HEDGING, SPECULATION, AND MARKET RESEARCH AS SELF PROTECTION*

Hideki Funatsu

Otaru University of Commerce
Otaru, Hokkaido 047 Japan

1. Introduction

Recent years have witnessed rapid expansion in the futures market\(^1\). The growth may indicate that the role of the futures market has now been widely recognized as a useful risk management tool. But at the same time the growing concerns over its highly volatile nature of the market have also emerged. Speculation in the futures market has been justified to form the price which reflects all the necessary information currently available so as to reduce the fluctuation of the future price around an unbiased estimate of the future spot price. If the futures market is properly functioning, it serves to provide a correct forecast of the future spot price on the average so that a risk averse producer can use it as a hedging device. Recently this orthodox view on the role of speculation in the futures market

---

* This research was completed when the author was research scholar at London School of Economics. An earlier version of this paper was presented at the 1991 annual conference of the Japan Society of Economics and Econometrics.

1) See Siegel and Siegel (1990) for recent development in the futures market in practice.
has been challenged by several authors who have demonstrated that speculation destabilizes the spot price due to various reasons such as the presence of noisy traders, incomplete information, manipulation, and so on. When this view is held, the price in the futures market could be a biased estimator with large variance. Under such a circumstance, risk averse producers cannot trust the price quoted in the futures market as often heard in the area of producing agricultural commodities, they have to protect themselves by another means against price uncertainty. Self protection may well take the form of ex ante market research such as purchasing detail weather forecast, reading economic news paper, and obtaining expert opinions.

The purpose of this study is to reconsider the behavior of risk averse firm which faces future spot price uncertainty but has a hedging opportunity in the futures market by incorporating ex ante market research activity as self protection. In the previous literature, Feder, Just, and Schmitz (1980, hereafter FJS) and Holthausen (1979) introduced the hedging opportunity into the model of a competitive firm under price uncertainty studied by Sandmo (1971) and found that in the presence of futures market the firm behaves just like under certainty and the output decision is completely separated from price uncertainty. In the case of an unbiased futures market price, the output level of risk averse firm is exactly same as that of risk neutral firm. On the other hand, Paroush (1981) introduced market research as self protection into the Sandmo's model. In his

2) For example, Hart and Kreps (1986), Newbery (1987), and De long, Schlerfer, Summers and Waldmann (1990).

3) A class of the negative exponential functions satisfy these properties.
model, the firm can reduce price uncertainty by collecting information via market research. Among other things, he derived the condition that allows the output decision to be independent of fixed cost. The present paper extends both the FJS and Holthausen model of hedging and the Paroush model of market research as self protection in a sense that a risk averse firm can engage into both hedging and self protection simultaneously. The main question to be addressed is whether hedging and market research are substitute or not to cope with price uncertainty. This question seems to be important especially when the expected price forecasted by the firm is different from the price quoted in the futures market. It turns out that hedging and self protection are indeed substitute each other in the sense that a reduction in the price of one activity increases the amount of other activity as long as the firm takes a long selling position. If the firm engages into speculation by taking a short selling position in the futures market, the volume of selling in the futures market and the amount of market research effort could become complement each other.

Section 2 describes the optimization problem of a risk averse firm. Section 3 analyzes boundary conditions of the problem and derives some propositions. Section 4 conducts comparative statics with respect to the effects of a small change in the level of initial asset, the price of market research, and interest rate on the optimal amount of futures market contract and the optimal amount of market research. Finally, Section 5 states conclusion.
2. The Model

Consider a competitive firm which faces future spot price uncertainty. The firm must make production decision before the price of the product becomes known. It is a price taker in all the markets and its behavior does not affect any market price. It has a subjective estimator of the future spot price, \( \hat{p} \), which can be parameterized by using the market research function \( g \) as follows:

\[
(1) \quad \hat{p} = \bar{p} + g(z)(p - \bar{p})
\]

where \( \bar{p} \) is the mean of a random variable \( p \) with a density function \( h \) and \( z \) is the amount of market research. The function \( g \) is assumed to be twice differentiable and to possess the following properties:

\[ g(0) = 1, \quad g(\infty) = 0, \quad g' < 0, \quad g'' > 0. \]

Without any market research activity, \( z = 0 \). Then \( \hat{p} = \bar{p} \), which is a subjective estimator without any prior information. In this model, market research causes mean preserving contraction\(^4\) of \( \hat{p} \) from the original distribution of random variable \( p \), which is the opposite of mean preserving spread\(^5\) defined as increase in risk by Rothchild and Stiglitz (1971). As the firm engages into more market research activity, it becomes more confident about its expectation on the future spot price and assigns more probability around the mean. In other words, market research serves to eliminate an extreme possibility so that the shape of the subjective estimator becomes sharper around the mean. It is easy to see that the variance of \( \hat{p} \) is a decreasing

---

\(^4\) The terminology is used in Hadar and Seo (1990).
\(^5\) See Davis (1989) for the income and substitution effects of the mean preserving spreads on the choice variable.
function of \( z \).

(3) \[ \text{Var}(\hat{\phi}) = g^2 \sigma_z^2 \]

where \( \sigma_z^2 \) is the variance of the original subjective distribution of the future spot price.

Let \( r \) denote the price of engaging into one unit of market research activity and \( p^* \) the realized future spot price. The firm's ex post profit without a futures market \( \pi_0 \) is written as

(4) \[ \pi_0 = p^*x - f(x) - b - rz \]

where \( x \) denotes the level of output, \( b \) fixed costs, and \( f(x) \) variable costs. The variable cost function is assumed to be twice differentiable and to possess positive first and second derivatives.

As for trading in the futures market, the firm must present margin to a broker which cannot earn a market rate of interest. The margin in the futures market is collateral against the possible loss. The margin rate \( \alpha \) varies from zero to one depending upon the condition of the market. The regulating authority imposes a high margin rate when the market is extremely speculative. In general, the firm must present cash equivalent to the futures market contract multiplied by a margin rate.

(5) \[ m = \alpha q |y|, \quad 0 \leq \alpha \leq 1, \]

where \( m \) denotes the margin, \( q \) the price in the futures market, and \( y \) the amount of the futures contract. Since we assume that a broker does not pay interest for margin, there is the transaction cost of engaging into the futures market. Let \( i \) denote the market rate of interest. Then the transaction cost in the form of opportunity cost is
simply im. Now the ex ante profit which is random variable can be written as

\[ \pi = \hat{\beta}x + (q - \hat{\beta})y - f(x) - b - rz - \text{im}. \]

Furthermore, the firm is assumed to possess a liquidable asset A which always satisfy the marign requirement: \( A > m \). Thus, the firm can trade in the futures market as much as it desires.

Consider a firm which is owned and managed by a person who possesses a von Neumann–Morgenstern utility function. The owner manager is assumed to be an expected utility maximizer and risk averter. Therefore, the first derivative of the utility function is positive and the second derivative negative. The firm’s objective is to maximize the expected utility over its terminal asset position with respect to the level of output, the amount of the futures contract, and the amount of market research.

\[ \text{Max } V(x, y, z) = \text{Eu}[ (1+i)A + \pi ] \]

If there exists an interior solution for this maximization problem, then the first order conditions take the following form:

\[ V_x = \text{Eu}' \cdot (\hat{\beta} - f') = 0 \]
\[ V_y = \text{Eu}' \cdot (q^* - \hat{\beta}) = 0 \]
\[ V_z = \text{Eu}' \cdot (g'(p - \hat{\beta})(x - y) - r) = 0, \]

where \( q^* = (1 - \alpha i)q \) is the net futures market price.

The second order conditions take the following form:
The inequality of the middle determinant imposes a further restriction on the market research function \( g \), that is:

\[
g'' \cdot (g')^2 > 0.
\]

3. Boundary Conditions

This section examines boundary conditions for each endogenous variable.

3 – 1 Output

First, we investigate into conditions for a positive output. Even if the firm does not produce any output, it can still trade in the futures market as a speculator. Thus, a condition for positive output is important to distinguish a hedging firm from speculator.

Since the second partial derivative of the objective function with respect to \( x \) is negative: \( V_{xx} = -f''Eu' + Eu'' \cdot (\hat{p} - f') < 0 \), the condition for a positive output is that the sign of first partial derivative evaluated at \( x=0 \) to be strictly positive.

\[
V_{x_1x_2} = Eu'[(1+i)A+\pi_1] \cdot (\hat{p} - f'(0)) > 0,
\]

where \( \pi_1 = (q^* - \hat{p})y - b - rz \).

After some manipulation, we obtain

\[
\hat{p} > f'(0) - \frac{cov(u, \hat{p})}{Eu_i}.
\]
If \( y > 0 \), then \( u_1' \) is an increasing function of \( p \) and \( \dot{p} \) is also an increasing function of \( p \). Thus, the sign of \( \text{cov} (u_1', \dot{p}) \) is positive. If \( y < 0 \), then \( u_1' \) is a non-decreasing function of \( p \) so that the sign of \( \text{cov} (u_1', \dot{p}) \) is non-positive, therefore, the marginal cost requirement for a positive output with a hedging opportunity is less stringent than otherwise.

### 3-2 Positive Hedging

Next, the boundary condition for a positive amount of hedging is considered. Since \( V_{yx} = \text{Eu}' \cdot (q^* - \dot{p})^2 < 0 \), the condition for \( 0 < y \) is that the first derivative evaluated at \( y = 0 \) is strictly positive.

\[
V_{yx, y=0} = \text{Eu}' \cdot \left[ (1+i)A + \pi_2 \right] \cdot (q^* - \dot{p}) > 0,
\]

where \( \pi_2 = \dot{p}x - f(x) - b - rz \).

After some manipulations, we obtain

\[
(13) \quad \dot{p} < q^* - \frac{\text{cov}(u_2', \dot{p})}{\text{Eu}_2}.
\]

Since \( u_2' \) is decreasing function of \( p \) and \( \dot{p} \) is increasing function of \( p \), \( \text{cov} (u_2', \dot{p}) < 0 \). Therefore, \( q^* < \dot{p} \) is allowed although there is a lower limit for \( q^* \).

Combining (12) and (13), we have the following condition for \( x, y > 0 \).

\[
(14) \quad f'(0) - \frac{\text{cov}(u_1', \dot{p})}{\text{Eu}_1} < \dot{p} < q^* - \frac{\text{cov}(u_2', \dot{p})}{\text{Eu}_2}.
\]

### 3-3 Full Hedging

We call partial hedging if \( 0 < y < x \), full hedging \( x = y \), and speculation if \( x < y \). As before, since the second partial derivative with res
pect to \( y \) is negative, the sign of the first partial derivatives evaluated at \( x=y \) determines the hedging position. \( V_{x=12} \frac{\partial}{\partial x} \geq 0 \) implies \( y \frac{\partial}{\partial x} x \).

\[
V_{x=12} = u_3' \cdot (q^* - \bar{p})
\]

where \( u_3' \equiv u'[(1+i)A+q^*y-f(x)-b-rz] \).

Therefore, \( x \frac{\partial}{\partial x} y \) according as \( q^* \frac{\partial}{\partial x} \bar{p}. \) The firm hedges full amount of output if and only if the net price quoted in the futures market is equal to the expected future spot price. If the net futures market price is lower than the expected future spot price, then the firm engages into partial hedging. On the other hand, if the net futures price is higher than the expected future spot price, the firm sells futures contract more than it intends to produce. Therefore, it engages into speculation.

3 - 4 Market Research as Self Protection

Boundary conditions derived so far are similar to those in the FJS and Holthausen. We are now in a position to present the first new result regarding on the boundary condition for market research as self protection.

**Proposition 1.**

If the net price quoted in the futures market is same as the future spot price expected by a firm, then the risk averse firm never engages into market research as self protection.

proof.

If \( q^* = \bar{p} \), then \( x = y \). Substitute \( x = y \) into equation (10), \( V_{x=12} = rEu' < 0 \). Since \( Vzz < 0 \) from the second order conditions, \( z = 0 \).
If the price in the futures market reflects all the necessary information available and it is an unbiased estimate of the future spot price, there is no need for self protection. The risk averse firm will hedge all the output and will not conduct any market research. In other words, when we observe the market research activity by a hedging firm, the price in the futures market is different from the firm's expectation.

As for necessary condition for $z > 0$, the positive sign of $V_{x_{1}}$ is required.

$$V_{x_{1}} = Eu_{3} \{ g'(0)(p-\hat{p})(x-y) - r \} > 0,$$

where $u_{3}' = u'[ (1+i)A+\hat{p}x+(q^{*-\tilde{p}})y-f(x)-b]$.  

After some manipulations,

$$r < \frac{\text{cov}[u_{3}', g'(0)(p-\hat{p})(x-y)]}{Eu_{3}'}$$

The covariance term is positive except $x=y$ because $\partial u_{3}'/\partial p$ and $\partial g'(0)(p-\hat{p})(x-y)/\partial p$ have same sign when $x \neq y$. Therefore, a sufficiently small price of market research activity will induce a risk averse firm to engage into self protection.

In sum, for an interior solution of all three variables: $x, y, z > 0$, equations (14) and (15) and $q^{*} \neq \tilde{p}$ must be satisfied.

4. Separation Property of an Interior Solution

If there exists an interior solution, then the first order conditions (8), (9), and (10) have to be satisfied in equality. Using equation (9) to eliminate $Eu'\hat{p}$ from Equation (8) yields:

$$q^* = f'.$$
HEDING, SPECULATION, AND MARKET RESEARCH AS SELF PROTECTION

A well-known separation property derived by FJS and Holthausen also holds in this model. The level of output is determined at a point where the marginal cost is equal to the price in the futures market. The risk and the attitude toward risk do not affect the output decision.

In the present model, there is another interesting separation property. Utilizing Equation (9) to eliminate Eu’p from Equation (10), we obtain

\[ \frac{g'}{g}(q^*-\bar{p})(x-y) = r. \]

The economic interpretation of Equation (17) should be clear. \( r \) is the price of one unit of market research activity so that it is the marginal cost of market research. On the other hand, the l.h.s. of (17) is the marginal benefit of market research. \( \frac{g'}{g} \) is a rate of mean preserving contraction and \((q^*-\bar{p})(x-y)\) is the monetary value of discrepancy between the price in the future market and the firm’s expected future spot price. As we have already seen, if \( q^*=\bar{p} \), then the l.h.s. is zero so that no market research is optimal at a positive i

The relation between optimal values of \( z \) and \( y \) can be derived by totally differentiating (17).

\[ \frac{dz}{dy} = \frac{g \cdot g'}{\{g'' \cdot g-(g')^2\}(x-y)} \]

\[ \frac{dz}{dy} < 0 \text{ if } x>y \text{ and } \frac{dz}{dy} > 0 \text{ if } x<y \text{ because } \{g'' \cdot g-(g')^2\}>0 \text{ from the second order condition.} \]

Namely, hedging and market research are gross substitute in the case of partial hedging and gross complement in the case of speculation. A special case of \( g(z)=e^{-\sqrt{z}} \), which satisfies

---

6) A similar separation property was derived by Funatsu (1986) in the context of export credit insurance.
fies all the restriction imposed on g, is illustrated in Figure 1. \( \frac{dy}{dx} = -\frac{2z}{(x-y)} \) in this case.

Insert figure 1 here.

![Figure 1. Hedging firm](image)

When the price in the future market is lower than the expected price, the firm hedges only partially. The amount of \( x-y \) remains to be unhedged so that it is exposed to risk. Thus, there is a need for market research. As the unhedged amount of production becomes larger, more demand for self protection. Clearly, hedging and market research are substitute. On the other hand, when the future price is higher than the expected future spot price, the firm will engage into speculation. Namely, the firm sells more future contract than it intends to produce on the belief that it can fulfill the contract of \( y-x \) by purchasing the good at a reduced price in the future spot market. Therefore, the firm essentially speculates the amount \( y-\)
x. In this case, the firm naturally demands more market research as it increases selling amount in the futures market. The pure speculator's case (x=0) is depicted in figure 2. Speculation and market research as self protection are complement.

Insert figure 2 here

\[ \text{Figure 2. Pure Speculator} \]

It is important to note that the result above holds for all risk averter because Equation (17) does not depend on the utility function.

Let \( \varepsilon = \frac{zg'}{g} \) called market research effectiveness and \( (q-p)(x-y) \) called monetary value of unhedged production. Multiplying \( z \) both sides in Equation (17), we have

(19) \[ \varepsilon(q-p)(x-y)=rz \]

Proposition 2

If the firm maximizes expected utility, the cost of engaging into market research is equal to the monetary value of unhedged produc-
tion multiplied by market research effectiveness.

5. Comparative Statics

This section studies effects of a small change in various parameters on optimal levels of hedging and market research. Since the optimal output is determined independently by Equation (10), we can use Equations (9) and (10) for comparative statics after substituting the optimal value of $x$. Let* denote the optimal value. Using a short hand notation, we can rewrite Equations (9) and (10) as follows;

\[(9') \quad V_y[y^*,z^*;A, r, i, \alpha] = 0, \]

\[(10') \quad V_z[y^*,z^*;A, r, i, \alpha] = 0. \]

5-1 Initial Asset

The effect of a small change in initial asset $A$ is same as that in fixed cost. Partially differentiating Equations (6') and (7') with respect $A$, we have

\[
\begin{bmatrix}
V_{yy} & V_{yz} \\
V_{zy} & V_{zz}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial y^*}{\partial A} \\
\frac{\partial z^*}{\partial A}
\end{bmatrix}
= \begin{bmatrix}
-V_{yA} \\
-V_{zA}
\end{bmatrix}
\]

Solving for $\frac{\partial y^*}{\partial A}$ and $\frac{\partial z^*}{\partial A}$ by using Cramer's rule yields

\[
\frac{\partial y^*}{\partial A} = (V_{yz}V_{yA} - V_{zy}V_{yA})/|V|, \]

\[
\frac{\partial z^*}{\partial A} = (V_{zy}V_{zA} - V_{yz}V_{zA})/|V|.
\]

Straightforward calculation yields

\[
V_{yz}V_{yA} - V_{zy}V_{yA} = -(1+i)\frac{g^*g-(g')^2}{g^2}(x-y)(q^*-\hat{p})
\]
$$V_x V_y = V_z = - (1 + i) \frac{g'}{g} (q^* - \hat{p}) Eu' \cdot Eu''(q^* - \hat{p})$$

The sign of expression $Eu''(q^* - \hat{p})$ is crucial.

**Lemma 1**

Under the assumption of decreasing absolute risk aversion,

$$Eu''(q^* - \hat{p}) \geq 0 \text{ according as } q^* \geq \hat{p}$$

Therefore, $\frac{\partial y^*}{\partial A} < 0$ as $q^* > \hat{p}$ and $\frac{\partial z^*}{\partial A} > 0$. An increase in initial asset makes the firm less risk averse so that it demands for less hedging in the case of partial hedging and it sells more future contracts in the case of speculation. In either case, it engages into more market research because the amount exposed to the future spot price is increased. Under the assumption of constant absolute risk aversion, the initial asset position does not affect optimal values of either hedging or market research.

**Proposition 3**

Under the assumption of decreasing absolute risk aversion, an increase in initial asset reduces (increases) the optimal amount of selling contract in the futures market if the price quoted in the futures market is less (more) than the expected future spot price and always increases the optimal amount of market research activity.

5 - 2 Price of Market Research

Using the same procedure, we obtain:

$$\frac{\partial y^*}{\partial r} = - \frac{V_{yz}}{|V|} Eu' - \frac{z}{1 + i} \frac{\partial y^*}{\partial A} > 0 \text{ according as } q^* > \hat{p},$$

$$\frac{\partial z^*}{\partial r} = \frac{V_{rz}}{|V|} Eu' - \frac{z}{1 + i} \frac{\partial z^*}{\partial A} < 0.$$
Proposition 4

A reduction in the price of market research activity reduces (increases) the optimal amount of selling contract in the futures market if the price quoted in the futures market is less (more) than the expected future spot price and always increases the optimal amount of market research activity.

As long as firm takes a long position in the futures market, hedging and market research are net substitute. If the firm takes a short selling position, speculation and market research are complement. Thus, the reduction in the cost of market research induces more speculation. The law of demand holds for market research.

5-3 Interest rate

The interest rate in this model is an opportunity cost for engaging into the futures market. The firm must deposit more cash to a broker as it sells more future contract. Therefore, interest rate can be regarded as the price of engaging into the futures market.

\[
\frac{\partial y^*}{\partial i} = \alpha q \text{Eu}' \frac{V_{zz}}{|V|} + \frac{A-m}{1+i} \frac{\partial y^*}{\partial A} < 0 \quad \text{if } q^* < \bar{p}
\]

\[
\frac{\partial z^*}{\partial i} = \alpha q \text{Eu}' \frac{V_{zz}}{|V|} + \frac{A-m}{1+i} \frac{\partial z^*}{\partial A} < 0 \quad \text{if } q^* < \bar{p}
\]

Proposition 5

If the price quoted in the futures market is less than the expected future spot price, an increase in the interest rate reduces the optimal amount of hedging and increases the optimal amount of market research activity.

As long as the firm takes a partial hedging position, the signs of comparative statics in this case are unambiguous. A higher interest
rate reduces the optimal amount of hedging and increases that of market research. Thus the law of demand holds and hedging and self protection are not substitutes. However, when the firm is induced to take a short selling position, results become ambiguous because the gross substitute term and the wealth term have opposite effects. A higher interest rate increases the value of total assets which makes the firm less risk averse so as to increase the amount of speculation. While the own gross substitution effect is always negative, the wealth effect might dominate. If this happens, just like the standard consumer theory, speculation becomes Giffen good. The same thing applies to the normal complementary relation between speculation and market research. If the wealth effect is large, speculation and market research could become complement.

5 - 4 Margin Requirement

Regulation authority in the futures market uses margin requirement to control the trading. When the market becomes highly speculative and the price seems to deviate from the equilibrium level, margin requirements are raised to cool off the excessive heat of the market. On the other hand, if the market becomes dull and the price movement is stagnant, margin requirement is reduced to encourage a new participation into the market. In the present model, the effect of $a$ on the optimal values of hedging (or speculation) and those of market research captures an element of reality.

$$\frac{\partial y^*}{\partial a} = q E u, \frac{V_{zz}}{|V|} - \frac{iqy^*}{1 + i} \frac{\partial y^*}{\partial A} < 0 \quad \text{if } q^* > \bar{p}$$

$$\frac{\partial y^*}{\partial a} = q E u, \frac{V_{zz}}{|V|} - \frac{iqy^*}{1 + i} \frac{\partial z^*}{\partial A} < 0 \quad \text{if } q^* > \bar{p}$$
Proposition 6

If the price quoted in the futures market is higher than the expected future spot price, a rise in the margin requirement unambiguously reduces the optimal amount of speculation and that of market research.

Although the effect of a reduction in margin requirement in the case of partial hedging is ambiguous, the regulation authority can successfully suppress the amount of speculation when the price is much higher than the firm's expectation.

6. Conclusion

This paper has studied the behavior of risk averse firm when both hedging and self protection opportunities are available. The firm engages into ex ante market research activity only if the price in the future market is different from its expected future spot price. As the future price deviates further from the expected spot price, there are more incentive for market research. In general, hedging and market research are gross substitute and speculation and market research are gross complement. However, the effect of interest rate change could be perverse in the case of speculation. A higher interest rate could enhance more speculation and increase market research activity under the assumption of decreasing absolute risk aversion. On the other hand, a reduction in the cost of market research unambiguously reduces the optimal amount of hedging and increases the amount of speculation. Therefore, a recent advance in information technology implies that the futures market is used increasingly not for hedging purpose but for speculation purpose. Finally, the regu-
lation authority can use margin requirement as a means to calm down excessive speculation.

Appendix

The signs of second partial derivatives

\[ V_{yy} = Eu'' \cdot (q^* - \hat{p})^2 < 0 \]

\[ V_{yz} = V_{zy} = Eu'' \cdot (q^* - \hat{p}) \cdot \{g'(p - \hat{p})(x - y) - r\} - g'Eu' \cdot (p - \hat{p}) \]

By substituting equation (11) into \( r \) in the first term, we have

\[ g'(p - \hat{p})(x - y) - r = \frac{g'}{g}(x - y)(\hat{p} - q^*) \]

From Equations (1) and (9),

\[ Eu'p = \frac{1}{g} \{q^* - (1 - g)\hat{p}\}Eu' \]

By using the expressions above, we have

\[ V_{yz} = V_{zy} = -\frac{g'}{g}\{(x - y)Eu''(q^* - \hat{p})^2 + (q^* - \hat{p})Eu'\} \geq 0 \text{ iff } q^* \geq \hat{p} \]

\[ V_{xz} = Eu'' \{g'(p - \hat{p})(x - y) - r\}^2 + g''Eu'(p - \hat{p})(x - y) \]

\[ = (g'/g)^2(x - y)^2 + (g''/g)(x - y)(q^* - \hat{p})Eu' < 0 \]

\[ V_{yz} = (1 + i)Eu''(q^* - \hat{p}) \]

\[ V_{xz} = (1 + i)Eu'' \{g'(p - \hat{p})(x - y) - r\} \]

\[ = -(1 + i)(g'/g)(x - y)Eu''(q - \hat{p}) \]
The proof of Lemma 1

Let $\pi_0 = (1+i)A + q^*y - f(x) - b - rz$ when $q^* = \bar{p}$

If $x > y(q^* < \bar{p})$, then for all $\bar{p} < p$, $\pi_0 < \pi$.

Under the assumption of decreasing absolute risk aversion,

$$\frac{-u''[A(1+i)+\pi]}{u'[A(1+i)+\pi]} < -\frac{u''[A(1+i)+\pi_d]}{u'[A(1+i)+\pi_d]}.$$  

Multiply both sides by $-u' \cdot (q^* - \bar{p}) > 0$.

$$u''[A(1+i)+\pi](q^* - \bar{p}) < \frac{u''[A(1+i)+\pi_d]}{u'[A(1+i)+\pi_d]} u'(q^* - \bar{p})$$

For all $p < \bar{p}$, then $\pi < \pi_0$

$$\frac{-u''[A(1+i)+\pi]}{u'[A(1+i)+\pi]} > -\frac{u''[A(1+i)+\pi_d]}{u'[A(1+i)+\pi_d]}.$$  

Multiply both sides by $-u'(q^* - \bar{p}) < 0$.

$$u''[A(1+i)+\pi](q^* - \bar{p}) < \frac{u''[A(1+i)+\pi_d]}{u'[A(1+i)+\pi_d]} u'(q^* - \bar{p})$$

Taking the expectation operator,

$$E u''(q^* - \bar{p}) < \frac{u''}{u'} E u''(q^* - \bar{p}) = 0$$

from the first order condition.

If $x < y$ $(q^* - \bar{p})$, then for all $\bar{p} < p$, $\pi < \pi_0$.

$$\frac{-u''[A(1+i)+\pi]}{u'[A(1+i)+\pi]} > -\frac{u''[A(1+i)+\pi_d]}{u'[A(1+i)+\pi_d]}.$$  

Multiply both sides by $-u' \cdot (q^* - \bar{p}) > 0$.

$$u''[A(1+i)+\pi](q^* - \bar{p}) > \frac{u''[A(1+i)+\pi_d]}{u'[A(1+i)+\pi_d]} u'(q^* - \bar{p})$$

For all $p < \bar{p}$, then $\pi_0 < \pi$.

$$\frac{-u''[A(1+i)+\pi]}{u'[A(1+i)+\pi]} < -\frac{u''[A(1+i)+\pi_d]}{u'[A(1+i)+\pi_d]}$$
Multiply both sides by \(-u' \cdot (q^*-\hat{p}) < 0\)

\[
u'[A(1+i)+\pi](q^*-\hat{p}) > \frac{u''[A(1+i)+\pi\hat{q}]}{u'[A(1+i)+\pi\hat{q}]}u'(q^*-\hat{p})
\]

Taking the expectation operator, 

\[Eu''(q^*-\hat{p}) > 0.\]

Comparative Statics

Price of Market Research

\[
\begin{align*}
\frac{\partial y}{\partial r} &= \frac{[V_{yr}V_{yr} - V_{yr}V_{yr}]}{|V|} \\
\frac{\partial z}{\partial r} &= \frac{[V_{zy}V_{yr} - V_{zy}V_{zy}]}{|V|} \\
V_{yr} &= -z Eu''(q^*-\hat{p}) \\
V_{zy} &= -Eu'z g'(x-y) Eu''(q^*-\hat{p})
\end{align*}
\]

Interest Rate

\[
\begin{align*}
\frac{\partial y}{\partial i} &= \frac{[V_{yi}V_{yi} - V_{yi}V_{yi}]}{|V|} \\
\frac{\partial z}{\partial i} &= \frac{[V_{zi}V_{yi} - V_{zi}V_{zi}]}{|V|} \\
V_{yi} &= (A-m)(g'/g)(x-y) Eu''(\hat{p}-q^*) \\
V_{zi} &= -a q Eu' + (A-m) Eu''(q^*-\hat{p})
\end{align*}
\]

Margin Rate

\[
\begin{align*}
\frac{\partial y}{\partial a} &= \frac{[V_{ya}V_{ya} - V_{ya}V_{ya}]}{|V|} \\
\frac{\partial z}{\partial a} &= \frac{[V_{za}V_{ya} - V_{za}V_{za}]}{|V|} \\
V_{ya} &= -iq Eu' - iqy Eu''(q^*-\hat{p}) \\
V_{za} &= iqy(g'/g)(x-y) Eu''(q^*-\hat{p})
\end{align*}
\]
References