[ V_{xx} = (p_1 - f')^2 \cdot Eu'' - f''Eu' < 0, \]  

(3.7a)

\[ V_{xx} \cdot V_{x_2 x_2} - (V_{x_2})^2 > 0, \]  

(3.7b)
The condition in (3.7a) is always satisfied under the assumption of risk aversion, and non-decreasing marginal cost. We assume that the condition in (3.7b) is satisfied.

We can obtain the optimal value of total output $x$ by solving Equation (3.5). It is important to note that the solution is independent of the utility function as well as the probability distribution of the foreign credit risk. The optimal value of $x$ depends entirely on the market price in the domestic market, and on the firm's cost conditions. Substituting this optimal total output into (3.6), we can solve for an optimal value of export. Obviously, the solution depends on prices in both markets, the attitude towards risk, and the evaluation of risk. This result is similar to those in studies of futures markets by Holtausen (1979) and Feder, Just, and Schmitz (1980).

We are now ready to introduce export credit insurance into the model. Suppose that export credit insurance is available with the proportional reimbursement method which was introduced in Section 2.2. We can write the profit function as follows:

$$\pi = p_1(x - x_2) + (1 - \alpha)p_2x_2 + \alpha y - f(x) - b - qy,$$

where $y$ is the amount of insurance coverage, and $q$ is the premium rate per dollar of coverage.

We must note that the firm cannot purchase insurance without exporting; in the case of $x_2 = 0$, the profit function should be written as $\pi = p_1x - f(x) - b$. We assume that the firm cannot purchase more
insurance than the value of exports. The firm's decision problem is described as follows:

$$\begin{align*}
\text{Max} \quad V(x, x_2, y) &= Eu(\pi), \\
\text{subject to} \quad 0 \leq y \leq p_2x_2.
\end{align*}$$

First we investigate the boundary condition for $x$. It is important to consider this condition first because there is no export and no demand for insurance if the firm does not produce at all. The first two partial derivatives with respect to $x$ are

$$
V_x = (p_1 - f') \cdot Eu',
$$

$$
V_{xx} = (p_1 - f')^2 \cdot Eu'' - f''Eu' < 0.
$$

The negativity of $V_{xx}$ follows from the assumptions of risk aversion and increasing marginal cost. The necessary and sufficient conditions for some positive output to be optimal is that $V_x$ evaluated at $x = 0$ is positive:

$$
V_x|_{x=0} = [p_1 - f'(0)] \cdot Eu'(-b) > 0.
$$

In order to avoid the trivial case (no output, no export, and no insurance), we assume that the condition $p_1 > f'(0)$ is always satisfied.

Next, we investigate the boundary condition for $x_2$. The first two partial derivatives with respect to $x_2$ are

$$
V_{x_2} = Eu' \cdot [(1 - \alpha)p_2 - p_1],
$$

$$
V_{x_2}x_2 = Eu'' \cdot [(1 - \alpha)p_2 - p_1]^2 < 0.
$$
From the same argument we made in the derivation of Proposition 3.1, the following condition is necessary and sufficient for some positive amount of export:

\[(1 - \bar{a})p_2 > p_1.\]  

(3.10)

Since the firm cannot purchase insurance without exporting, the opposite case of the condition (3.10) \(((1 - \bar{a})p_2 \leq p_1)\) is a sufficient condition for no insurance purchase \((y = 0)\).

Finally, we investigate the boundary condition for \(y\). The first two partial derivatives with respect to \(y\) are

\[V_y = Eu' \cdot (\alpha - q),\]  

(3.11)

\[V_{yy} = Eu'' \cdot (\alpha - q)^2 < 0.\]

Using the same argument as in Section 2.3, we can show that a fair or more than fair premium \((q < \bar{a})\) is a necessary and sufficient condition for full coverage, and an unfair premium \((q > \bar{a})\) is a necessary condition for no coverage. We now have two conditions for no coverage. One is a sufficient condition, that is

\[(1 - \bar{a})p_2 \leq p_1,\]  

(3.12)

and the other is a necessary condition, that is,

\[\bar{a} < q.\]  

(3.13)

These conditions are significant when we later summarize the solutions for optimal levels of exports and insurance coverage according to the values of parameters.
Now we assume that the solutions for $x$ and $y$ are interior, and then show that the solution for $x_2$ cannot be interior; the firm either exports all its product or none according to the parameters of the model ($p_1$, $p_2$, and $q$). If the solutions for $x$ and $y$ are interior, the following equations must hold:

$$P_1 - f' = 0, \quad (3.8')$$

$$Eu' \cdot (\alpha - q) = 0. \quad (3.11')$$

The sufficient conditions for the maximization problem are

$$V_{xx} < 0, \quad \begin{vmatrix} V_{xx} & 0 \\ 0 & V_{x2x2} \end{vmatrix} > 0, \quad \begin{vmatrix} V_{xx} & 0 & 0 \\ 0 & V_{x2x2} & V_{x2y} \\ 0 & V_{yx2} & V_{yy} \end{vmatrix} < 0, (3.14)$$

where these co-factors are evaluated at the optimum solution so that $V_{xx} = V_{x2x} = V_{xy} = V_{yx} = 0$ from (3.8') and (3.11'). If we assume that $V_{x2x2} V_{yy} - V_{x2y} V_{yx2} = \{Eu'' \cdot [(1 - \alpha)p_2 - p_1]^2 \} \cdot \{Eu'' \cdot (\alpha - q)^2 \} - \{Eu'' \cdot [(1 - \alpha)p_2 - p_1] \cdot (\alpha - q)\}^2 > 0$, then the conditions in (3.14) are satisfied.

From (3.11'), we have $Eu' \alpha = qEu'$. Substituting $qEu'$ in the place of $Eu' \alpha$ in Equation (3.9), we obtain

$$V_{x2} = [(1 - q)p_2 - p_1] \cdot Eu'.$$

Since the marginal utility is positive, the sign of $V_{x2}$ is the same as
that of \([(1 - q)p_2 - p_1]\). The terms \((1 - q)p_2\) and \(p_1\) are the marginal revenues from the sales in the foreign and domestic markets, respectively. Let us call the expression \((1 - q)p_2\), the net foreign price. If the net foreign price is larger than the domestic price, \(V_{x_2}\) is always positive, so that the solution is at the upper bound \((x_2 = x)\). On the other hand, if the net foreign price is less than the domestic price, \(V_{x_2}\) is always negative, so that the solution is at the lower bound \((x_2 = 0)\).

We now have three important values for the parameters in the model: \(p_1\), the domestic price; \((1 - q)p_2\), the net foreign price; \((1 - \bar{a})p_2\), the expected foreign price. According to these parameters, we can summarize the optimal solutions for export and insurance coverage as in Table 3.1. In order to see the effect of export credit insurance, the optimal level of export when the insurance is not available is also included in the table.

It was argued in Section 2.3 that a fair premium rate is a necessary and sufficient condition for full coverage when there is no domestic market. Case 2 in Table 3.1 provides a counter example to this assertion. If the domestic price is larger than the expected foreign price or the net foreign price, the firm does not export at all. Hence, it does not buy any insurance. The presence of the domestic market imposes a significant limitation on the effectiveness of export credit insurance as a means of promoting exports. The price in the foreign market must be sufficiently higher than the domestic price. Otherwise, the introduction of export credit insurance does not have any effect on the level of exports. Cases 4 - 6 in Table 3.1 provide the instances in which export credit insurance could be effective to stimulate the level of exports.
<table>
<thead>
<tr>
<th>Case</th>
<th>Condition</th>
<th>Premium Rate</th>
<th>Export</th>
<th>Export</th>
<th>Insurance Coverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((1-q)p_2 &lt; (1-\alpha)p_2 &lt; p_1)</td>
<td>unfair</td>
<td>none</td>
<td>none</td>
<td>none</td>
</tr>
<tr>
<td>2</td>
<td>((1-\alpha)p_2 \leq (1-q)p_2 \leq p_1)</td>
<td>fair</td>
<td>none</td>
<td>none</td>
<td>none</td>
</tr>
<tr>
<td>3</td>
<td>((1-q)p_2 &lt; p_1 &lt; (1-\alpha)p_2)</td>
<td>unfair</td>
<td>some or all</td>
<td>some or all</td>
<td>none</td>
</tr>
<tr>
<td>4</td>
<td>((1-\alpha)p_2 \leq p_1 \leq (1-q)p_2)</td>
<td>fair</td>
<td>none</td>
<td>all</td>
<td>full</td>
</tr>
<tr>
<td>5</td>
<td>(p_1 \leq (1-\alpha)p_2 \leq (1-q)p_2)</td>
<td>fair</td>
<td>some or all</td>
<td>all</td>
<td>full</td>
</tr>
<tr>
<td>6</td>
<td>(p_1 &lt; (1-q)p_2 &lt; (1-\alpha)p_2)</td>
<td>unfair</td>
<td>some or all</td>
<td>all</td>
<td>partial</td>
</tr>
</tbody>
</table>
The introduction of export credit insurance induces the firm to export all of its product to the foreign market. The results are summarized verbally in the following proposition.

Proposition 3.2

a) If the domestic price is higher than both the net foreign price and the expected foreign price, the firm does not export at all even though the insurance is available at a fair rate.

b) If the domestic price is higher than the net foreign price but lower than the expected foreign price, the firm exports at least some of its product without purchasing insurance.

c) If the net foreign price is higher than the domestic price, the firm exports all of its product to the foreign market and purchases at least some amount of insurance.

Finally, we must make a comment on the price adjustment in the domestic market. When export credit insurance is not available, the competitive firm engages in sales in both markets. However, in the presence of insurance, if the net foreign price is higher than the domestic price, the firm starts exporting all of its product to the foreign market, which will reduce the total supply in the domestic market. If the other conditions in the economy stay the same, the domestic market price must rise presumably until it becomes equivalent to the net foreign price. Therefore, although our analysis was carried out in a partial-equilibrium setting, we can suspect that the domestic consumer might suffer some welfare loss as a result of the introduction of export credit insurance. However, in order to discuss this issue,
one must solve the problem in a general equilibrium model, which we do not pursue in this study.

3.3 Domestic Monopoly

In this section we study the case in which the firm has monoplastic power in the domestic market, while being a price taker in the foreign market. Assuming that effective price discrimination is physically and legally possible, the monopolist's problem is to choose the optimal distribution of its output between the two markets.

Retaining the assumptions and notation in Section 3.2, we can write the profit function as follows:

\[ \pi = x_1 h(x_1) + (1 - \alpha)p_2 x_2 + \alpha y - f(x_1 + x_2) - b - qy, \quad (3.15) \]

where \( h \) is the inverse demand function in the domestic market, and \( x_i \) is the amount of the product sold in the domestic (\( i = 1 \)) or foreign (\( i = 2 \)) market.

The firm's decision problem is to choose optimal values of \( x_1, x_2, \) and \( y \) in order to maximize expected utility from profit,

\[
\max_{x_1, x_2, y} V(x_1, x_2, y) = Eu(\pi).
\]

If there exists an interior maximum for this problem, then the necessary conditions are

\[
V_{x_1} = [h(x_1) + x_1 h' - f'] \cdot Eu' = 0, \quad (3.16)
\]
\[
V_{x_2} = Eu' \cdot [(1 - \alpha)p_2 - f'] = 0, \quad (3.17)
\]
\[
V_y = Eu' \cdot (\alpha - q) = 0. \quad (3.18)
\]
The second order conditions take the following form:

\[
\begin{vmatrix}
V_{x_1^2} & V_{x_1 x_2} \\
V_{x_2 x_1} & V_{x_2^2}
\end{vmatrix} < 0,
\begin{vmatrix}
V_{x_1 x_1} & V_{x_1 x_2} & V_{x_1 y} \\
V_{x_2 x_1} & V_{x_2 x_2} & V_{x_2 y} \\
V_{y x_1} & V_{y x_2} & V_{y y}
\end{vmatrix} < 0.
\]

We assume that the above conditions are satisfied.

As before, from (3.17) and (3.18), we can derive the following equation:

\[(1 - q)p_2 - f' = 0.\]  \(\text{(3.19)}\)

We can rewrite the condition (3.16) as follows:

\[(1 - \eta)p_1 - f' = 0,\]  \(\text{(3.20)}\)

where \(\eta\) is the inverse of the absolute value of the demand elasticity in the domestic market which is also known as Lerner's measure of monopoly power. Solving (3.19) and (3.20) simultaneously, we can obtain the optimal values for \(x_1\) and \(x_2\). These optimal solutions are obviously independent of the attitude toward risk and the probability distribution function. We can once again confirm the validity of Proposition 2.3; if export credit insurance is available with the proportional reimbursement method, the optimal level of export is independent of the attitude toward risk and the distribution of the random variable.

From (3.19) and (3.20), we obtain \((1 - \eta)p_1 = (1 - q)p_2\). Then the next proposition follows immediately.
Proposition 3.3

The domestic monopolist charges a higher domestic price than the foreign price if and only if the premium rate of export credit insurance is less than Lerner's measure of monopoly power, provided that it sells in both markets.

Under certainty, the firm never charges a lower domestic price than the foreign price. (Refer to Caves and Jones (1973) and Davies and McGuinness (1982) about this point.) However, when there exist risks in the foreign market, the firm may set the domestic price lower than the foreign price. This will happen if the premium rate is relatively high (which may imply that it is very risky to export), and the firm's monopolistic power is not relatively strong. The economic reason for this result follows from the fact that the risk averse firm is willing to give up some opportunities to make profit in order to avoid a risky situation. But one should remember that the domestic price is always higher than the marginal revenue from the foreign market even in our model as it is illustrated in Figure 3.1. The total output, \( x \), is determined at a point of intersection between the marginal revenue from the foreign market, \((1 - q)p_2\), and the marginal cost curve. Furthermore, the point of intersection between \((1 - q)p_2\) and the domestic marginal revenue curve determines the amount of output which should be distributed in the domestic market, \( x_1 \). Then the amount \( x_2 \) is exported to the foreign market. Since the marginal revenue curve always lies below the demand curve, the domestic price \( p_1 \) must always be greater than the marginal revenue from the foreign market.
Figure 3.1. Export of domestic monopoly
We can also see from Figure 3.1 that a point of intersection between MC and MR from the domestic market determines the lower bound for the net foreign price \((1-q)p_2\). If \((1-q)p_2\) is lower than that price, the firm never exports, and distributes all its product to the domestic market.

Next we study the effect of a small change in the premium rate on the level of export and the amount of sales in the domestic market. Differentiating (3.16) and (3.19) implicitly with respect to \(q\), we obtain the following system of equations:

\[
\begin{bmatrix}
2h' + x_1h'' - f'' & -f'' \\
-f'' & -f''
\end{bmatrix}
\begin{bmatrix}
\frac{\partial x_1}{\partial q} \\
\frac{\partial x_2}{\partial q}
\end{bmatrix} = \begin{bmatrix} 0 \\ p_2 \end{bmatrix}.
\]

Using Cramer's rule, we obtain

\[
\frac{\partial x_1}{\partial q} = \frac{1}{D} p_2 f'', \quad (3.21)
\]

\[
\frac{\partial x_2}{\partial q} = \frac{1}{D} p_2 (2h' + x_1h'' - f''), \quad (3.22)
\]

where \(D = -f''(2h' + x_1h'')\).

We assume that the marginal revenue from the domestic market is a decreasing function of \(x_1\), which is equivalent to assuming that \(2h' + x_1h'' < 0\). Hence, we have \(D > 0\), \(\frac{\partial x_1}{\partial q} > 0\), and \(\frac{\partial x_2}{\partial q} < 0\). Since \(h' < 0\), \(\frac{\partial x_1}{\partial q} > 0\) implies that \(\frac{\partial p_1}{\partial q} < 0\). Let \(x\) denote the level of total output. From (3.21) and (3.22), we have
We have established the following proposition.

**Proposition 3.4**

As the premium rate of export credit insurance becomes lower, the monopolist produces more, increases the level of exports, and charges a higher price in the domestic market.

This result is quite similar to the one obtained in the case of a competitive firm. The government can promote the level of export by setting a lower premium rate, but it will induce an increase in the domestic price. Again domestic consumers might suffer some welfare loss.

The effect of a change in the premium rate on the insurance coverage can also be studied in this model. However, the essential feature of the analysis and conclusions are the same as those in the next section. In order to avoid an unnecessary repetition of the same analysis, we do not analyze it in this section.

### 3.4 Monopoly in the Two Markets

This section studies the case in which the firm has monopoly power in both markets. The optimal policy of this type of price discriminating firm which faces uncertainty in the foreign market has recently been studied by Katz, Paroush, and Kahana (1982). They showed, among other things, the following results:

1. Uncertainty in the foreign market will reduce exports, increase domestic sales, and reduce total sales.
2. Given decreasing absolute risk aversion, an increase in fixed costs will reduce the sales in the foreign market.

3. An increase in a domestic sales tax may or may not reduce the sales in the foreign market.

We will show in this section that their results no longer hold in the presence of export credit insurance.

Using the same notation as in the previous sections, we write the profit function as follows:

\[ \pi = (1 - t)S_1(x_1) + (1 - \alpha)S_2(x_2) + \alpha y - f(x_1 + x_2) - qy - b, \]  

(3.23)

where \( x_i \) is the amount of the product sold in the domestic (\( i = 1 \)) or foreign (\( i = 2 \)) market, \( S_i \) is sales in the domestic (\( i = 1 \)) or foreign (\( i = 2 \)) market, and \( t \) is the sales tax in the domestic market.

It is assumed that the sales functions are twice differentiable, and have the following derivatives:

\[ S'_1 > 0, \quad S''_1 < 0, \quad i = 1, 2. \]

The remaining assumptions are the same as in the previous sections.

Now, the firm's decision problem is written as follows:

\[
\text{Max} \quad V(x_1, x_2, y) = \text{Eu}(\pi).
\]

\[ x_1, x_2, y \]

The first order conditions for an interior maximum are

\[ V_{x_1} = \text{Eu}' \cdot [(1 - t)S'_1 - f'] = 0, \]  

(3.24)

\[ V_{x_2} = \text{Eu}' \cdot [(1 - \alpha)S'_2 - f'] = 0, \]  

(3.25)
\[ V_y = \text{Eu}^{t} \cdot (\alpha - q) = 0. \]  \hspace{1cm} (3.26)

The second order conditions take the following form:

\[
\begin{vmatrix}
V_{x_1x_1} & V_{x_1x_2} \\
V_{x_2x_1} & V_{x_2x_2}
\end{vmatrix} < 0,
\begin{vmatrix}
V_{x_1x_1} & V_{x_1x_2} & V_{x_1y} \\
V_{x_2x_1} & V_{x_2x_2} & V_{x_2y} \\
V_{yx_1} & V_{yx_2} & V_{yy}
\end{vmatrix} > 0.
\]

We assume that these conditions are satisfied. We are ready to state the next proposition.

**Proposition 3.5**

If export credit insurance is available with the proportional reimbursement method, a change in fixed costs, the attitude toward risk, or the distribution of the random variable, do not affect optimal sales in either market.

The proof is simple. From (3.24), we have

\[(1-t)S'_{1} - f' = 0.\] \hspace{1cm} (3.24')

Using the same procedure as in the previous sections, we can obtain the following from (3.25) and (3.26):

\[(1-q)S'_{2} - f' = 0.\] \hspace{1cm} (3.27)

Then we can obtain the optimal values of \(x_1\) and \(x_2\) by solving (3.24') and (3.27) simultaneously. Since neither (3.24') nor (3.27) contains...
the fixed costs \( b \), the attitude toward risk, or the distribution of the random variable, the proposition follows.

Next we conduct a comparative statics analysis of a change in the premium rate and the domestic sales tax. Differentiating (3.24') and (3.27) with respect to \( q \), we obtain the following system of equations:

\[
\begin{bmatrix}
(1 - t)S_1'' - f'' & - f'' \\
- f'' & (1 - q)S_2'' - f''
\end{bmatrix}
\begin{bmatrix}
\frac{\partial x_1}{\partial q} \\
\frac{\partial x_2}{\partial q}
\end{bmatrix}
= \begin{bmatrix}
0 \\
S_2'
\end{bmatrix}.
\]

Let \( D \) denote the Jacobian determinant of Equations (3.24') and (3.27). Using Cramer's rule, we obtain

\[
\frac{\partial x_1}{\partial q} = \frac{1}{D} \cdot f'' \cdot S_2' > 0,
\]

\[
\frac{\partial x_2}{\partial q} = \frac{1}{D} \cdot S_2' \cdot [(1 - t)S_1'' - f''] < 0,
\]

where \( D = (1 - t)(1 - q)S_1''S_2'' - f' \cdot [(1 - t)S_1'' + (1 - q)S_2''] > 0 \). If we let \( x \) denote total output, then we have,

\[
\frac{\partial x}{\partial q} = \frac{\partial x_1}{\partial q} + \frac{\partial x_2}{\partial q} = \frac{1}{D} \cdot (1 - t)S_1''S_2' < 0.
\]

In the same manner, we can derive

\[
\frac{\partial x_1}{\partial t} = \frac{1}{D} \cdot S_1' \cdot [(1 - q)S_2'' - f''] < 0,
\]

\[
\frac{\partial x_2}{\partial t} = \frac{1}{D} \cdot S_1' \cdot f'' > 0,
\]
We have proved the following proposition.

**Proposition 3.6**

In the presence of export credit insurance, a price discriminating firm increases foreign sales, decreases domestic sales, and increases total sales as the premium rate of insurance becomes lower.

Once again, we confirm the conclusion that the government can promote exports by reducing the premium rate of insurance, and the resulting increase in exports is obtained partly at the expense of a reduction in the amount of domestic sales. An increase in the domestic sales tax has the same effect on foreign and domestic sales as a reduction in the premium rate, but it is less preferable to a reduction in the premium rate in the sense that it reduces the total output. We must also note that if the domestic price is set initially higher than the foreign price, then the price differential is widened as a result of export promotion policies as long as both demand curves are negatively sloped. If we define "dumping" as the case in which the firm charges a lower price in the foreign market than in the domestic market, then one might also argue that the dumping practice is amplified through the export credit insurance program.

Finally, we study the effect of a small change in fixed costs and in the premium rate on the optimal amount of insurance coverage. In order to do so, we use the following procedure. After solving (3.24') and (3.27) for the optimal values of $x_1$ and $x_2$, we substitute those
solutions into (3.26). We use the following shorthand notation for Equation (3.26):

\[ V_y = \psi[x_1, x_2, y; b, q] = 0 \]  \hspace{1cm} (3.26')

Differentiating the above expression implicitly with respect to b, we have

\[ \psi x_1 \frac{\partial x_1}{\partial b} + \psi x_2 \frac{\partial x_2}{\partial b} + \psi y \frac{\partial y}{\partial b} + \psi b = 0 \]  \hspace{1cm} (3.34)

Since \( \frac{\partial x_1}{\partial b} = \frac{\partial x_2}{\partial b} = 0 \) from Proposition 3.5, we obtain

\[ \frac{\partial y}{\partial b} = -\frac{\psi b}{\psi y}. \]  \hspace{1cm} (3.35)

Going back to the original expression in (3.26), we have

\[ V_{yy} = \psi_y = E u'' \cdot (\alpha - q)^2 < 0, \]  \hspace{1cm} (3.36)

\[ V_{yb} = \psi_b = -E u'' \cdot (\alpha - q). \]  \hspace{1cm} (3.37)

Under the assumption of decreasing absolute risk aversion, it can be shown that \( \psi_b > 0 \) (see Appendix A for the proof). Therefore, \( \frac{\partial y}{\partial b} > 0. \)

**Proposition 3.7**

An increase (reduction) in fixed costs increases (reduces) the demand for export credit insurance.

Proposition 3.7 holds for all the previous models which utilize the proportional reimbursement method. An implication of this
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proposition is significant. Generally speaking, in order to encourage exports, it is essential to subsidize the infrastructure, such as Research and Development, and other types of fixed costs. However, as we have shown, a fixed subsidy (i.e., a decrease in fixed costs) not only does not change the level of export as in the certainty case, but decreases the purchase of export credit insurance which is also supposed to encourage exports. This possible inconsistency between the two policies should be carefully noted by policy makers.

Turning to the effects of changes in the premium rate, we differentiate (3.26') implicitly with respect to \( q \), and get

\[
\psi'_x \frac{\partial x_1}{\partial q} + \psi'_x \frac{\partial x_2}{\partial q} + \psi_y \frac{\partial y}{\partial q} + \psi_q = 0. \tag{3.38}
\]

From the original expression in (3.26), we can write \( \psi'_x \) as

\[
\psi'_x = [(1-t)S_1 - f'] \cdot \text{Eu}''(\alpha - q). \tag{3.39}
\]

From (3.24'), \( \psi'_x = 0 \). Then we can solve (3.38) for \( \frac{\partial y}{\partial q} \),

\[
\frac{\partial y}{\partial q} = - \frac{1}{\psi_y} \left( \psi'_x \frac{\partial x_2}{\partial q} + \psi_q \right).
\]

Since \( \psi_y \) is strictly negative from (3.36), the sign of \( \frac{\partial y}{\partial q} \) is the same as the term in parentheses. We can write \( \psi'_x \) and \( \psi_q \) explicitly as follows:

\[
V_{x_1} = \psi_x = \text{Eu}'' \cdot (\alpha - q) \cdot [(1 - \alpha)S_2' - f'], \tag{3.40}
\]

\[
V_{yq} = \psi_q = - \text{Eu}' - y \cdot \text{Eu}''(\alpha - q). \tag{3.41}
\]
The sign of $\psi_{x_2}$ is always nonnegative. The proof is simple. When $\alpha = q$, we have $(1-q)S_2' - f' = 0$ by (3.27). For $\alpha < q$, $(1-\alpha)S_2' - f' > 0$ and for $\alpha > q$, $(1-\alpha)S_2' - f' < 0$. Therefore, $(\alpha - q) \cdot [(1-\alpha)S_2' - f'] \leq 0$ for all values of $\alpha$. Multiplying by $u''$, and taking expectation, we have $Eu'' \cdot (\alpha - q) \cdot [(1-\alpha)S_2' - f'] \geq 0$.

Since $\frac{\partial x_2}{\partial q}$ is negative from (3.29), $\psi_{x_2} \frac{\partial x_2}{\partial q} \leq 0$. An interpretation of this term is that an increase in the premium rate reduces the foreign sales, regardless of the degree of risk aversion or the riskiness of the situation, so that the necessity for insurance is reduced.

On the other hand, we have an ambiguous sign for $\psi_q$. The first term in the right-hand side of (3.41) is clearly negative, while the second term is positive from the proof in Appendix A. As in the standard consumer theory, we may call the first term the price effect, and the second term the wealth (income) effect. The second term is positive, zero, or negative accordingly as the Arrow-Pratt absolute risk aversion function is decreasing, constant, or increasing. In our problem the usual assumption of decreasing absolute risk aversion means that an increase in the premium rate decreases the profit level at the original point, which makes the firm more risk averse in a sense that it asks for a higher risk premium for the same risky prospect. Hence it demands more coverage. If we define the normal case as the one in which the subsitution effect dominates the wealth effect, we have proved the following proposition.
Proposition 3.8

If the substitution effect dominates the income effect, then an increase (decrease) in the premium rate of insurance decreases (increases) the optimal level of coverage.

3.5 Summary

We have studied the firm's demand for export credit insurance for the case in which there exists a riskless domestic market. Three different types of firms are discussed: 1) a price taker in the two markets, 2) a monopolist in the domestic market but a price taker in the foreign market, and 3) a monopolist in the two markets. Several important implications have emerged from the analyses.

If the domestic price of the product is higher than the marginal revenue from foreign sales, which is the foreign price discounted by a premium rate of insurance, the competitive firm never exports. In this case, even if the premium rate is more than fair, the introduction of export credit insurance is ineffective as a means of promoting exports. On the other hand, if the domestic price is lower than the net foreign price, a competitive firm exports all of its product to the foreign market. This will eventually increase the domestic price so that the domestic consumer must suffer some welfare loss. This point also holds in the case of monopoly. The government agency can induce the monopolist to raise the level of export by setting a lower premium rate. The monopolist does, indeed, increase foreign sales, but he charges a higher domestic price at the same time. In Section 2.3 it was argued that unless the premium rate is intentionally set at a more than fair rate, the role of export credit insurance is only to make the cost of
the risk explicit. However, in the presence of a domestic market, the export promotion effect has a negative side effect on the domestic consumers in the form of an increased domestic price, which could be regarded as a hidden tax.

In this chapter we have confirmed the important result derived in Chapter II, that is, if export credit insurance is available with proportional reimbursement method, the exporting firm's production decision is separated from risk management. The property was shown to be robust to the introductions of a domestic market and monopoly power.

In the presence of export credit insurance, the domestic monopolist may charge a lower price than the foreign price. This seemingly perverse result arises when the premium rate of insurance is relatively high, and the firm's monopolistic power is relatively weak in the domestic market.

As for the model in which the firm is a monopolist in both markets, it was shown that a decrease in fixed costs does not affect optimal sales in both markets, and reduces the demand for export credit insurance. Since a fixed subsidy to the exporting companies which utilize a large capital stock is common in various countries, the policy maker must realize that such a fixed subsidy reduces the effectiveness of export credit insurance. There might be a possible inconsistency among policies which were designed to serve the same purpose.
CHAPTER IV

SELF-PROTECTION

4.1 Introduction

In the previous chapters we have assumed that the probability distribution function of export credit risk would not be affected by the behavior of the firm or the presence of export credit insurance. We now modify the model by assuming that the probability distribution of the loss of revenues can be affected by certain self-protective activities on the part of the firm. Such activities, for example, may make it possible for the firm to identify importers with a lower risk of default. When we incorporate this feature into the model, we are essentially dealing with the so-called "moral hazard" problem.

The problem of moral hazard has long been recognized in the literature of insurance. According to Dionne (1983), there exist two types of moral hazards in the economic literature on insurance. The first type is defined as the reduction of self-protection by the insured due to the difficulty on the part of the insurer in observing this type of activity, and in using this information for the selection of an appropriate premium. Since the insurance premium does not vary with the effort to prevent loss, the insured has less incentive to undertake preventive activities. The studies of Ehrlich and Becker (1972), Pauly (1974), Helpman and Laffont (1975), Marshall (1976), and Shavell (1979) belong to this type. The second type of moral hazard
is defined as the increase in the consumption of insured services due to a decrease in the price paid by the insured for these services. The insured is subsidized by the insurance coverage and continues to spend for the services after the marginal benefit falls below marginal cost. In that case, the insurer can observe the amount of loss, but cannot verify, in a costless fashion, the state of the world that has created this expense. This interpretation of moral hazard was first proposed by Pauly (1968) in a comment on a paper by Arrow (1963).

Our analysis in this chapter is more closely related to the first type of moral hazard. Introducing self-protection behavior by exporting firms into the model, we study its effect on the demand for export credit insurance. We are especially interested in the effects of changes in various parameters on the optimal level of self-protection and the demand for export credit insurance. Ehrlich and Becker showed that, contrary to the moral hazard argument, market insurance and self-protection could be complements not only in the sense that the availability of the former could increase the demand for the latter, but also in the sense that an increase in the productivity of self-protection or a decrease in the real cost of market insurance would increase the demand for both. We investigate whether their assertion holds in our model or not.

From the point of view of a policy maker, the problem of moral hazard is important. If the presence of export credit insurance severely discourages the firm from spending on self-protection, the expected insurance payment may exceed the premium revenue. If the deficit is paid from the government budget, the government explicitly subsidizes the exporting firms.
This chapter is organized as follows: Section 2 presents the basic model and assumptions; Section 3 examines the boundary conditions for self-protection and the demand for export credit insurance; Section 4 studies the separation property and comparative statics with respect to various parameters in the model; and finally, a summary is presented in Section 5.

4.2 The Model and Assumptions

When we introduce self-protecting behavior into the model, the analysis becomes considerably more complex. In order to make the analysis tractable and to derive meaningful conclusions, we must make several simplifying assumptions. First, we assume that the firm exports all its product to a foreign market as was the case in Chapter II. In this way we can reduce the number of endogenous variables, and it will be easier to investigate the effect of self-protecting behavior on the level of export and the demand for insurance. Second, it is assumed that during any specified time interval, total sales will either be completely lost with probability $\gamma$, or the firm will suffer no loss at all with probability $1-\gamma$. In other words, we assume that only two states of the world are possible. This kind of simplifying assumption has been made in previous studies of moral hazard (Ehlich and Becker, Shavell). The probability of loss $\gamma$ is assumed to be a decreasing function of self-protecting activity.

In our model, the risk is divided into two types; exogenous and endogenous risk. Examples of the former type are political risks such as war, coup d'etat, and import restrictions, which are usually beyond the control of a private company. On the other hand, certain risks can
be reduced by the exporting firm's effort. The risk of default by an importer is a good example. The firm can minimize such a risk by engaging in market research and information gathering. Let \( w \) denote the amount of such self-protecting activity. We now write the probability of total loss \( \gamma \) in two terms as follows:

\[
\gamma = \phi(w) + \lambda, \tag{4.1}
\]

where \( \phi \) is endogenous risk and \( \lambda \) is exogenous risk. We further assume that the function \( \phi \) is positively valued, decreasing, convex, and bounded by \( 1 - \lambda \), that is,

\[
\phi(w) > 0, \quad \phi' < 0, \quad \phi'' > 0, \quad \text{and} \quad \phi(w) < 1 - \lambda. \tag{4.2}
\]

The insurance arrangement in this model is relatively simple. The exporting firm can choose the amount of coverage, \( y \), which should not exceed the value of total sales, \( px \). In case of total loss, the firm receives the amount of \( y \) from the insurance agency. In order to receive this insurance service, the firm must pay a premium payment of \( qy \), where \( q \) is the premium rate per dollar of coverage.

Retaining the notation used in the previous chapters, we can write the random profit function as follows:

\[
\pi = \begin{cases} 
\pi_1 = px - f(x) - b - qy - rw & \text{with probability } 1 - \gamma \\
\pi_2 = y - f(x) - b - qy - rw & \text{with probability } \gamma,
\end{cases}
\]

where \( r \) denotes the price per unit of self-protecting activity. As in the previous chapters, the exporting firm maximizes the expected utility of profit. The optimization problem is described as follows:
Max \[ V(x,y,w) = Eu(\pi) = [(1 - \phi(w) - \lambda) \cdot u(\pi_1) + [\phi(w) + \lambda] \cdot u(\pi_2), \]

subject to \[ 0 \leq w \text{ and } 0 \leq y \leq px. \quad (4.3) \]

4.3 Boundary Conditions

Suppose that the firm produces some output. We are interested in the boundary conditions for self-protection and the demand for insurance coverage.

First, we investigate the conditions under which the exporting firm chooses no insurance coverage. Differentiating the objective function (4.3) twice with respect to \( y \), we obtain

\[ V_y = - q [1 - \phi(w) - \lambda]u_1' + (1 - q)[\phi(w) + \lambda]u_2', \quad (4.4) \]

\[ V_{yy} = q^2 [1 - \phi(w) - \lambda]u_1'' + (1 - q)^2[\phi(w) + \lambda]u_2'' < 0, \quad (4.5) \]

where \( u_1' \) and \( u_1'' \) are \( u'(\pi_1) \) and \( u''(\pi_1) \), respectively. Since the objective function is concave with respect to \( y \), the necessary and sufficient condition for the firm to choose no insurance coverage requires the first partial derivative evaluated at \( y = 0 \) to be non-positive, that is,

\[ V_y|_{y=0} = - q [1 - \phi(w) - \lambda]u'(\pi_3) + (1 - q)[\phi(w) + \lambda]u'(\pi_4) \leq 0, \quad (4.6) \]

where \( \pi_3 \) is \( px - f(x) - b - rw \) and \( \pi_4 \) is \( -f(x) - b - rw \). Solving (4.6) for \( q \), we obtain

\[ \frac{[\phi(w) + \lambda]u_4'}{[1 - \phi(w) - \lambda]u_3' + [\phi(w) + \lambda]u_4'} \leq q. \quad (4.7) \]

Let \( \theta \) denote the left-hand side of (4.7). The expression \( \theta \) is the
highest value for the premium rate which can induce the firm to purchase insurance. We are now interested in how the value of \( \theta \) is affected by changes in the amount of self-protection. Differentiating \( \theta \) with respect to \( w \), we have

\[
\frac{d\theta}{dw} = \frac{u_3' \cdot u_4' \{\phi' + [\phi(w) + \lambda][1 - \phi(w) - \lambda]R(\pi_4) - R(\pi_3)\}}{\{[1 - \phi(w) - \lambda]u_3' + [\phi(w) + \lambda]u_4'\}^2}
\]

where \( R = -\frac{u''}{u'} \) denotes the Arrow-Pratt measure of absolute risk aversion. Since \( \pi_4 < \pi_3 \), it follows that \( R(\pi_3) < R(\pi_4) \) under the usual assumption of decreasing absolute risk aversion. Thus, the sign of \( \frac{d\theta}{dw} \) is ambiguous. In the special case of constant absolute risk aversion, it is strictly negative, which means that the likelihood of condition (4.7) being satisfied increases as the amount of self-protection increases. In this case, we might say that self-protection and the demand for insurance coverage are substitutes in the sense that a larger amount of self-protection reduces the likelihood of purchasing some amount of insurance coverage. However, under the assumption of decreasing absolute risk aversion, the sign of \( \frac{d\theta}{dw} \) can be positive.

Insurance purchase and self-protection can be complements in the sense that an increase in self-protection might increase the likelihood for the firm to purchase some insurance coverage.

Similarly, we can derive and examine the necessary and sufficient condition for full coverage. It is that the first derivative evaluated at \( y = px \) is non-negative:

\[
V_y|_{y=px} = [\phi(w) + \lambda - q] \cdot u'(\pi_5) \geq 0
\]
where \( \pi_5 = (1 - q(px - f(x) - b - rw) \). Since \( u' > 0 \), the condition is reduced to the following:

\[
\phi(w) + \lambda \geq q. 
\] (4.8)

Condition (4.8) is similar to the one studied in Section 2.2; that is, the firm takes full coverage if and only if the premium is fair or more than fair. However, the probability of total loss here is a function of self-protection, while in the previous chapters it was completely exogenous. Therefore, whether or not the premium is fair or not depends in part on the firm's choice of \( w \). We can consider two polar cases. If the exogenous risk \( \lambda \) is greater than \( q \), condition (4.8) holds irrespective of the level of self-protection. Therefore, \( \lambda > q \) is a sufficient condition for the firm to purchase full coverage. On the other hand, if \( \phi(0) + \lambda < q \), then the firm never purchases full coverage. Thus, \( \phi(0) + \lambda \geq q \) is a necessary condition for full coverage.

Next we investigate the condition for no self-protection. Differentiating the objective function (4.3) partially with respect to \( w \), we have

\[
V_w = \phi'(u_2 - u_1) - r\{[1 - \phi(w) - \lambda]u_1' - [\phi(w) + \lambda]u_2'\}, 
\] (4.9)

\[
V_{ww} = - \phi''(u_1 - u_2) + 2r\phi'(u_1' - u_2') + r^2\{[1 - \phi(w) - \lambda]u_1'' + [\phi(w) + \lambda]u_2''\}, 
\] (4.10)

where \( u_i \) is \( u(\pi_i) \). The second partial derivative is not necessarily negative. Under the assumption of risk aversion, we have \( u_1' < u_2' \), so that the second term of the right-hand side in (4.10) is positive (since \( \phi' < 0 \)). However, the other two terms are negative. If we
assume that the magnitudes of the negative terms dominate that of the positive term, which guarantees that \( V_{ww} < 0 \), then the necessary and sufficient condition for no self-protection is as follows:

\[
V_{w|w=0} = -\phi'(0) \cdot (u_5 - u_6) - ru_5' + r(\phi(0) + \lambda)(u_5' - u_6') \leq 0 , \quad (4.11)
\]

where \( \pi_5 = px - f(x) - b - qy \) and \( \pi_6 = y - f(x) - b - qy \). Solving (4.11) for \( r \), we have

\[
\frac{-\phi'(0) \cdot (u_5 - u_6)}{u_5' - (\phi(0) + \lambda)(u_5' - u_6')} \leq r . \quad (4.11')
\]

The assumptions of \( u' > 0 \) and \( u'' < 0 \) imply that \( u_5 > u_6 \), and \( u_5' < u_6' \) since \( \pi_6 < \pi_5 \). Therefore the left-hand side of (4.11') is always a positive number. If the firm purchases full coverage, i.e., \( y = px \), then \( \pi_5 = \pi_6 = (1-q)px - f(x) - b \), in which case the left-hand side of (4.11') is zero. Thus, the condition for no self-protection is always satisfied. The above discussion can be summarized in the following proposition.

**Proposition 4.1**

If the premium rate of export credit insurance is less than the exogenous risk, the firm chooses full coverage and no self-protection.

The proposition illustrates the extreme case of moral hazard. Without the availability of export credit insurance, it would be optimal for the firm to engage in some self-protective activities. But the presence of export credit insurance at a low premium rate works as a disincentive for taking self-protection. This point has a significant implication for the operation of an export credit insurance program.
One main purpose of the program is to promote export. In order to do so, as it has been shown in the previous chapters, the government agency must set the premium rate as low as possible. If the agency underestimates the negative side effect of the insurance in the form of a reduced self-protection, it might incur an unexpected deficit in the account of export credit insurance.

4.4 The Separation Property and Comparative Statics

In Chapters II and III, we have shown that the firm's production decision is separated from the risk management when export credit insurance is available with the proportional reimbursement method. First, in this section, we investigate whether this separation property still holds in the model with self-protection. Then we conduct comparative statics analyses on the various parameters in the model.

If the objective function in (4.3) has a regular interior maximum, then we have the following first-order conditions:

\[ V_x = [1 - \phi(w) - \lambda]u_1'(p - f') - [\phi(w) + \lambda]u_2'f' = 0, \]  
\[ V_y = - q[1 - \phi(w) - \lambda]u_1' + (1 - q)[\phi(w) + \lambda]u_2' = 0, \]  
\[ V_w = \phi'(u_2 - u_1) - r[1 - \phi(w) - \lambda]u_1' + [\phi(w) + \lambda]u_2' = 0. \]  

The second-order conditions can be stated as follows:

\[ V_{xx} < 0, \quad V_{xy} > 0, \quad V_{yx} < 0, \quad V_{yy} < 0, \quad V_{xx} V_{xy} V_{xw} \quad V_{yx} V_{yy} V_{yw} \quad V_{wx} V_{wy} V_{ww} < 0. \]
In the case of risk neutrality, both the 2x2 and 3x3 determinants are always positive, so that an interior maximum does not exist. Even under the assumption of risk aversion the 3x3 determinant may be positive. We will, however, assume that the conditions in (4.15) are satisfied.

We now show that the production decision is indeed separated from the risk management in this model.

From (4.13), we have

\[ [\phi(w) + \lambda]u_2 = \frac{q[1 - \phi(w) - \lambda]u_1}{1 - q}. \] (4.13')

Since \( \frac{[1 - \phi(w) - \lambda]u_1}{1 - q} \) is positive, the following equation must hold:

\[ (1 - q)p - f' = 0. \] (4.17)

The expression in (4.17) is exactly the same as the one derived in Chapter II in which export credit insurance is available with the proportional payment method. The production decision is clearly separated from the risk management. The optimal level of export is independent of the attitude toward risk, the probability of total loss, prices of insurance and self-protection, and the fixed costs. We must also note that the level of export is larger as the premium rate becomes lower. As was mentioned before, the government agency can promote exports by reducing the premium rate.

Next, we conduct comparative statics analyses with respect to various parameters in the model. From (4.17), it is quite obvious that the optimal level of export is negatively related to the level of the premium rate and positively related to the commodity price. The other
parameters of the model do not affect the optimal level of export. Therefore, our main task is to find out the effect of a small change in each parameter on the optimal level of self-protection and the optimal amount of insurance coverage.

We can solve (4.17) for the optimal value of $x$;

$$x = g^X(p,q), \quad g^X_p > 0, \quad \text{and} \quad g^X_q < 0,$$

(4.18)

where $g^X_i$ denotes the partial derivative with respect to parameter $i$, $i = p, q, b, r, \lambda$. Substituting the optimal value of $x$ into (4.13) and (4.14), we can solve for the optimal values of $y$ and $w$. We express them as functions of the parameters of the model as follows:

$$y = g^Y(b, r, \lambda, p, q),$$

(4.19)

$$w = g^W(b, r, \lambda, p, q).$$

(4.20)

We write (4.13) and (4.14) in the reduced form as follows:

$$V_y(x, y, w; b, r, \lambda, p, q) = 0,$$

(4.13')

$$V_w(x, y, w; b, r, \lambda, p, q) = 0.$$

(4.14')

Substituting (4.18), (4.19) and (4.20) into (4.13') and (4.14'), we have

$$V_y[g^X(\cdot), g^Y(\cdot), g^W(\cdot); b, r, \lambda, p, q] = 0$$

(4.13'')

$$V_w[g^X(\cdot), g^Y(\cdot), g^W(\cdot); b, r, \lambda, p, q] = 0.$$  

(4.14'')
Fixed Costs

Differentiating (4.13") and (4.14") with respect to b, we have the following system of equations:

\[
\begin{bmatrix}
V_{yy} & V_{yw} \\
V_{wy} & V_{ww}
\end{bmatrix}
\begin{bmatrix}
g^y_b \\
g^w_b
\end{bmatrix} =
\begin{bmatrix}
-V_{yb} \\
-V_{wb}
\end{bmatrix}. \tag{4.21}
\]

From the second-order conditions we have \( V_{yy} < 0, V_{ww} < 0 \), and \( D = V_{yy}V_{ww} - V_{yw}V_{wy} > 0 \). We can write \( V_{yw} \) explicitly as follows:

\[
V_{yw} = V_{wy} = \phi'[qu_1 + (1-q)u_2^t] + r[q[1-\phi(w)-\lambda]u_1^t - (1-q)[\phi(w) + \lambda]u_2^t]. \tag{4.22}
\]

From (4.13), we have

\[
(1-q)[\phi(w) + \lambda] = q[1-\phi(w) - \lambda] \frac{u_1^t}{u_2^t}. \tag{4.13'}
\]

Substituting (4.13') into (4.22), we obtain

\[
V_{yw} = V_{wy} = \phi'[qu_1 + (1-q)u_2^t] + rq[1-\phi(w) - \lambda]u_1^t(R_2 - R_1), \tag{4.22'}
\]

where \( R \) denotes the Arrow-Pratt measure of absolute risk aversion. In the special case of constant absolute risk aversion, the second term of the right-hand side in (4.22') is zero, so that the sign of \( V_{yw} \) is negative. In general, under the usual assumption of decreasing absolute risk aversion, the sign is ambiguous because \( R_2 > R_1 \) implies that the second term in (4.22') is positive.

In consumer theory, one of the usual definitions of substitutability or complementarity of two commodities is given by the sign of the cross
substitution term of the Slutsky equation. Since insurance and self-protection are different from ordinary commodities in the sense that both of them serve as a device for dealing with an uncertain situation, it will be justifiable to introduce the following definition.

**Definition 4.1**

a) If \( V_{yw} \leq 0 \), then insurance coverage and self-protection are weak stochastic substitutes.

b) If \( V_{yw} > 0 \), then insurance coverage and self-protection are stochastic complements.

The terminology of stochastic substitutes (or complements) was used by Hiebert (1983) in the context of production uncertainty. Definition 4.1 plays an important role in the determination of the comparative statics results.

Differentiating (4.13) and (4.14) with respect to \( b \), and using (4.13'), we can write \( V_{yb} \) and \( V_{wb} \) explicitly as follows:

\[
V_{yb} = q[1 - \phi(w) - \lambda]u_1'' - (1 - q)[\phi(w) + \lambda]u_2'' = q[1 - \phi(w) - \lambda]u_1'(R_2 - R_1),
\]

\[
V_{wb} = \phi'(u_1' - u_2') + r[1 - \phi(w) - \lambda]u_1'' + [\phi(w) + \lambda]u_2''.
\]

The sign of \( V_{yb} \) is positive under the assumption of decreasing absolute risk aversion, while the sign of \( V_{wb} \) is ambiguous in general. However, if the firm purchases an amount of insurance coverage which is close to total sales (which might usually be the case), then the difference between \( u_1' \) and \( u_2' \) will be relatively small, so the effect of the second term in (4.24) will be dominant. We will assume that \( V_{wb} \) is negative.
Solving (4.21) for \( g_b^y \) and \( g_b^w \), we obtain,

\[
g_b^y = \frac{1}{D} (V_{yw}V_{wb} - V_{yb}V_{ww}), \tag{4.25}
\]

\[
g_b^w = \frac{1}{D} (V_{wy}V_{yb} - V_{yy}V_{wb}). \tag{4.26}
\]

If the sign of \( V_{yw} \) is negative, we obtain determinate results, as stated in the following proposition.

**Proposition 4.2**

If insurance coverage and self-protection are weak stochastic substitutes, then an increase in fixed costs increases the optimal amount of insurance coverage and reduces the optimal level of self-protection.

We must note that if insurance coverage and self-protection are stochastic complements, then the above effects are ambiguous. We must also note that a change in fixed costs has no effect on the level of export. In Chapter III it was argued that a fixed subsidy which is intended to promote exports would be unsuccessful and inconsistent in the presence of export credit insurance. In the present model we have obtained the same result. But here we also have a positive side effect inasmuch as a fixed subsidy could induce the exporting firm to spend more on self-protection. This has a favorable effect on the account of the insurance agency, since it reduces the expected amount of reimbursement.

**Price of Self-Protection**

Differentiating (4.13") and (4.14") with respect to \( r \), we have the following system of equations:
Differentiating (4.13) and (4.14) with respect to $r$, and using (4.13'), we have

$$
\begin{bmatrix}
  V_{yy} & V_{yw} \\
  V_{wy} & V_{ww}
\end{bmatrix}
\begin{bmatrix}
  g^y_r \\
  g^w_r
\end{bmatrix}
= 
\begin{bmatrix}
  - V_{yr} \\
  - V_{wr}
\end{bmatrix}.
\tag{4.27}
$$

Under the assumption of decreasing absolute risk aversion, we have $V_{yr} > 0$. After rearranging some of the terms, we can rewrite (4.29) as follows:

$$
V_{wr} = \phi' w (u_1' - u_2') - [1 - \phi(w) - \lambda] u_1' - [\phi(w) + \lambda] u_2' \\
+ rw[1 - \phi(w) - \lambda] u_1'' + rw[\phi(w) + \lambda] u_2''.
\tag{4.29}
$$

Solving (4.27) for $g^y_r$ and $g^w_r$, we have

$$
g^y_r = \frac{1}{D} (V_{wr} V_{yw} - V_{yr} V_{ww}),
$$
$$
(-) (-) (+) (-)
$$
\[ g_r^S = \frac{1}{D} (V_{wy}V_{yr} - V_{yy}V_{wr}) . \]

The condition \( V_{yw} < 0 \) implies that \( g_r^Y > 0 \) and \( g_r^W < 0 \), and so we have derived the following proposition.

**Proposition 4.3**

If insurance coverage and self-protection are weak stochastic substitutes, then an increase in the price of self-protection increases the optimal amount of insurance coverage, and reduces the optimal level of self-protection, provided that \( \varepsilon_w \leq 1 \).

In this case, substitutability according to Definition 4.1 implies that insurance is a gross substitute for self-protection according to the usual definition in consumer theory. However, if insurance and self-protection are stochastic complements, the signs of \( g_r^Y \) and \( g_r^W \) are ambiguous in general. Therefore, there is no one to one relation between Definition 4.1 and the conventional definition.

**Exogenous Risk**

Differentiating (4.13') and (4.14') with respect to \( \lambda \), we have the following system of equations:

\[
\begin{bmatrix}
V_{yy} & V_{yw} \\
V_{wy} & V_{ww}
\end{bmatrix}
\begin{bmatrix}
g_{\lambda}^Y \\
g_{\lambda}^W
\end{bmatrix}
= 
\begin{bmatrix}
-V_y \\
-V_w
\end{bmatrix} .
\]

(4.30)

From the original expressions in (4.13) and (4.14), we have

\[ V_{y\lambda} = qu'_1 + (1 - q)u'_2 > 0 , \]
Solving (4.30) for \( g^Y_\lambda \) and \( g^W_\lambda \), we have

\[
V_{w_\lambda} = r(u'_1 - u'_2) < 0.
\]

The condition \( V_{yw} < 0 \) implies that \( g^Y_\lambda > 0 \) and \( g^W_\lambda < 0 \), and so we have derived the following proposition.

**Proposition 4.4**

If insurance coverage and self-protection are weak stochastic substitutes, an increase in the exogenous risk increases the optimal amount of insurance coverage and reduces the optimal level for self-protection.

This result is consistent with our intuition. If there is an increase in the political risk such as war, coup d'etat, or nationalization which an exporting firm cannot control, the firm will increase the insurance coverage, and reduce the expenditure on self-protection. However, we must remember that this result also depends on stochastic substitutability between insurance and self-protection. If they are complements, a perverse case might arise.

**The Commodity Price**

Differentiating (4.13") and (4.14") with respect to \( p \), we have the following system of equations:
Differentiating (4.13) and (4.14), we can write $V_{yx}$, $V_{wx}$, $V_{yp}$ and $V_{wp}$ explicitly as follows:

$$V_{yx} = - q[1 - \phi(w) - \lambda](p - f')u_1'' - (1 - q)[\phi(w) + \lambda]f''u_2'' > 0 , \quad (4.32)$$

$$V_{wx} = - \phi'(p - f')u_1' - \phi'f'u_2' - r[1 - \phi(w) - \lambda](p - f')u_1'' + r[\phi(w) + \lambda]f''u_2'' , \quad (4.33)$$

$$V_{yp} = - q[1 - \phi(w) - \lambda]u_1''x > 0 , \quad (4.34)$$

$$V_{wp} = - \phi'u_1'x - r[1 - \phi(w) - \lambda]u_1''x > 0 . \quad (4.35)$$

The sign of $V_{wx}$ is indeterminate in general. The first three terms on the right-hand side of (4.33) are positive, and the last term is negative. We will assume that the positive terms always dominate the negative terms.

Solving (4.31) for $g^y_p$ and $g^w_p$, we have

$$g^y_p = \frac{1}{D} \left[ (V_{wx} g^x_p + V_{wp})V_{yw} - (V_{yx} g^x_p + V_{yp})V_{ww} \right],$$

$$(+) (+) (+) (+) (+) (-)$$

$$g^w_p = \frac{1}{D} \left[ (V_{yx} g^x_p + V_{yp})V_{wy} - (V_{wx} g^x_p + V_{wp})V_{yy} \right].$$

$$(+) (+) (+) (+) (+) (-)$$
If insurance coverage and self-protection are stochastic substitutes, the signs of \( g_p^Y \) and \( g_p^W \) are indeterminate. However, if they are stochastic complements, both of them are positive. If insurance and self-protection are combined to serve for the same purpose, the demands for both insurance and self-protection increase as a result of a rise in the commodity price.

The Premium Rate

Differentiating (4.13) and (4.14) with respect to \( q \), we have the following system of equations:

\[
\begin{bmatrix}
V_{yy} & V_{yw} \\
V_{wy} & V_{ww}
\end{bmatrix}
\begin{bmatrix}
g_q^Y \\
g_q^W
\end{bmatrix}
= 
\begin{bmatrix}
-V_{yx} g_q^X - V_{yq} \\
-V_{wx} g_q^X - V_{wq}
\end{bmatrix}.
\]  
(4.36)

Differentiating (4.13) and (4.14), we can write \( V_{yq} = V_{wq} \) explicitly as follows:

\[V_{yq} = -[1 - \phi(w) - \lambda]u_1^i - [\phi(w) + \lambda]u_2^i + yq[1 - \phi(w) - \lambda]u_1^i - (1 - q)[\phi(w) + \lambda]u_2^i,\]  
(4.37)

\[V_{wq} = y\{\phi'(u_1^i - u_2^i) + r[(1 - \phi(w) - \lambda)u_1^i + (\phi(w) + \lambda)u_2^i]\}.\]  
(4.38)

As it was done in Equation (4.22), we can rewrite (4.37) as follows:

\[V_{yq} = -[1 - \phi(w) - \lambda]u_1^i - [\phi(w) + \lambda]u_2^i + yq[1 - \phi(w) - \lambda]u_1^i(R_2 - R_1).\]  
(4.37')

Under the assumption of decreasing absolute risk aversion, the third term
is positive while the first two terms are negative. As in Section 3.4, we can regard the first two terms as the price effect and the third term as the wealth effect. As before, we will assume that the price effect dominates the wealth effect, so $V_{yq}$ is negative.

From (4.24), we can write $V_{wq} = yV_{wb}$. The sign of $V_{wq}$ is the same as that of $V_{wb}$ which was assumed to be negative before.

Solving (4.36) for $g^y_q$ and $g^w_q$, we obtain

$$g^y_q = \frac{1}{D} [(V_{wx} g^x_q + V_{wq}) V_{yw} - (V_{yx} g^x_q + V_{yq}) V_{ww}],$$

$$g^w_q = \frac{1}{D} [(V_{yx} g^x_q + V_{yq}) V_{wy} - (V_{wx} g^x_q + V_{wq}) V_{yy}].$$

Once again, results are ambiguous if insurance coverage and self-protection are stochastic substitutes. If they are stochastic complements, we obtain that $g^y_q < 0$, and $g^w_q < 0$ which implies that insurance coverage and self-protection are gross complements according to the conventional definition in consumer theory.

4.5 Summary

We have studied the role of export credit insurance in a model in which the self-protecting effort by the exporting firm, such as market research, can reduce the probability of sales loss in the foreign market. In a simplified setting of the nature of uncertainty (i.e., we assumed that only two states of the world are possible; either no loss or total loss), we showed that export credit insurance can still
separate the firm's production decision from the risk management and the decision on self-protection.

It was also demonstrated that the government agency can promote the level of export by quoting a relatively low premium rate, which is a result that also held in the previous chapters. However, a lower premium rate tends to induce the exporting firm to purchase full coverage and not to engage in self-protection at all. One extreme case occurs when the premium rate is lower than the exogenous risk which is defined as the risk that self-protection cannot eliminate. In this case, the firm never engages in self-protection. Therefore, there exists a problem of moral hazard. The introduction of export credit insurance with a low premium rate would reduce the firm's incentive for self-protection.

Comparative statics analyses were conducted with respect to various parameters in the model. In order to obtain determinate results, we had to introduce a definition of substitutability between insurance coverage and self-protection which is slightly different from the conventional definition in consumer theory. It turns out that insurance coverage and self-protection could also be complements in the conventional sense insofar as an increase in the price of insurance coverage might reduce the demand for self-protection and vice versa. The effect of an increase in fixed costs or exogenous risk has the same impact on the optimal amount of insurance coverage and the optimal level of self-protection. It is likely to increase the demand for insurance and reduce the self-protecting activities.
CHAPTER V

FURTHER EXTENSIONS AND GENERAL CONCLUSIONS

5.1 Extensions

In the previous three chapters we have studied various aspects of export credit insurance, mainly from the exporting firm's perspective. However, we can investigate the problem from the other side of the coin, namely, from the insurance agency's point of view. In this section, as extensions of the analyses in the previous chapters, we will discuss two important issues concerning the operation of the insurance program: the optimal premium structure and the use of nonlinear pricing.

As it was presented in Section 1.2, the export credit insurance programs in most of the countries are run by the government. There are two reasons why the insurance operation is undertaken by the government.

First, some of the risks, especially those associated with catastrophic events, may not be adequately served by private insurers. Even though the probability of the occurrence of such events is very small, the resulting losses can be extremely large, making it necessary for the insurer to set the premium at a level at which the exporting firms may be reluctant to purchase any insurance. Furthermore, should a disastrous event occur, a private insurer may find it difficult to reimburse the exporting firms for their losses.
Second, because of imperfect information, the problem of "adverse selection" may arise. This situation is caused by the insurer's inability to distinguish those exporting firms who are facing high risks from those facing low risks. Then, so long as the insurer quotes a single premium rate, some firms may not purchase any insurance at all, while others will elect to take full coverage. This may lead to a loss of profits to the insurer, and could possibly induce him not to offer any export credit insurance. It is this type of market failure which justifies the government's involvement in the insurance market.

In this section we set up a model of an export credit insurance program directly operated by the government. The government agency's objective is assumed to promote the level of exports. We analyze the case in which exporting firms are divided into two types according to the riskiness of the foreign markets to which they export. We examine the conditions for how the government agency should set the premium rate for each type of exporting firm under a self-supporting budget constraint. We also discuss the use of nonlinear pricing which has been proposed by Stiglitz (1977) and Schlesinger (1983) as a device for solving the adverse selection problem.

Optimal Premia

We study how the government insurance agency should set the premium rates in order to maximize the total level of exports. Exporting firms are classified into two types according to the riskiness of the foreign market to which they export. We call them high-risk firms and low-risk firms. We assume that the two types of firms are identical in all other respects; namely, they produce the same commodity by using the
same production technology, and have the same attitude toward risk.
The government agency's objective is to maximize the combined level of
exports under a budget constraint which requires that, on the average,
all the insurance payments should be paid only from the premium revenues.
If two firms are identical even in their riskiness, the answer to this
problem is to set a fair premium rate, as was shown in Section 2.3.
However, when the two firms face different degrees of risk, should the
insurance agency still quote a fair premium for each firm? Or should
it quote the same premium rate for both? We try to answer these ques­tions by setting up a simple model of the insurance agency.

We assume that the insurance agency provides export credit insur­
ance with the proportional reimbursement method which was discussed in
Section 2.3. One of the important results under the proportional
reimbursement method is that the firm's production decision is separated
from the risk management. We assume that the insurance agency knows
the export supply function, and the demand function for insurance, as
follows:

\[ x = f(q), \] 
\[ y = h(q, \gamma), \] 

where \( x \) is the level of exports, \( y \) is the demand for insurance, \( q \) is
the premium rate per dollar of insurance, and \( \gamma \) is the expected value
of the proportion of total sales that is lost, which is the same as \( \bar{\alpha} \)
in Section 2.3. The higher the value of \( \gamma \) is, the riskier the foreign
market is.
Furthermore, we assume that the above functions are twice differentiable, and the derivatives satisfy

\[ f' < 0, \quad f'' < 0, \quad (5.3) \]

\[ h_q < 0, \quad h_\gamma > 0, \quad h_{qq} < 0. \quad (5.4) \]

Let the subscripts 1 and 2 denote the high-risk firm and the low-risk firm, respectively. By definition, \( \gamma_1 > \gamma_2 \). The insurance agency is assumed to be risk neutral, and to choose the optimal premium rates \( q_1 \) and \( q_2 \) in order to maximize the combined level of exports \( x \), subject to the budget constraint.

\[
\max_{q_1, q_2} x = x_1 + x_2 \quad \text{subject to} \quad q_1y_1 + q_2y_2 = \gamma_1y_1 + \gamma_2y_2.
\]

Using (5.1) and (5.2), and the Lagrangian form, we can rewrite the above maximization problem as follows:

\[
\max_{q_1, q_2} V = f(q_1) + f(q_2) + \lambda[(q_1 - \gamma_1)h(q_1, \gamma_1) + (q_2 - \gamma_2)h(q_2, \gamma_2)].
\]

Assuming the existence of an interior maximum, we have the following first order conditions:

\[
V_1 = \frac{\partial V}{\partial q_1} = f'(q_1) + \lambda[h(q_1, \gamma_1) + (q_1 - \gamma_1)h_q(q_1, \gamma_1)] = 0, \quad (5.5)
\]

\[
V_2 = \frac{\partial V}{\partial q_2} = f'(q_2) + \lambda[h(q_2, \gamma_2) + (q_2 - \gamma_2)h_q(q_2, \gamma_2)] = 0, \quad (5.6)
\]

\[
V_\lambda = \frac{\partial V}{\partial \lambda} = (q_1 - \gamma_1)h(q_1, \gamma_1) + (q_2 - \gamma_2)h(q_2, \gamma_2) = 0. \quad (5.7)
\]

The second order condition for this problem is given by the following inequality involving the bordered Hessian determinant:
The condition is reduced to the following:

\[-V_{11}^2 V_{22} - V_{22}^2 V_{11} > 0.\]

If $V_{11} < 0$ and $V_{22} < 0$, the condition is satisfied.

Eliminating $\lambda$ from (5.5) and (5.6), we have

\[
\frac{f'(q_1)}{f'(q_2)} = \frac{h(q_1, \gamma_1) + (q_1 - \gamma_1)h_q(q_1, \gamma_1)}{h(q_2, \gamma_2) + (q_2 - \gamma_2)h_q(q_2, \gamma_2)}. \tag{5.8}
\]

Solving (5.7) and (5.8) simultaneously, the insurance agency can determine the optimal premium rates.

We will show that it is never optimal to quote the same premium rate for both the high-risk and low-risk exporting firms. Evaluating (5.8) at $q_1 = q_2 = q$, we can immediately see that the l.h.s. is equal
to one. At the same premium rate, \( h(q, \gamma_1) > h(q, \gamma_2) \) from the assumption of \( h_\gamma > 0 \). From the budget constraint we know that \( \gamma_2 < q < \gamma_1 \), otherwise there is a deficit or surplus in the insurance agency's account. Then we can see that \((q - \gamma_1)h_q(q, \gamma_1)\) is positive and \((q - \gamma_2)h_q(q, \gamma_2)\) is negative. Thus, \( h(q, \gamma_1) + (q - \gamma_1)h_q(q, \gamma_1) > h(q, \gamma_2) + (q - \gamma_2)h_q(q, \gamma_2) \). Therefore, the r.h.s. of (5.8) is greater than one. The equality in (5.6) never holds in this case. Quoting the same premium rate for both types of firms is never optimal.

Another possible way of pricing is to quote the fair premium to each firm; namely, \( q_1 = \gamma_1 \) and \( q_2 = \gamma_2 \). Intuitively, this pricing scheme seems to be appropriate for the insurance agency because it always satisfies the budget constraint. However, in general, this solution is not optimal. As it was shown in Section 2.3, the exporting firm purchases full coverage at a fair premium rate under the proportional reimbursement method. Therefore, evaluating (5.8) at \( q_1 = \gamma_1 \) and \( q_2 = \gamma_2 \), we have

\[
\frac{f'(\gamma_1)}{f'(\gamma_2)} = \frac{px_1}{px_2} = \frac{f(\gamma_1)}{f(\gamma_2)}. \tag{5.9}
\]

Obviously, the export supply function \( f \) does not necessarily satisfy the condition (5.9) at given levels of risks \( \gamma_1 \) and \( \gamma_2 \). In general, quoting a fair premium rate to each type of the exporting firms is not optimal.

Next, we conduct comparative statics with respect to a small change in \( \gamma_1 \). For simplicity, we assume that the solutions are initially \( q_1 = \gamma_1 \) and \( q_2 = \gamma_2 \). Differentiating the first order conditions
implicitly with respect to $\gamma_1$, we have the following system of equations:

$$
\begin{bmatrix}
V_{11} & 0 & h(q_1, \gamma_1) \\
0 & V_{22} & h(q_2, \gamma_2) \\
h(q_1, \gamma_1) & h(q_2, \gamma_2) & 0
\end{bmatrix}
\begin{bmatrix}
\frac{\partial q_1}{\partial \gamma_1} \\
\frac{\partial q_2}{\partial \gamma_1} \\
\frac{\partial \lambda}{\partial \gamma_1}
\end{bmatrix}
= \begin{bmatrix}
-\lambda h_{\gamma}(q_1, \gamma_1) + h_q(q_1, \gamma_1) \\
\cdot \\
h(q_1, \gamma_1)
\end{bmatrix}.
$$

Let $D$ denote the bordered Hessian determinant. Solving (5.10) for $\frac{\partial q_1}{\partial \gamma_1}$ and $\frac{\partial q_2}{\partial \gamma_1}$, we obtain

$$
\frac{\partial q_1}{\partial \gamma_1} = \frac{1}{D} \left\{ [h(q_2, \gamma_2)]^2 \cdot [\lambda h_{\gamma}(q_1, \gamma_1) - h_q(q_1, \gamma_1)] - V_{22}[h(q_1, \gamma_1)]^2 \right\} > 0,
$$

$$
\frac{\partial q_2}{\partial \gamma_1} = \frac{1}{D} \left\{ h(q_1, \gamma_1) \cdot h(q_2, \gamma_2) [h_q(q_1, \gamma_1) - \lambda h_{\gamma}(q_1, \gamma_1) - V_{11}] \right\} \geq 0.
$$

As a result of an increase in risk faced by firm 1, the insurance agency should increase the premium rate for firm 1, but may or may not raise the premium rate for firm 2.

**Nonlinear Premium Schedule**

If a monopolist has enough information about the preferences or demands of the consumers in the market, he can extract some additional profits by charging different prices for different quantities purchased. This price setting practice is called nonlinear pricing. A typical example is a quantity discount given in a supermarket. The price per unit of the product becomes less as the amount of the purchases
increases. The consumer may be induced to buy a larger quantity than in the case of a single price, so that total sales might increase. Ramsey and Hadar (1974) studied nonlinear pricing in the commodity market. One of their conclusions is that nonlinear pricing prevails, unless the market demand function has the same elasticity at all levels of purchase.

The use of nonlinear pricing is common in many insurance policies. In such cases, the premium payment per unit of coverage varies according to the level of total coverage. The prevalence of nonlinear pricing in insurance contracts seems to be based on a slightly different reason from that in the case of an ordinary commodity. Stiglitz (1977) and Schlesinger (1983) proposed nonlinear pricing as a screening device which discriminates among buyers facing different risks. If the insurer does not have enough information about the background of the insured, or if it is too costly to obtain such information, setting a single premium rate gives rise to the adverse selection problem. Nonlinear pricing could solve this problem by letting the insured choose the optimal combination of coverage and premium rate.

In this subsection we will discuss the use of nonlinear pricing in the export credit insurance program. While the previous papers analyzed nonlinear pricing in the context of a monopolist's profit maximization problem, the insurance agency's objective in our problem is to provide the insurance service for the exporting firms in order to increase the level of exports. Therefore, nonlinear pricing might not be so useful in our problem. We examine the effect of nonlinear pricing by introducing a premium function into the model. We shall continue
to assume that indemnity payments are made according to the propor­tional method.

We will write the premium rate \( q \) as a function of the level of coverage \( y \) as follows:

\[
q = \phi(y). \tag{5.11}
\]

Retaining all the other assumptions and notations from Section 2.3, we can write the profit function of the exporting firm as follows:

\[
\pi = (1 - \alpha)px + \alpha y - f(x) - b - \phi(y) \cdot y. \tag{5.12}
\]

The firm's decision problem is to choose the optimal level of exports \( x \) and the optimal amount of insurance coverage \( y \) in order to maximize the expected utility from profits.

\[
\text{Max } V(x, y) = Eu(\pi), \quad \text{subject to } 0 \leq y \leq px.
\]

First we investigate the condition for no coverage. Differentiating the objective function twice with respect to \( y \), we have

\[
V_y = Eu'(\alpha - \phi - \phi' y),
\]

\[
V_{yy} = Eu''(\alpha - \phi - \phi' y)^2 < 0.
\]

Since the second derivative is negative under the assumption of risk aversion, a necessary and sufficient condition for the firm to choose no coverage is that the first derivative at \( y = 0 \) be nonpositive, that is,

\[
V_y \bigg|_{y=0} = Eu'[(1 - \alpha)px - f(x) - b] \cdot [\alpha - \phi(0)] \leq 0. \tag{5.13}
\]
We can rewrite the r.h.s. of (5.13) as follows:

\[ V_y \big|_{y=0} = E u'[(1 - \alpha) px - f(x) - b] \cdot [\bar{\alpha} - \phi(0)] + \text{cov} [u', \alpha - \phi(0)] \leq 0. \]  

(5.13')

It can be shown that \( \text{cov} [u', \alpha - \phi(0)] \) is positive because both \( u' \) and \( \alpha - \phi(0) \) are increasing functions of \( \alpha \). Thus in order for the inequality in (5.13) to hold, the first term of the r.h.s. in (5.13') must be negative. Hence, we see that \( \bar{\alpha} \leq \phi(0) \) is necessary for no coverage.

Similarly, a necessary and sufficient condition for the firm to take full coverage is that the first derivative at \( y = px \) be nonnegative, that is,

\[ V_y \big|_{y=px} = u'[(1 - \phi(px)) px - f(x) - b] \cdot [2 - \phi(px)] - \phi'(px) \cdot px \geq 0. \]

Since \( u' > 0 \), the condition is reduced to the following:

\[ \bar{\alpha} \geq \phi(px) \cdot [1 + \epsilon(px)], \]

(5.14)

where \( \epsilon = \frac{\rho}{q} \cdot \frac{dq}{dy} \) is the elasticity of the premium function \( \phi(y) \). In the case of a single premium rate, the condition was simply \( \bar{\alpha} \geq q \).

An important difference here is that the r.h.s. in (5.14) depends on the level of export \( x \). This fact indicates that there exists an interdependence between the production decision and the risk management in the case of nonlinear pricing. This point is further investigated by examining the case of an interior solution.

Assuming the existence of an interior maximum for this optimization problem, the first order conditions take the following form:
\[ V_x = E'[ (1 - \alpha)p - f'] = 0, \]  
\[ V_y = Eu'[ \alpha - \phi(1 + \epsilon)] = 0. \]  
The second order conditions are

\[ V_{xx} = Eu''[ (1 - \alpha)p - f']^2 - f''Eu' < 0, \]
\[ V_{xx} V_{yy} - V_{xy} V_{yx} = \{ Eu''[ (1 - \alpha)p - f']^2 - f''Eu' \}Eu''[ \alpha - \phi(1 + \epsilon)]^2 \]
\[ - \{ Eu''[ (1 - \alpha)p - f'][ \alpha - \phi(1 + \epsilon)] \}^2 > 0. \]

From (5.16), we have

\[ Eu'\alpha = \phi(1 + \epsilon)Eu'. \]  

We rewrite (5.15) as \( pEu' - pEu'\alpha - f'Eu' = 0 \), and using (5.17) to replace the term \( Eu'\alpha \), we have

\[ [1 - \phi(1 + \epsilon)]p = f'. \]  

In Section 2.3, in which we analyzed a similar model, the corresponding condition was

\[ p(1 - q) = f'. \]  

In Chapter II we concluded that the optimal level of export is independent of the attitude toward risk and the distribution of the random variable. This property does not hold when a nonlinear pricing scheme is used, since (5.18) involves both \( x \) and \( y \). Totally differentiating (5.18) with respect to \( y \), we have
Unless \( \phi' = 0 \) and \( \phi'' = 0 \), the optimal level of export would never be independent of the choice of insurance coverage which, in turn, depends on the risk and the attitude toward risk. In other words, any change in a parameter that causes a change in \( y \), will also bring about a change in \( x \).

Another effect of nonlinear pricing is the creation of uncertainty about the premium revenue. If a single premium rate is quoted, the premium revenue is simply \( qy \). In the case of the proportional reimbursement method, the expected insurance payment is \( \bar{a}y \). At any level of insurance coverage, the insurance agency can predict whether the insurance account will on the average be in a deficit or surplus, according as the premium rate is higher or lower than the expected value of \( \bar{a} \). This point is illustrated in Figure 5.1. If the firm happens to choose the level of coverage \( y_1 \), the insurance agency will have a surplus of \( (q_1 - \bar{a})y_1 \). On the other hand, if the firm chooses the level of coverage \( y_2 \), the insurance agency must incur a deficit of \( (q_2 - \bar{a})y_2 \) on the average. Since the insurance agency's objective is not to make a profit, but to provide insurance service at the lowest cost under an appropriate budget constraint, this uncertainty about a deficit in the account could be a serious obstacle to achieving the desired goal.

### 5.2 Conclusions

The problem of export credit insurance has not been the subject of a rigorous study in either the economics or the finance literature.
Figure 5.1. Nonlinear premium schedule
despite its growing importance in the real world. Serious debt problems in developing countries and rising protectionism in the developed countries in the 1980's have created uncertain circumstances for exporters in the world. Export credit insurance has a potential for helping exporters to cope with uncertainty. The purpose of this dissertation was to develop theoretical frameworks which can shed some light on the role of this relatively unknown device in international business. We have derived the following main results.

1. If the proportional reimbursement method is used by an insurance agency, export credit insurance can make the exporting firm's production decision independent of the risk in the foreign market or the attitude toward risk. However, this property is sensitive to the type of reimbursement method used as well as to the pricing scheme employed by the insurance agency.

2. The insurance agency can increase the level of exports by setting the premium rate at a relatively low level. This promotional effect is larger the more risk averse the exporting firm is. If the premium rate per dollar of coverage is set at a fair rate, a risk averse firm will export as much as a risk neutral firm does in the absence of insurance.

3. If the insurance agency operates under a self-supporting budget constraint, it will require a fairly large number of risk averse firms for the sound operation of the program. Under the non-proportional reimbursement method, the insurance agency's account is always in deficit if a risk neutral firm purchases insurance.
4. The presence of a domestic market restricts the effectiveness of export credit insurance. If the domestic price is higher than the net foreign price (foreign price multiplied by one minus the premium rate), the firm does not export at all even though the insurance is available at a fair rate.

5. As a result of an increase in the level of exports which is created by the introduction of export credit insurance, the price in the domestic market is likely to increase. This will cause a shift in the resource allocation from the non-exportable good sector to the exportable good sector.

6. If a price discriminating firm practices dumping in the foreign market, the introduction of export credit insurance will amplify this practice, namely, the firm will lower the foreign price, and raise the domestic price.

7. The exporting firms will purchase full coverage of export credit insurance if the premium rate is set at a fair or a more-than-fair rate. When they do purchase full coverage, they do not spend any money on the self-protecting activities to reduce the export credit risk. Therefore, there exists a problem of moral hazard.

8. A fixed subsidy to the exporting firm does not change the level of exports, and decreases the purchase of export credit insurance. Since it is essential to subsidize the infrastructure, Research and Development, and other types of fixed costs in order to promote exports, there is a possible inconsistency between export incentive policies. Since the government agency can increase the level of exports by setting a low premium rate, we ought to mention the implication
of the export credit insurance program on the trade policy. As long as the insurance agency operates under the self-supporting constraint, one could argue that the government does not really subsidize the exporting firm. Since the premium revenue is paid by the exporting firm, the role of the government agency is only to make the cost of uncertainty explicit in the form of the premium payment. However, if the government agency intentionally sets the premium rate at a more-than-fair rate, there will be a deficit in the insurance agency's account. In this case the government agency actually subsidizes the exporting firm in the form of a reduced premium payment.

Our analysis in Chapter IV shows that the domestic price is likely to increase as a result of the higher level of exports promoted through export credit insurance. This result suggests that there will be a shift in the resource allocation from the non-exportable sector to the exportable sector in the economy. However, the partial equilibrium setting of our model is not suitable for a complete analysis of this issue. The casting of this problem in a general equilibrium framework is an important extension of the current study.

Further research should also incorporate the financial sector into the model. Since the default risk of the exporting firm will be directly related to the foreign credit risk, export credit insurance will also be the interest of the financial institutions which lend money to the exporting firms. These financial institutions usually reduce the lending rate if the exporting firm purchases export credit insurance because the default risk of the exporting firm is reduced
by the credit insurance cover. It is, therefore, of interest to construct a more general model capable of capturing the financial effects.
APPENDIX

In this appendix it is shown that under the assumption of decreasing absolute risk aversion, we have

\[ -Eu'' \cdot (\alpha - q) > 0 \text{ at } (x^*_1, x^*_2, y^*), \]

where \( x^*_1, x^*_2, \) and \( y^* \) are the optimal values for \( x_1, x_2, \) and \( y. \)

Proof

Let \( R(\pi) = -\frac{u''(\pi)}{u'(\pi)} \) denote the Arrow-Pratt measure of absolute risk aversion, and let \( \pi_0 \) denote the profit level when \( \alpha = q; \) that is,

\[
\pi_0 = (1-t)S_1(x^*_1) + (1-q)S_2(x^*_2) + qy^* - f(x_1^* + x_2^*) - qy^* - b.
\]

If \( q < \alpha, \) then \( \pi < \pi_0. \) Under the assumption of decreasing absolute risk aversion,

\[
-\frac{u''}{u'} > R(\pi_0), \text{ for } q < \alpha.
\]

Multiplying both sides by \( u' \cdot (\alpha - q) \) gives

\[
- u'' \cdot (\alpha - q) > R_0 \quad u' \cdot (\alpha - q)
\]  \hspace{1cm} (A.1)

where \( R_0 = R(\pi_0). \)

If \( q > \alpha, \) then \( \pi > \pi_0. \) Multiplying both sides by \( u' \cdot (\alpha - q) \) gives \( (A.1) \) again. So the inequality of \( (A.1) \) holds for all values of \( \alpha. \)
Taking the expectation on both sides of (A.1), we get

\[- Eu'' \cdot (\alpha - q) > R_0 \cdot Eu' \cdot (\alpha - q) = 0.\]

But from the first order condition we have \( Eu' \cdot (\alpha - q) = 0 \) at \((x_1^*, x_2^*, y^*)\), so that \(- Eu'' \cdot (\alpha - q) > 0.\)


