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ON THE ROLE OF ASYMMETRIC INFORMATION IN THE AGGREGATE MATCHING FUNCTION

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When agents in the labour market are fully known, labour matching process is expected to match all agents in the smaller group as the size of the labour market becomes large. Moreover, its convergence occurs fairly quickly. Its implication is that there will be no prolonged unemployment, contradicting the observation. It implies some kind of disruption of perfect information plays a critical role in job matching. The present paper shows that a matching process which arises when there is a cost to collect information about agents exhibits desirable properties that fit data: incomplete matching, emergence of involuntary unemployment, constant returns to scale of matching function, instability of Beveridge curve, and counterclockwise trajectory on the uv plane.

1. Introduction

It is natural to consider that stable matching will arise as an equilibrium when agents in the matching market have perfect information about other agents. The original setting of this problem by ?, called stable marriage problem, is that there are two groups of agents, men and women, each having preference over the agents in the other group and being single. The objective of this problem is to make matches between men and women, one by one, in a sustainable way. Namely, the *matching*, i.e. a pattern of matches, should satisfy such a property that, in equilibrium, there should be no agents who are not matched as a pair but both of whom have incentive to form a new pair changing from the current partner. We call such a matching ***stable matching***. If a matching is not stable, there is at least one pair of man and woman who wants to and will deviate from the current matching. In the current literature, it is usually assumed that not only agents in the matching market but also their preferences are common knowledge. This change of setting from the original is motivated by the fact that there are multiple stable matching equilibria in general and those equilibria are ranked by each group. Therefore, there may be incentive for an agent to shift the market outcome from one equilibrium to the other. Crawford and Knoer (1981) introduced a notion of equilibrium, called ***bargaining equilibrium***, when utility is transferable between agents in a matched pair. This transferable utility case applies to the matching in the labour market because part of utility of an agent in a matched pair can be transferred to the other in the form of wage payments. In the bargaining equilibrium, a pair equally divides the surplus of total payoff of this pair over the sum of their threats and distribute it on each threat. The threat is the payoff the agent receives in a new matching once the original pair becomes unavailable. Since we need to know the threat in the new matching game, we need to calculate threat recursively until we reach to a trivial matching game in which only one pair is available. The above operation determines the pattern of utility transfer within all possible pairs, which transforms the utility profile of each agent when utility is not transferable into another utility profile. The bargaining equilibrium is a stable matching equilibrium upon this transformed utility.

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Then, the question arises: can the bargaining equilibrium explain the data observed in the labour market? The answer is unfavourable. Dagsvik (2000) and Yokota (2001) showed that, under certain probability distributions that generate evaluation of the counterpart, such a matching process leads to *complete matching*[1] when the market becomes large in size. It implies that there should be no observed prolonged unemployment. Dagsvik (2000) assumed that the utility that an agent feels from a potential partner is drawn from a special case of extreme value distribution. The utility to be single is also drawn from the same distribution but multiplied by a scaling factor. Namely, when the utility of potential partners is a random variable U, the utility to be single is $\alpha U'$ where U' has the same distribution function as U and $\alpha > 0$ is a scaling factor. The great contribution of Dagsvik (2000) is that he obtained an asymptotic but analytical formula for the number of matches.[2] As far as only number of matches concerns, the assumption on the probability distribution is harmless because what matters is the preference order of each agent and the level of utility does not affect the pattern of matching. However, the assumption that utility of being single is random may be arguable. In search context, the factors which affect the decision to stay single are the expected utility to match in the next period, unemployment benefits and intertemporal discount rate, which are all not random. However, Yokota (2001) confirmed that the same result holds for the cases where the utility of being single is constant. He also found that the convergence to complete matching occurs fairly quickly and under empirically conceivable size of matching market, all agents in the smaller group should be matched. Yokota (2004) also found that if a researcher fits Cobb-Douglas function to the outcome of a stable matching process, the estimated returns to scale is larger than 1.4 which is much larger than the actual observation and strongly rejects the hypothesis of constant returns to scale. The common findings by empirical literature such as **?** are that the hypothesis of constant returns to scale is barely rejected. It casts strong doubt on the hypothesis that the matching process in the labour market can be described by a stable matching process.

These stochastic models assume independence of probability distributions from which evaluation of an agent by another is drawn. Since utility is transferred within a pair in the form of wages, this is a fictitious story under the job matching situation. Yokota (2004) showed that the bilateral utilities within any pair after the transfer of utility will exhibit positive correlation. It has the effect to raise the matching probability and it only strengthens the tendency to reject the hypothesis that job matching is described by a stable matching process. Or, one may criticise that there is ranking within workers or jobs in reality. Graduates of good universities tend to be accepted from many firms compared to those who are not, and jobs which are considered to have socially high status will attract many applicants. Introduction of such ranking does not change large sample results as far as there is strictly positive probability to be accepted for any agents. Our calculation shows that it does not change small sample results as well significantly.

All the above results suggest that some kind of disruption of perfect information plays a key role in job matching. Therefore, we want to see what happens if searching agents cannot find others costlessly. We will find that the matching process which arises from there brings prolonged unemployment and unfilled vacancies, constant returns to scale of the aggregate matching function, instability of the Beveridge curve, and counterclockwise trajectory on the uv plane. Traditionally, it is argued that deterioration of matching efficiency are the cause of the shift of the Beveridge curve after 1970's. Our result places precautious warning to interpret the position of estimated Beveridge curve directly as an indicator of efficiency. The apparent "shift" of the Beveridge curve may occur without any deterioration of matching efficiency. There are some empirical studies which cast doubt on the validity of the Beveridge curve to represent structural

[1] I borrowed this term from the context of the graph theory with a slight modification. Here, we define complete matching to be a matching which matches all agents in the smaller group. The term used in graph theory literature is as follows. In a bipartite graph G comprised of sets A and B, if all elements in A is matched to the elements in B, it is said that there is *complete matching from A to B*. If all elements in B is matched to the elements in A, there is *complete matching from B to A*. Since it is obvious that the number of matches M does not exceed the minimum number between A and B, when we simply say there is *complete matching* in this paper, we imply that $M = \min\{|A|, |B|\}$.
The tackle on the problem of matching on a graph precedes ?. One of the major results, Hall's theorem, which showed the necessary and sufficient condition that complete matching exists was obtained in 1935. Matching on a graph can be regarded as a special case that the preference of market participants is binary, "acceptable" and "not acceptable". It is not necessarily stable in the sense of Gale and Shapley.

[2] He mentioned that if the scaling factor α is close to zero and the number of each group to be matched is equal, all agents are matched. But this result is more general. Even if α is not zero, all agents are matched as the market size becomes large. Moreover, all agents in the smaller group are matched even if the number of each group is not equal.

or stable relationship in the labour market. Using the panel data of nine regions in West Germany, B?rsch-Supan (1991) showed that the observed shift of the Beveridge curve occurred too quickly to be explained by structural changes and that the change in the composition of unemployment pool, i.e. structural factors on the side of workers, have little explanatory power of the shifts. He raised a question on the stability of the Beveridge curve under the presence of business cycles. He pointed out the possibility that, after a shock, the trajectory on the uv plane may not converge back to the original Beveridge curve in general, since parameters change. In the same spirit, we will find that, given the existence of fluctuation in inflows to the labour market, the apparent shift of the Beveridge curve is inevitable. In our case, what makes the trajectory converge to a different point is the fact that the market is in a different phase of the inflow cycle when the market crosses the same vu ratio from above and below. It makes the trajectory on the uv plane form a cycle, especially in the counterclockwise direction. Counterclockwise behaviour of the trajectory has been found in a number of countries. Hansen (1970) pointed out the cyclical behaviour from the data of United Kingdom from 1953 to 1964. He explained the emergence of the cyclical behaviour using a model, but failed to show why the direction of the cycle is counterclockwise, not clockwise.[3] To explain the direction of the cycle, Bowden (1980) assumed that vacancies responds more quickly to shocks than unemployment. Pissarides (1985) explained it from the asymmetry between matching process and breakup process. He assumed that breakup of a job-worker pair can be undertaken immediately, whereas matching is a time-consuming process. We will show in this paper that the counterclockwise behaviour can arise solely from the matching process regardless the breakup process. It comes from the time lag until change in flows (inflow of jobs and workers) is reflected in stocks (u and v).

Throughout the paper, words job-seeker and worker are used interchangeably. The same applies to words (unfilled) (job) vacancies and firms. It is assumed that each job is independent and there is no interrelations between jobs in the same firm. Thus, a firm can be decomposed into units of jobs, enabling the interchangeability of words and notion between a job and a firm. We will set up a simple matching model under asymmetric information in Section 2. Section 3 illustrates the implication of asymmetric information to the Beveridge curve. Section 4 proves the existence of cycles in the uv plane when inflows to the labour market fluctuate periodically. It also proves a sufficient condition for the cycle to be counterclockwise. Section 5 proves the behaviour of the trajectory in the uv plane when the inflows to the labour market are not balanced. Section 6 obtains a general result without assuming periodicity in inflows. Section 7 provides some examples for various inflow patterns. Section 8 gives a strict definition of the Beveridge curve which is mentioned in Section 3, and studies its relation to the trajectory in the uv plane. Section 9 provides some concluding remarks.

2. The Model

There are two groups of agents in the labour matching market, job-seekers (workers) and job vacancies (firms). The objective of the matching market is to form pairs from these two groups, consisting of one from each. However, *properties of each agent are not revealed automatically.* Communication between two agents, in which initiative can be taken from either side, is required to collect information about agents on the other side. This setting would be thought natural if one observes the difficulty to distinguish job-seekers from a crowd in a theatre only by sight. Under such an environment, an *advertisement process*, which we define below, arises naturally as a matching procedure as shown in Appendix. At the beginning of a period, all firms that hold a job vacancy place an advertisement for a worker with the properties of the job. Then, job-seekers get information about job vacancies in the market through the advertisement. Next, a job-seeker applies to his best choice among advertising firms. At the end of the period, he receives

[3] A simplified version of Hansen's model is

$$\begin{aligned} u(t) &= 1 - e(t) \\ v(t) &= \delta(t) - e(t) \end{aligned}$$

where $u(t)$ is the unemployment rate at time t, $v(t)$ is the vacancy rate, $e(t)$ is the employment rate and $\delta(t)$ is the rate of labour demand in terms of men to be employed (desired employment). He assumed

$$\delta(t) = \alpha \cos t + \beta \tag{1}$$

where α and β are parameters to obtain counterclockwise cycles in the uv plane. However, as easily observed, $\delta(t) = \alpha \sin t + \beta$, which is obtained by simply shifting the initial time from zero to $\pi/2$ in equation (1), will give clockwise cycles and there is no reason to assume the former version.

a notification of appointment or rejection. If he is appointed, he leaves the labour market and engages in production from the next period. If he is rejected, he remains in the labour market in the next period. For simplicity, we put a memoryless assumption. A worker who has rejected by a firm can apply to the same job next period if the vacancy remains unfilled. This simplification would be harmless when the market size is sufficiently large, or it may be thought as a case in which passage of time changes the evaluation of the worker. We assume that a firm will assign one of applicants to a job if and only if the evaluation of the best applicant exceeds the threshold value. Job-seekers will apply if and only if his evaluation of the best job posted exceeds his threshold value. Their threasholds are affected by the discount factor of each agent, unemployment benefits or cost to keep a vacancy, and the expected value of being unmatched in the next period.

When an agent obtains the profile of a matching candidate, he makes evaluation of the candidate in the form of utility. The evaluation by a particular agent is drawn from an i.i.d. probability distribution for any encounter. Agents in the same group are ex ante homogeneous before a matching session begins, in the sense that they share the same probability distribution. We denote a firm's evaluation of a worker by x and a worker's evaluation of a job by y. The threshold value of a firm is denoted by x^* and that of a worker is denoted by y^*. If evaluation of the best applicant by a firm is $x \geq x^*$, it will employ him. If $x \leq x^*$, it will employ nobody and leave the position vacant for another period. The same holds for a worker. We assume that each agent is risk neutral. x and y are random variables that have i.i.d. probability distributions $F(\cdot)$ and $G(\cdot)$ respectively. We denote the number of unfilled job vacancies in the labour market by v and that of job-seekers by u. The threasholds x^* and y^* increase when the number of candidates is expected to increase in the next period and decreases when it is expected to decrease under the assumption of constant returns to scale of the aggregate matching function. If the number of candidates is expected to increase, the probability to match does not change when matching technology shows constant returns to scale. However, conditional expectation of utility after match increases because the utility of the best candidate follows distribution $F(x)^n$ or $G(y)^n$ where n is the number of candidates. Therefore, x^* continues to rise and y^* continues to fall in recession and vice versa in boom. The pattern of the fluctuation depends on the shape of F and G.

Proposition 1. *There are v vacancies and u job-seekers in the labour market. On each encounter, the evaluation x of a worker by a firm is drawn from an i.i.d. distribution F and the evaluation y of a firm by a worker is drawn from an i.i.d. distribution G. If the evaluation of the best candidate is lower than the threshold value x^* or y^*, the agent chooses not to match and stays in the labour market. Then, probability for a vacancy to match P_v is given by*

$$P_v = 1 - \left[1 - \frac{(1 - F(x^*))\,(1 - G(y^*)^v)}{v}\right]^u . \tag{2}$$

Furthermore, P_v converges to

$$\bar{P}_v = 1 - \exp\left(-\frac{1 - F(x^*)}{\theta}\right) \tag{3}$$

as the number of agents increases for given vu ratio θ.

Proof. The probability that the evaluation of a job by a worker is above the threshold is $1 - G(y^*)$. When there are v job vacancies, a worker will apply to one of them with probability $1 - G(y^*)^v$. Let's call A the event that exactly m job-seekers apply to one of the vacancies when there are u job-seekers in the market. Then, the probability of event A to occur is $\Pr(A) = \binom{u}{m}\left[1 - G(y^*)^v\right]^m G(y^*)^{v(u-m)}$. On the other hand, each vacancy is homogeneous *ex ante,* in the sense that its evaluation is drawn from the same probability distribution. Conditional upon having m effective job-seeking applicants in the whole labour market ($u - m$ workers do not apply), the probability that n workers apply to a particular vacancy is equal for all vacancies and $\Pr(B \mid A) = \binom{m}{n}(1/v)^n(1 - 1/v)^{m-n}$, where B is the event that a vacancy receives n applicants. The firm assigns one of those n applicants whose evaluation is the highest, only if his evaluation exceeds the threshold. We denote by C the event that a firm assigns a job-seeker out of applicants. Then, a firm approves a worker with conditional probability $\Pr(C \mid A \wedge B) = 1 - F(x^*)^n$.

The event that m job-seekers actually apply and n among them apply to a particular firm is exclusive each other for different (m, n) pairs. Thus, the probability that a firm assigns a job to a worker is given by

$$\text{(4)} \quad \sum_{m=1}^{u}\sum_{n=1}^{m} \Pr(A)\Pr(B \mid A)\Pr(C \mid A \wedge B) = \sum_{m=0}^{u}\sum_{n=0}^{m} \Pr(A)\Pr(B \mid A)\Pr(C \mid A \wedge B)$$
$$= 1 - \left[1 - \frac{(1 - F(x^*))(1 - G(y^*)^v)}{v}\right]^u$$

The first line uses a convention $1 - F(x^*)^n = 0$ when $n = 0$.[4]

The limit of equation (4) as $u, v \to +\infty$ keeping $\theta = v/u$ constant gives equation (3). □

Corollary 1. *Probability for a job-seeker to match P_u is given by $P_u = (v/u)P_v$. As $u, v \to +\infty$ keeping θ constant, P_u converges to $\bar{P}_v = \theta \bar{P}_u$.*

Corollary 2. *The aggregate matching function μ is given by*

$$\text{(5)} \quad \mu(u, v;\, F(x^*)) = P_v v = \left[1 - \exp\left\{-\frac{u}{v}[1 - F(x^*)]\right\}\right] v$$

Corollary 3 (Existence of involuntary unemployment). *Probability that a firm receives at least one application and rejects all applicants is given by*

$$\text{(6)} \quad \left[1 - \frac{(1 - F(x^*))(1 - G(y^*)^v)}{v}\right]^u - \left[1 - \frac{1 - G(y^*)^v}{v}\right]^u$$

Furthermore, this probability becomes

$$\text{(7)} \quad e^{-(1-F(x^*))/\theta} - e^{-1/\theta}$$

when market size goes infinity for given vu ratio θ.

Proof. The probability is calculated by $\sum_{m=1}^{u}\sum_{n=1}^{m} \Pr(A)\Pr(B \mid A)\Pr(\bar{C} \mid A \wedge B)$. Taking a limit of equation (6) as $v \to \infty$ keeping θ constant, we have $e^{-(1-F(x^*))/\theta} - e^{-1/\theta}$. □

Note that, if $F(x^*) > 0$, probability for a firm to receive at least one application and reject all of its applicants is strictly positive even asymptotically. When this event occurs, *job-seekers are involuntary unemployed* since they applied because they had a will to work and rejected. On the other hand, a firm is voluntarily unmatched.

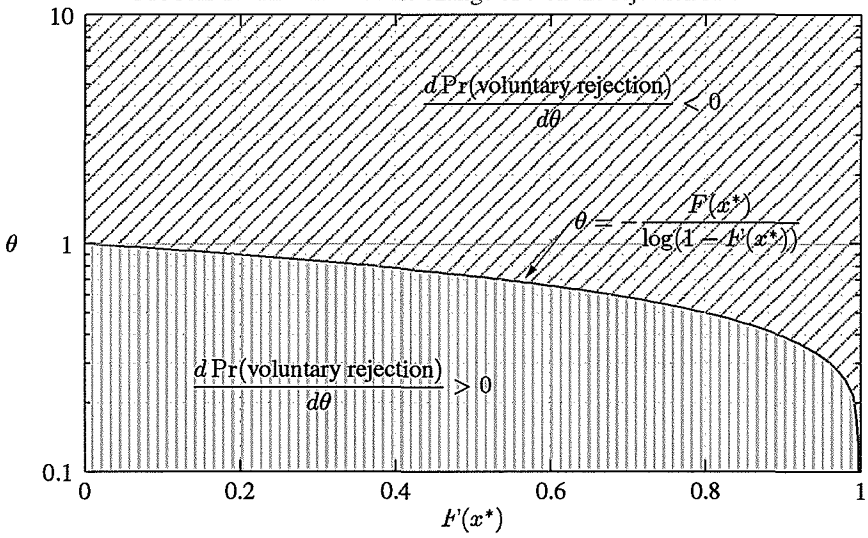
One may expect that a decrease of θ induces a firm to accept workers more easily, since it tends to have more candidates. However, it is not true. Figure 1 shows that the asymptotic probability of rejection (7) is maximised at $\theta = -F(x^*)/\log(1 - F(x^*))$. If θ is sufficiently high, a decrease of θ increases the probability of rejection since the vacancies which previously had no applicants become to receive applications *and* reject. This effect is larger than the effect to reduce the probability of rejection by having more applicants in those vacancies which previously had some applicants. We immediately have to add that, in this analysis, we considered only direct effects when the change of θ is not expected. If it is expected, a firm will update their threshold value x^*. If it is expected to rise, then it reduces x^* since passing over an applicant at hand becomes more costly, because the conditional expected utility after match in the next period decreases. It results in the decrease of $F(x^*)$. Therefore, the expected change of θ going upward in the graph induces the change of $F(x^*)$ going leftward in the graph. It compensates the direct impact of the decrease in the probability of voluntary rejection.

Proposition 2 (Asymptotic non-existence of voluntary unemployment). *If $G(y^*) < 1$, probability that all job-seekers apply converges to one as u and v go infinity for given θ.*

Proof. Obviously, $u > 0$ and $v > 0$ for the market to exist. The probability that all workers apply to a vacancy is $[1 - G(y^*)^v]^u$. We define a notation $r := G(y^*)^\theta$. As u and v go infinity keeping θ constant,

[4] The case when $F(x^*) = 0$ is safely included by setting $F(x^*) \to +0$.

FIGURE 1. The effect of the change of θ on the rejection rate



Note: The vertical axis is measured in log scale.

the probability that all workers apply to a vacancy becomes

$$\begin{aligned}
\lim_{u,v\to+\infty} \left[1 - G(y^*)^v\right]^u \Big|_{v/u=\theta} &= \lim_{u\to+\infty} \left[1 - \left\{G(y^*)^\theta\right\}^u\right]^u \\
&= \exp\left\{\lim u \log\left(1 - r^u\right)\right\} \\
&= \exp\left\{\frac{\lim u}{\lim \frac{1}{\log(1-r^u)}}\right\} \\
&= \exp\left\{\lim \frac{(1 - r^u)\left[\log(1 - r^u)\right]^2}{r^u \log r}\right\} \\
&= \exp\left\{\lim_{k\to+0} \frac{(1 - k)\left[\log(1 - k)\right]^2}{k \log r}\right\} \\
&= \exp\left\{-\frac{\lim\left[\log(1 - k)\right]^2 + 2\lim\log(1 - k)}{\lim\log r}\right\} \\
&= 1
\end{aligned}$$

In the fourth line, $0 \le r < 1$ holds from $G(y^*) < 1$ and $v > 0$. Thus, $\lim_{u\to+\infty} r^u$ is equivalent to $\lim_{k\to+0} k$ defining $k := r^u$. □

The increase of market size has two effects on the side of job-seekers. Increase of v has an effect to offer more choices to a job-seeker, which makes it easier to find a candidate above the threashold. On the other hand, increase of u means increase of evaluation trials, which brings higher probability that *at least one* job-seeker's evaluation falls below the threshold. The above proposition says that the former effect is larger. It implies that involuntary unemployment tends to disappear as the market size becomes larger for any $\theta > 0$. A small sample result is shown in Figure 2 for $G(y^*) = 0.1$ and $\theta = 1/2$, 1, 2. Almost all Japanese historical data falls in the region $1/2 \le \theta \le 2$. Considering the fact that a typical graduate of Japanese universities samples more than seventy jobs, the probability for voluntary unemployment to exist is literally negligible. The pathological cases, which is observed in reality, such as NEAT and the homeless should be understood as having exceptionally high threshold y^* so that the assumption of the above proposition $G(y^*) < 1$ breaks down. It is probablly brought by their too high personal unemployment benefit. They are

FIGURE 2. Probability that there exists voluntary unemployment when $G(y^*) = 0.1$

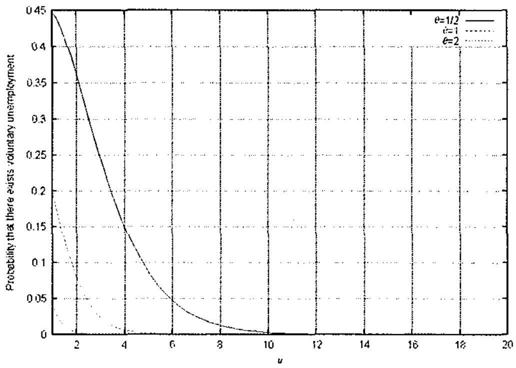
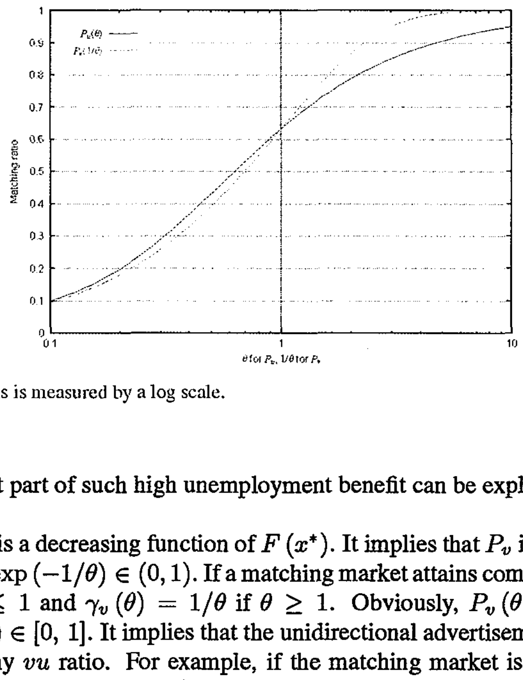


FIGURE 3. Matching ratios of a job-seeker and a vacancy



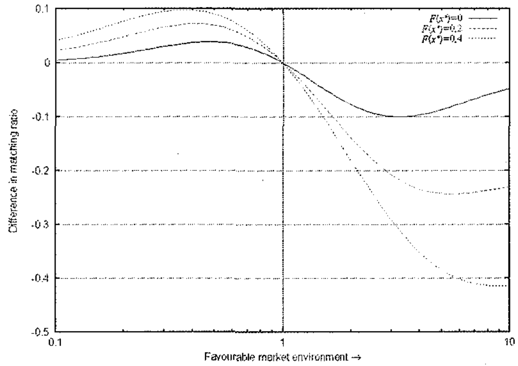
Note: The horizontal axis is measured by a log scale.

pathological because most part of such high unemployment benefit can be explained only by psycological factors.

From equation (3), P_v is a decreasing function of $F(x^*)$. It implies that P_v is maximised at $F(x^*) = 0$ with maximum value $1 - \exp(-1/\theta) \in (0, 1)$. If a matching market attains complete matching, $P_v = \gamma(\theta)$ where $\gamma_v(\theta) = 1$ if $\theta \leq 1$ and $\gamma_v(\theta) = 1/\theta$ if $\theta \geq 1$. Obviously, $P_v(\theta, F(x^*)) < \gamma_v(\theta)$ for all $\theta \in (0, \infty)$ and all $F(x^*) \in [0, 1]$. It implies that the unidirectional advertisement process does not attain complete matching for any vu ratio. For example, if the matching market is balanced, a unidirectional advertisement process matches only $1 - 1/e = 63.21$ percent of agents. The other 36.79 percent was rejected if he is a worker, or either rejected applicants or did not receive any applications if he is a firm. The probability for a firm to reject applicants is given by Corollary 3. The probability that it does not receive any applications is given by $\Pr(\text{no apps}) = \left[\left(v - 1 + G(y^*)^v\right)/v\right]^u$ for small samples. If $G(y^*) = 0$, it is minimum at $\left[(v-1)/v\right]^u > 0$ for any v and u. In large samples, $\lim_{u,v \to +\infty} \left[(v-1)/v\right]^u\big|_{v/u=\theta} = \exp(-1/\theta) > 0$. Therefore, $\Pr(\text{no apps}) > 0$ for any θ in both small and large samples.

$P_v\left(1/\theta;\, F(x^*)\right)$ and $P_u\left(\theta;\, F(x^*)\right)$ when $F(x^*) = 0$ are drawn in Figure 3. P_v and P_u for other values of $F(x^*)$ are smaller than this. To emphasise the asymmetry between P_v and P_u, the graph of P_v is horizontally flipped. The horizontal axis of P_u is measured by a log scale of θ whereas that of P_v is measured by a log scale of $1/\theta$. The rightward direction of the axis shows a favourable market environment for the particular agent. The matching ratio of job-seekers P_u is strictly increasing in θ and the matching ratio of vacancies P_v is strictly decreasing (strictly increasing in the graph as it is flipped). Note that they are solely determined by the vu ratio and $F(x^*)$. Particularly, the threshold of workers y^* does not affect

FIGURE 4. $P_u - P_v$ for different $F(x^*)$'s



Note: The horizontal axis is measured in a log scale.

the matching ratio at all whereas the threshold of firms x^* does. It is because all workers apply any of firms in a large market.

The effect of θ on each matching ratio is asymmetric for given $F(x^*)$. In general, deviation from $\theta = 1$ affects more the matching ratio of vacancies than the matching ratio of job-seekers. P_u remains better when θ changes from 1 to 1/2, compared to the change of P_v when θ changes from 1 to 2. Contrary, the increase of P_u when θ changes from 1 to 1/2 is not so large compared to the case of P_v when θ changes from 1 to 2. Roughly speaking, the matching ratio of job-seekers under slump is still better than the matching ratio of vacancies under boom, but it is not so good in boom as the matching ratio of vacancies under slump.[5] The asymmetry appears more sharply when vu ratio becomes favourable to either agent (the rightward direction of the graph). Matching ratio of vacancies quickly converges to one as vu ratio goes to the right pane of the graph, but the convergence of matching ratio of job-seekers is not so quick. If vu ratio fluctuates symmetrically around one, it means that unemployment cannot decrease with the same pace as vacancies since, when the economy is in slump, unemployment cannot absorb the relative loss of efficiency when the market environment was favourable. It in turn makes the vu ratio impossible to fluctuate symmetrically around one. It suggests that even if the fluctuation of the inflow into the labour market fluctuates symmetrically, the market environment expressed by vu ratio tends to spend more time unfavourably to job-seekers. The relative deterioration of the matching efficiency for a job-seeker when the market environment is favourable in comparison to the case of a vacancy appears more significantly when $F(x^*)$ becomes positive. Figure 4 shows the difference between the matching ratio of job-seekers and of vacancies for some values of $F(x^*)$. If $F(x^*)$ is strictly positive, the matching ratio of job-seekers does not converge to one even if vu ratio goes to infinity. This is because x and y are independently distributed. The increase of the vu ratio means that for a job-seeker, there will be less competitors. As it goes infinity, it is more likely that he is the only applicant to a particular vacancy. And, since the applied vacancy is the best position for him, the increase of the vu ratio means that he evaluates the position even more highly for each given size of the market. However, since x is independent from y, he still has the possibility to be rejected with probability $F(x^*)$.

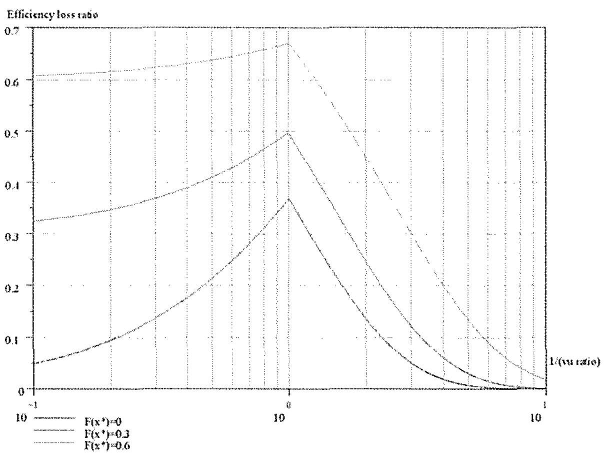
Figure 5 compares the matching efficiency between a stable matching process and a unidirectional advertisement process. The matching ratio of vacancies in a stable matching process is given by $\max\{1/\theta, 1\}$ for a large market, which always exceeds the matching ratio in a unidirectional advertisement process. We write $P_v^* := \max\{1/\theta, 1\}$. The graph shows the efficiency loss ratio which is defined by $-(P_v - P_v^*)/P_v^*$. In the graph, three examples for different values of $F(x^*)$ are drawn. The loss of matching efficiency is

[5] Calculating the following equation

$$\left|\frac{dP_v}{d\theta}\right| - \left|\frac{dP_u}{d\theta}\right| = 1 - \left(1 + \frac{1}{\theta} + \frac{1}{\theta^2}\right) e^{-1/\theta} > 0,$$

we have $|dP_v/d\theta| > |dP_u/d\theta|$.

FIGURE 5. Deterioration of matching efficiency caused by disruption of perfect information



Note: The horizontal axis is measured in a log scale.

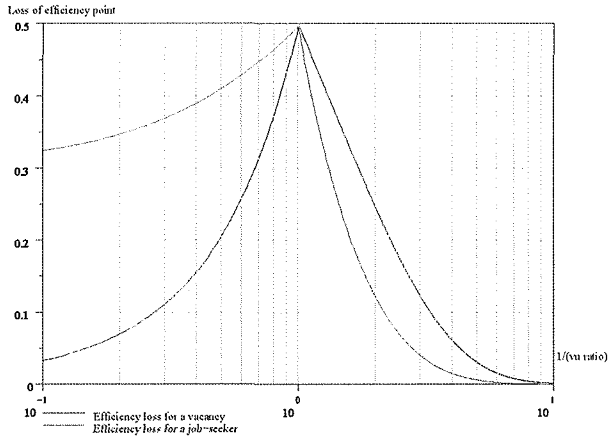
largest when u and v are equal for every instances of $F(x^*)$. The loss of the efficiency decreases significantly if $1/\theta$ becomes larger and away from one, and is obviously asymmetric taking one as the centre. When $F(x^*)$ is strictly positive, the efficiency loss ratio doesn't converge to zero but converges to $F(x^*)$ as $1/\theta$ goes to infinity. Thus, $F(x^*)$ can be regarded as the efficiency loss when the friction between job-seekers becomes negligible. If the vu ratio becomes smaller than one ($1/\theta$ larger than one), the complete information case gives complete matching for vacancies and even for the incomplete information case, the probability that all vacancies receive some applications and there is at least an applicant who is above the threshold level becomes closer to one as the vu ratio converges to zero. If the vu ratio becomes larger than one, the matching ratio for the complete information case is $1/\theta < 1$ as the number of match cannot exceed $\min\{u, v\}$. Therefore, the absolute amount of the loss becomes zero as the vu ratio goes infinity, although the efficiency loss ratio doesn't.

The same Figure 5 applies to the loss of matching efficiency for job-seekers too, since $-(P_v - P_v^*)/P_v^* = -(P_u - P_u^*)/P_u^*$ where $P_u^* := \max\{\theta, 1\}$ is the matching ratio of job-seekers in a stable matching process.[6] That is, the efficiency loss ratio is completely symmetric between the cases for vacancies and job-seekers. When $F(x^*)$ is away from zero, the matching loss ratio of job-seekers remains strictly positive when the market environment becomes favourable for him, although the matching loss ratio of vacancies is away from zero when the market environment becomes unfavourable for it. This symmetry does not hold for the *amount* of loss. Note that the loss in this region should not occur for job-seekers in a stable matching process. It happens when the market gives one hundred percent matching for job-seekers, so it means that the loss doesn't disappear when the market environment becomes infinitely favourable in terms of *both* the matching loss ratio *and* the absolute percentage point of the loss. Figure 6 shows the amount of the efficiency loss for the case of $F(x^*) = 0.3$.

[6] It is obtained by

$$
\begin{aligned}
-\frac{P_v - P_v^*}{P_v^*} &= 1 - \frac{1 - \exp\left\{-\frac{1}{\theta}\left(1 - F(x^*)\right)\right\}}{\min\left\{\frac{1}{\theta}, 1\right\}} \\
&= 1 - \frac{\theta\left[1 - \exp\left\{-\frac{1}{\theta}\left(1 - F(x^*)\right)\right\}\right]}{\min\{1, \theta\}} \\
&= -\frac{P_u - P_u^*}{P_u^*}.
\end{aligned}
$$

FIGURE 6. Amount of matching efficiency loss



Note: The horizontal axis is measured in a log scale.

In Proposition 1, note that P_u and P_v depend on θ, but not on the value of u or v themselves. The following proposition shows that most of ordinary presumption about an aggregate matching function are supported but not all.

Proposition 3. *The asymptotic aggregate matching function $\mu(u, v)$ has the following properties:*

(1) μ is homogeneous of degree one,
(2) $\mu(u, 0) = \mu(0, v) = 0$
(3) $m < \min(u, v)$
(4) $\dfrac{\partial \mu}{\partial u} > 0, \dfrac{\partial \mu}{\partial v} > 0, \dfrac{\partial^2 \mu}{\partial u^2} < 0$
(5) $\dfrac{\partial^2 \mu}{\partial v^2} < 0$ if and only if $\dfrac{u}{v} > \dfrac{2}{1 - F(x^*)}$

Proof. Property (1) is obvious from $\mu(nu, nv) = P_v(nu/nv) \cdot nv = nP_v(u/v)\, v = n\mu(u, v)$ for arbitrary n in the asymptotic aggregate matching function (5). Since $0 \leq P_v \leq 1$, property (2) is obtained from $m = \mu(u, 0) = P_v \cdot 0 = 0$. Also, from $P_v(0) = 0$, we have $\mu(0, v) = P_v(0) \cdot v = 0$. Property (3) is obvious from $P_v < 1$. Property (4) is obtained as follows. Since

$$\begin{aligned} \frac{\partial P_v}{\partial u} &= \frac{1 - F(x^*)}{v} \exp\left\{-\frac{u}{v}(1 - F(x^*))\right\} > 0 \\ \frac{\partial P_v}{\partial v} &= -\frac{1}{v} \cdot \frac{u}{v}(1 - F(x^*)) \exp\left\{-\frac{u}{v}(1 - F(x^*))\right\} < 0, \end{aligned}$$

we have

$$\begin{aligned} \frac{\partial \mu}{\partial u} &= v\frac{\partial P_v}{\partial u} = (1 - F(x^*)) \exp\left\{-\frac{u}{v}(1 - F(x^*))\right\} > 0 \\ \frac{\partial \mu}{\partial v} &= \frac{\partial P_v}{\partial u} v + P_v = 1 - \left[1 + \frac{1}{v} \cdot \frac{u}{v}(1 - F(x^*))\right] \exp\left\{-\frac{u}{v}(1 - F(x^*))\right\} \\ &= 1 - \left(1 + \frac{A}{v}\right) e^{-A} > 0. \end{aligned}$$

Here, we use the fact that $A = \frac{u}{v}(1 - F(x^*)) > 0$ and the last line is obtained fact that $\left(1 + \frac{A}{v}\right) e^{-A}$ is a decreasing function of v in the region $A \geq 0$. Furthermore,

$$\frac{\partial^2 \mu}{\partial u^2} = -\frac{(1 - F(x^*))^2}{v} \exp\left\{-\frac{u}{v}(1 - F(x^*))\right\} < 0$$

For property (5), from

$$\frac{\partial^2 \mu}{\partial v^2} = -\left[\frac{u}{v}(1 - F(x^*)) - 2\right] \frac{1}{v^2} \frac{u}{v}(1 - F(x^*)) \exp\left\{-\frac{u}{v}(1 - F(x^*))\right\},$$

we have $\frac{\partial^2 \mu}{\partial v^2} \lesseqgtr 0$ as $\frac{u}{v}(1 - F(x^*)) \gtreqless 2$. □

Since most of aggregate matching models are based on the assumption of homogeneity of degree one of the matching function, property (1) of Proposition 3 may be of special interest. It gives a justification of such a presumption. It excludes the possibility of multiple equilibria caused by increasing returns to scale of the matching function.

3. Implication to the Beveridge Curve

In Section 2, we obtained matching "technology" constructed on information asymmetry. We are now going to investigate the effect of such technology on statistics which can be summarised as a plot on a uv plane. Investigation of the behaviour of the statistics is not only of statistical interest but also because it has real effects through the wage bargaining process. When friction exists in the labour market, wage rate is determined by the current status of v and u. If the matching technology causes nontrivial behaviour of the uv plot, it can raise nontrivial behaviour of real factors.

Important characteristics of a matching process is that it matches two *stocks* and outputs flow. It is analogous to a vector field on the uv plane. If there is no inflow to stocks, the vector field autonomously determines intertemporal behaviour of the uv plot. One might naturally want to know the ultimate terminal of the plot for given starting point. The set of such terminal points for various starting points are called a Beveridge curve. To be exact, a Beveridge curve thus defined is a theoretical correspondence to the empirically obtained Beveridge curve. It has a name of "curve" since the plot of historical data on the uv plane was roughly a downward sloping curve. However, a theoretically-defined Beveridge curve is not guaranteed to be a smooth curve as will be shown immediately. In addition, note that it is a set of stable fixed points in an *autonomous* vector field. When there is forced oscillation to the system, the set of stable fixed points is not the same as those in an autonomous vector field.[7] And, there is such forced oscillation in the empirical labour market in the form of inflows of new job-seekers and vacancies. It casts doubts on applicability of the notion of the Beveridge curve, *since we are applying a notion defined on an autonomous system to a forced oscillation system.*[8] We will study how addition of forced oscillation changes the "Beveridge curve" in Section 4 and 5. Before proceeding, I will show how traditionally defined Beveridge curve differs for a stable matching process and a unidirectional advertisement process.

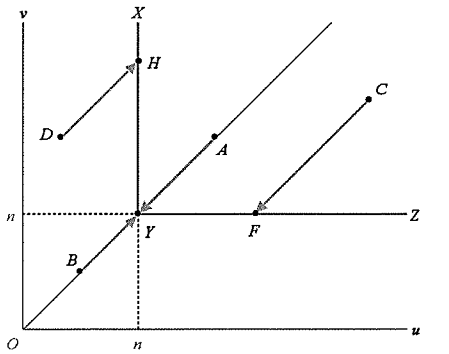
First, we begin with a stable matching process. We assume that inflows are stationary so that $\eta = \xi = n$ where η is inflow of vacancies, ξ is inflow of job-seekers and n is some constant. A stable matching process attains complete matching. If the initial state of stocks is balanced $v = u = n$, all of v and u are matched with no leftover. At the end of the period, there is no one in the labour market. At the beginning of the next period, new inflows restore $v = u = n$ immediately. Therefore, measuring at the beginning of each period, there are always n vacancies and n job-seekers in the labour market.

Now, suppose that one time shock occurred in unemployment so that $\tilde{\xi} \neq n$ only for one period. If $\tilde{\xi} > n$, $\left|\tilde{\xi} - n\right|$ job-seekers cannot match (a match is a one-to-one pair!). They are passed over to the subsequent

[7]If the external force is not periodic, we cannot expect to have stable fixed points.

[8]Moreover, a matching process without inflows is a *damping* system. The effect of adding forced oscillation on a phase is more fundamental for a damping system than the case of non-damping system. Consider a simple example of pendulum. Under the presence of gravity, its swing is damped and stops at bottom finally. However, if the experimenter periodically shakes his hand supporting the pendulum, it will continue to swing. If there were not gravity, change of the phase is not so drastic. Without gravity, the pendulum continues to swing without shaking hand.

FIGURE 7. Dynamics of a stable matching process

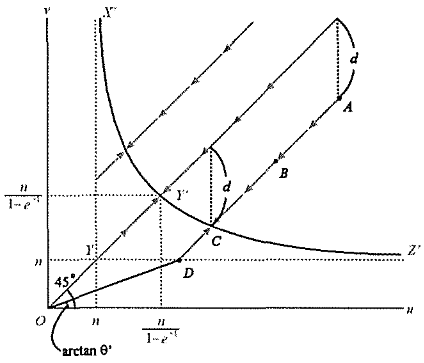


period. If $\tilde{\xi} < n$, $\left|\tilde{\xi} - n\right|$ vacancies are carried over to the next period. The simple but important fact is that *these unmatched agents are not resolved forever* unless a new shock arrives. In the former case, number of job-seekers next period is $u = n + \left|\tilde{\xi} - n\right| = \tilde{\xi}$ because of carry-over. Number of vacancies is $v = n$ since all vacancies are matched. Thus, the same state as previous period is restored. For the latter case, we simply swap vacancies and job-seekers of the former case. Therefore, after one time shock, the labour market always stays at $(u, v) = (n + \left|\tilde{\xi} - n\right|, n) = (\tilde{\xi}, n)$ if $\tilde{\xi} > n$ and $(u, v) = (n, n + \left|\tilde{\xi} - n\right|) = (n, 2n - \tilde{\xi})$ if $\tilde{\xi} < n$. Thus, after a history of various shocks in inflows, the Beveridge "curve" will be on the kinked line XYZ in Figure 7. The initial state with $u = v = n$ is point Y in the figure. Any balanced shocks to, say, point A or B are absorbed immediately and return to point Y. However, an asymmetric shock preserves the difference between η and ξ. A shock to point C today leads to point F for infinite future. A shock to point D leads to point H. Therefore, except those periods a shock occurred, the market will stay on the kinked line XYZ. The position on the kinked line depends on the history of shocks.

Any shocks have absorbed immediately for the stable matching process. The adjustment process of a unidirectional advertisement process is much slower than this. Suppose that initially $u = v = 0$ and $\eta = \xi = n$ in each period. We assume $F(x^*) \equiv 0$ for simple representation. By Theorem 1, probability to match is $1 - e^{-1}$ for both firms and workers. e^{-1} of agents do not match for large samples. In period 0, ne^{-1} vacancies and job-seekers are left over. There are $u = v = (1 + e^{-1})n$ vacancies and job-seekers at the beginning of period 1. Again, e^{-1} of those u and v are passed over to period 2. Thus, $u = v = (1 + e^{-1} + e^{-2})n$ at the beginning of period 2. Such a multiplier process leads to point Y' in Figure 8. The coordinate of Y' is $\left(n/(1 - e^{-1}), n/(1 - e^{-1})\right)$. The multiplier coefficient equals the inverse of matching ratio. Obviously, Y' is located upper right of Y. Similar process works for an arbitrary initial value of (u, v). The imbalance of the initial (u, v) is preserved forever. Since total number of matches is equal for job-seekers and vacancies, uv plot must be always on a 45 degree line offset from the origin which passes through the initial point. When the initial point is A in the figure, the market is always on line AC during the adjustment process which is parallel to the 45 degree line OY'. It is convenient to focus on a set of parallel lines to the 45 degree line in the following analyses. We shall call those lines *isoinflux lines*. For initial point A, the market converges to point C which locates lower right of Y'. Beginning from any initial point on the same isoinflux line like B, the path converges to the same point C.

Matching ratio depends only on θ once $F(x^*)$ is given. If θ is higher, the matching ratio of vacancies becomes smaller. Suppose that the initial value of v is strictly positive while the initial value of u stays zero. Then the initial point is D instead of Y in Figure 8. The matching ratio of vacancies varies each period since θ is not constant every period. However, for any periods, it is higher than the case when the initial values of u and v are zero since $\theta < 1$ always holds. It appears a smaller angle of elevation $\arctan\theta$ at each point. The higher matching ratio of vacancies implies lower multiplier, making the v coordinate of point C smaller than that of Y'. Since C is on the isoinflux line AD which is right to the 45 degree line, point C must be lower right of Y'. All paths with any initial values on the same isoinflux line converge to the same point. Thus, the set of convergent points becomes $X'Y'Z'$ in the figure. As is shown later,

FIGURE 8. Dynamics of an unidirectional advertisement process and the Beveridge curve



the curve is downward sloping and convex to the origin. Its shape is similar to the empirically obtained Beveridge curve.

Note that the Beveridge curve is not a function which relates unemployment with vacancies. It is only a set of stable points. For the Beveridge curve to be visible on the graph, inflows must be always fluctuating going off balance. But such fluctuation itself impedes the convergence to the Beveridge curve. There is a contradiction here. Inflows may fluctuate regularly keeping balanced average with very long cycles. In such a case, the trajectory of uv plots provides a good idea of the existence of Beveridge curve. Especially, if cycle is long enough including sufficiently many matching sessions within a cycle, the plot will not go very far away from the Beveridge curve. However, if the cycle is not regular, if it contains subcycles or if cycle period is too short, the trajectory can be complicated or going far away from the Beveridge curve. Sticking to the notion of the Beveridge curve in such a case is not very useful. The data Beveridge curve predicts contain too much error.

4. Inflow Cycles

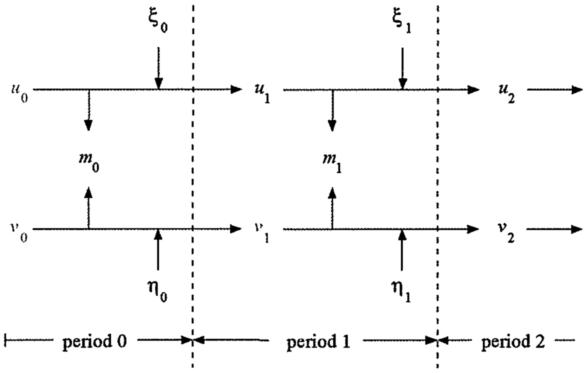
Empirical data show that inflows of new job-seekers and vacancies to the labour market fluctuate persistently and fairly regularly along with business cycles. I said "*fairly* regularly" but in general, it is hopeless for the inflows to have *strictly* regular cycles. Nevertheless, it is still useful to focus on a case in which inflows have regular cycles, since what is important is the recurrence nature of cycles. If we leave inflows arbitrary, it is very difficult to obtain a general result. At most what we can do would be simply to trace the trajectory for each inflow pattern by doing calculation. We need some kind of restriction imposed on external forces which affect the system. In this section, we assume inflows η and ξ to fluctuate and to have same averages over a cycle. They are balanced in long run although the length of a cycle may not coincide. Then, a trajectory on the uv plane beginning from any initial point converges to a unique cycle, the period of which is the least common multiple of the period of vacancy inflow and the period of job-seeker inflow. In the subsequent proof, we use the following extension of the contraction mapping theorem.

Lemma 1. *Let* $\{\mathsf{T}^{(0)}, \mathsf{T}^{(1)}, \dots, \mathsf{T}^{(k-1)}\}$ *be a sequence of contraction mappings with modulus* $\beta_\kappa \in (0, 1)$ *(*$\kappa = 0, 1, \dots, k-1$*) from a Banach space* X *to itself. For any initial value* $x_0 \in X$*, we define a dynamics by*

$$x_{t+1} = \mathsf{T}^{(t \bmod k)} x_t. \qquad (t = 1, 2, \dots)$$

Then, the sequence $\{\mathsf{T}^{(\kappa)}\}_{\kappa=1}^{k-1}$ *has a unique and globally stable cycle* $\{\bar{x}^{(0)}, \bar{x}^{(1)}, \dots, \bar{x}^{(k-1)}\} \in X^k$ *where* $\bar{x}^{(\kappa)}$ *(*$\kappa = 0, 1, \dots, k-1$*) is a fixed point of the mapping* $\mathsf{T}^{(\kappa-1)} \circ \mathsf{T}^{(\kappa-2)} \circ \cdots \mathsf{T}^{(0)} \circ \mathsf{T}^{(k)} \circ \mathsf{T}^{(k-1)} \circ \cdots \circ \mathsf{T}^{(\kappa)}$.

FIGURE 9. Timing of inflow and outflow



Proof. A composite mapping of contraction mappings is a contraction mapping. Namely, for any $k = 0, 1, \ldots$, $\mathrm{T}^{(k-1)} \circ \mathrm{T}^{(k-2)} \circ \cdots \circ \mathrm{T}^{(0)}$ is a contraction mapping. For any $x, y \in X$,

$$\begin{aligned} & \left\| \mathrm{T}^{(k-1)} \circ \mathrm{T}^{(k-2)} \circ \cdots \circ \mathrm{T}^{(0)} x - \mathrm{T}^{(k-1)} \circ \mathrm{T}^{(k-2)} \circ \cdots \circ \mathrm{T}^{(0)} y \right\| \\ \leq\ & \beta_{k-1} \left\| \mathrm{T}^{(k-2)} \circ \mathrm{T}^{(k-3)} \circ \cdots \circ \mathrm{T}^{(0)} x - \mathrm{T}^{(k-2)} \circ \mathrm{T}^{(k-3)} \circ \cdots \circ \mathrm{T}^{(0)} y \right\| \\ \leq\ & \cdots \\ \leq\ & \beta_{k-1}\beta_{k-2} \cdots \beta_0 \left\| x - y \right\| \end{aligned}$$

holds. Applying the contraction mapping theorem, sequence $\{x_0, x_k, x_{2k}, \ldots\}$ has a unique fixed point $\bar{x}^{(0)} \in X$ which is globally stable.

The above result holds for any mappings which can be obtained by permutation of the above mapping: $\mathrm{T}^{(0)} \circ \mathrm{T}^{(k-1)} \circ \mathrm{T}^{(k-2)} \circ \cdots \circ \mathrm{T}^{(1)}$, $\mathrm{T}^{(1)} \circ \mathrm{T}^{(0)} \circ \mathrm{T}^{(k-1)} \circ \cdots \circ \mathrm{T}^{(2)}$, $\cdots$. By the same logic, $\{x_i, x_{k+i}, x_{2k+i}, \ldots\}$ has a unique and globally stable fixed point $\bar{x}^{(i)} \in X$ for any $i = 1, 2, \ldots, k-1$. □

We denote the state of the labour market at the beginning of each period $t = 0, 1, 2, \ldots$ by $(u_t, v_t) \in \mathbb{R}^2_{++}$ where u_t is the number of job-seekers and v_t is the number of unfilled vacancies in the labour market. The initial value (u_0, v_0) is arbitrarily given. At the beginning of each period, firms advertise their vacancies and job-seekers apply to a vacancy which they think best. Successfully matched m_t pairs of a job-seeker and a vacancy exit the market and the rest remains in the market. At the end of the period, new inflow of job-seekers ξ_t and new inflow of vacancies η_t are added to the remaining stocks, forming the stock of job-seekers and vacancies (u_{t+1}, v_{t+1}) in the next period. The timing is shown in Figure 9. We denote by $\mathrm{LCM}(x, y)$ the least common multiple of integers x and y.

Theorem 1. *Suppose that the inflow of new vacancies into the labour market has a cycle of period $S \in \mathbb{N}$. The inflows with in a cycle are given by $\{\eta^0, \eta^1, \ldots, \eta^{S-1}\} \in \mathbb{R}^S_+$. The inflow of new job-seekers into the labour market has a cycle of period $T \in \mathbb{N}$. The inflows within a cycle are given by $\{\xi^0, \xi^1, \ldots, \xi^{T-1}\} \in \mathbb{R}^T_+$. Then, if the condition*

$$\frac{\sum_{i=0}^{S-1} \eta^{(i)}}{S} = \frac{\sum_{i=0}^{T-1} \xi^{(i)}}{T} \tag{8}$$

is satisfied, a trajectory on the uv plane beginning from any initial value $(u_0, v_0) \in \mathbb{R}^2_+$ converges to a cycle of period $\mathrm{LCM}(S, T)$.

Proof. The transition of (u, v) plot is given by

$$v_{t+1} = v_t + \eta^{(t \bmod S)} - m_t \tag{9}$$

$$u_{t+1} = u_t + \xi^{(t \bmod T)} - m_t \tag{10}$$

for any $t = 0, 1, \ldots$. Denoting the least common multiple of S and T by M, it implies

$$v_{t+M} = v_t + \frac{M}{S}\sum_{i=0}^{S-1}\eta^{(i)} - \sum_{i=0}^{M-1} m_{t+i} \tag{11}$$

$$u_{t+M} = u_t + \frac{M}{T}\sum_{i=0}^{T-1}\xi^{(i)} - \sum_{i=0}^{M-1} m_{t+i}. \tag{12}$$

Subtracting equation (12) from equation (11) side by side and using equation (8), we obtain $u_{t+M} = u_t - v_t + v_{t+M}$. Recursive operation gives

$$u_{\kappa+Mn} = u_\kappa - v_\kappa + v_{\kappa+Mn}. \tag{13}$$

for any $\kappa = 0, 1, \ldots, M-1$ and for any $n \in \mathbb{N}$. Namely, the data picked up from time series with interval M stay on the same line. As obvious from derivation, it is one of isoinflux lines.

From equation (5), for any t,

$$\begin{aligned} v_{t+1} &= \eta^{(t \bmod S)} + \exp\left\{-\frac{u_t}{v_t}\left[1 - F(x^*)\right]\right\} v_t \\ &= \eta^{(t \bmod S)} + \exp\left\{-\left(1 - \frac{v_{t \bmod M} - u_{t \bmod M}}{v_t}\right)\left(1 - F(x^*)\right)\right\} v_t \end{aligned} \tag{14}$$

where $v_{t \bmod M} - u_{t \bmod M}$ is a constant obtained by $v_{t \bmod M} - u_{t \bmod M} = v_0 - u_0 + \sum_{i=0}^{t \bmod M-1}\left(\eta^{(i)} - \xi^{(i)}\right)$.

Equations (13) and (14) describe the dynamics. Since (13) is continuous, u converges as v does. Defining the right-hand side of (14) by $\mathrm{T}^{(t \bmod M)} v_t$, we obtain

$$\begin{aligned} \frac{d\mathrm{T}^{(t \bmod M)} v_t}{dv_t} &= \left\{\left(1 - F(x^*)\right)\left(1 - \frac{v_{(t \bmod M)} - u_{(t \bmod M)}}{v_t}\right) + F(x^*)\right\} \\ &\quad \times \exp\left\{-\left(1 - F(x^*)\right)\left(1 - \frac{v_{(t \bmod M)} - u_{(t \bmod M)}}{v_t}\right)\right\} \\ &= (A + F)\exp(-A) \in \left(0,\ e^{-\{1-F(x^*)\}}\right] \subset (0,\ 1) \end{aligned} \tag{15}$$

where $A := \left(1 - F(x^*)\right) u_t/v_t > 0$. For any $a, b \in \mathbb{R}_{++}$, there exists $c \in (a, b)$ such that $\mathrm{T}^{(t \bmod M)} a - \mathrm{T}^{(t \bmod M)} b = (d\mathrm{T}^{(t \bmod M)} c/dc)(a - b)$ by the mean-value theorem. However, $0 < c < \exp\{-(1 - F(x^*))\} < 1$. Thus, for any a and b, $|\mathrm{T}^{(t \bmod M)} a - \mathrm{T}^{(t \bmod M)} b| < \beta|a - b|$ where $0 < \beta < 1$, i.e. $\mathrm{T}^{(t \bmod M)}$ is a contraction mapping.

From Lemma 1, the sequence of mappings $\{\mathrm{T}^{(\kappa)}\}_{\kappa=0}^{M-1}$ has a unique cycle $\{\bar{v}^{(0)}, \bar{v}^{(1)}, \ldots, \bar{v}^{(M-1)}\} \in \mathbb{R}_+^M$ and the paths from any initial value (u_0, v_0) converge to it. Furthermore, $\bar{v}^{(\kappa)}$ $(\kappa = 0, 1, \ldots, M-1)$ is a fixed point of $\mathrm{T}^{(\kappa)}$. This proves that the asymptotic cycle has a period no greater than M. However, the period of the cycle should be exactly M as shown below.

Proof. Suppose that the period of the cycle is $M' < M$. Then, M' must be a divisor of M. Otherwise, it contradicts the above fact that any $v \pmod{M}$ share the same fixed point. For such M' to exist, $\left\{\bar{v}^{(0)}, \bar{v}^{(1)}, \ldots, \bar{v}^{(M'-1)}\right\} = \left\{\bar{v}^{(M')}, \bar{v}^{(M'+1)}, \ldots, \bar{v}^{(2M'-1)}\right\}$ and $\left\{\bar{u}^{(0)}, \bar{u}^{(1)}, \ldots, \bar{u}^{(M'-1)}\right\} = \left\{\bar{u}^{(M')}, \bar{u}^{(M'+1)}, \ldots, \bar{u}^{(2M'-1)}\right\}$ where $\bar{u}^{(n)} := u_{(n)} - v_{(n)} + \bar{v}^{(n)}$ for $n = 0, 1, \ldots, M-1$ must hold. It implies that $u_{(n)} - v_{(n)} = u_{(M'+n)} - v_{(M'+n)}$ for given n. From equation (14), for $\left(\bar{u}^{(n+1)}, \bar{v}^{(n+1)}\right) = \left(\bar{u}^{(M'+n+1)}, \bar{v}^{(M'+n+1)}\right)$ to hold, the condition $\eta^{(n)} = \eta^{(M'+n)}$ must be satisfied. On the other hand, $u_{(n)} - v_{(n)} = u_{(M'+n)} - v_{(M'+n)}$ requires $\sum_{i=0}^{M'-1}\eta^{(i)} = \sum_{i=0}^{M'-1}\xi^{(i)}$ for all n, which implies $\xi^{(n)} = \xi^{(M'+n)}$. The result $\left(\eta^{(n)}, \xi^{(n)}\right) = \left(\eta^{(M'+n)}, \xi^{(M'+n)}\right)$ contradicts the fact that M is the *least* common multiple of S and T. □

□

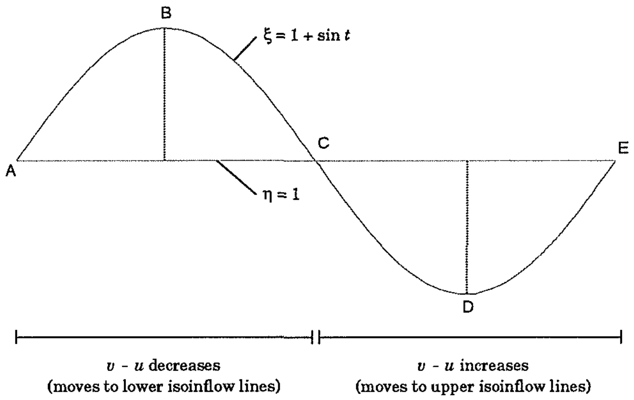
It is an established fact that the trajectory on the uv plane empirically moves around the Beveridge curve. During a recession, u gradually increases whereas v decreases. Thus, the trajectory moves to the lower right of the uv plane. On recovery, opposite pressure carries the trajectory to the upper right. However, when recovery takes place, u does not recover quickly whereas v begins to increase. Thus, the trajectory during a recovery phase goes above the one during a recession phase. Often, this asymmetric response of the labour market to business cycles is attributed to loss of skills of unemployed workers. In recession, the average length of the unemployed period becomes longer. Unemployed workers lose more skills as the unemployed period becomes longer. It makes difficult for unemployed workers to be matched since they tend to be evaluated low and rejected. There would be such an effect working. However, Theorem 1 shows that cycle in the uv plane can emerge even without the existence of loss of skills. Only cyclical fluctuations of new vacancies and new job-seekers causes the emergence of a cycle in the uv plane. There are indications that the deviation of the cycle from the Beveridge curve is too large to be explained by loss of skills. First of all, skills may not deteriorate much even if unemployment period becomes longer. They are not a simple pile of pieces of knowledge. Essence of skills is tacit synthesis of them. Ostensive knowledge and their ostensive synthesis may support it, but opposite is impossible. Accumulation of ostensive knowledge does not create skills. In general, memory of ostensive knowledge fades away relatively quickly, but tacit knowledge to synthesise ostensive knowledge doe not. Even if ostensive knowledge is lost, they can be quickly recovered if tacit knowledge is alive. Second, even if the skill of workers has deteriorated, it is unlikely that a firm can detect such deterioration. Unless a firm imposes a period of trial employment or an examination, direct observation of loss of skills is quite difficult.

Theorem 1 does not tell anything about the shape of the cycle. Especially, it does not tell to which direction the cycle circulates. We will study conditions that bring counterclockwise trajectory on the uv plane now. The dynamics when inflows periodical fluctuates are given by equation (14). Note that $\mathrm{T}^{(t \bmod M)}$ depends only on $v_{(t \bmod M)} - u_{(t \bmod M)}$ and $\eta^{(t \bmod M)}$. $v_{(t \bmod M)} - u_{(t \bmod M)}$ determines the position of the isoinflux line to which sequence $\left\{\left(u_{(t \bmod M)}, v_{(t \bmod M)}\right), \left(u_{2(t \bmod M)}, v_{2(t \bmod M)}\right), \left(u_{3(t \bmod M)}, v_{3(t \bmod M)}\right), \dots\right\}$ belongs to. The initial point in the sequence $v_{(t \bmod M)} - u_{(t \bmod M)}$ is calculated by the equation

$$v_{(t \bmod M)} - u_{(t \bmod M)} = v_0 - u_0 + \sum_{i=0}^{t \bmod M} \left(\eta^{(i)} - \xi^{(i)}\right). \tag{16}$$

Once the isoinflux line is determined above, $\mathrm{T}^{(t \bmod M)}$ solely depends on $\eta^{(t \bmod M)}$.

Consider a simple example that the inflow of vacancies is fixed at $\eta = 1$ and the inflow of job-seekers follows $\xi = 1 + \sin t$. This is a good example to begin with since constant η makes mapping $\mathrm{T}^{(t \bmod M)}$ fixed for all periods. Note that it has balanced inflows. Suppose that there are four matching sessions within a cycle of the inflows as in the figure below.



The matching market opens at points A, B, C, D and E. We assume that initial point is balanced: $v = u$. We plot the status of the matching market that corresponds to points A-E in Figure 10 that shows recursive application of functions . We label corresponding plots in the figure as points A-E as well. Point A in the uv plane (it represents all periods 0 mod 4) is on the 45 degree line $v = u$. At point B (period 1 (mod 4)), job-seeker inflow $\xi^{(1)}$ is larger than vacancy inflow $\eta^{(1)}$ by one. Thus, the labour market is on isoinflux line $v = u - 1$. At point C (period 2 (mod 4)), inflows are balanced. It does not change the isoinflux line the point belong to, so point C stays on the same line as point B. At point D (period 3 (mod 4)), vacancy inflow $\xi^{(3)}$ is less than job-seeker inflow $\eta^{(3)}$ by one, thus the isoinflux line shifts up by one returning back to the 45 degree line. In sum, point A and D and point B and C are on the same isoinflux line respectively. We use the following small lemma.

Lemma 2. *Let $X \subset \mathbb{R}$. If $\mathsf{T} : X \to X$ is a contraction mapping, then $\mathsf{T}x - x$ is x a strictly decreasing function.*

Proof. If $\mathsf{T}x - x$ is not a decreasing function of x, then there exist $a, b \in X$ such that $a > b$ and $\mathsf{T}a - a \geq \mathsf{T}b - b$. However, it implies $\mathsf{T}a - \mathsf{T}b \geq a - b$. Since $a > b$, it means $\|\mathsf{T}a - \mathsf{T}b\| \geq \|a - b\|$. This contradicts the fact that T is a contraction mapping. Therefore, the lemma holds. □

We characterise the limit cycle by the following proposition.

Proposition 4. *Let $\mathfrak{T} = \{\mathsf{T}^{(0)}, \mathsf{T}^{(1)}, \dots, \mathsf{T}^{(k-1)}\}$ be a sequence of contraction mappings from $X \subset \mathbb{R}$ to itself with modulus $\beta_\kappa \in (0, 1)$ ($\kappa = 0, 1, \dots, k-1$). Suppose each mapping is an increasing function. Consider a dynamics defined by*

$$x_{t+1} = \mathsf{T}^{(t \bmod k)} x_t, \qquad (t = 1, 2, \dots)$$

for any initial value $x_0 \in X$. Let $\bar{y}^{(\kappa)}$ ($\kappa = 0, 1, \dots, k-1$) be a fixed point of $\mathsf{T}^{(\kappa)}$ and at least for one pair of $i, j = 0, 1, \dots, k-1$ ($i \neq j$), $\bar{y}^{(i)} \neq \bar{y}^{(j)}$. Then the unique stable limit cycle $\{\bar{x}^{(0)}, \bar{x}^{(1)}, \dots, \bar{x}^{(k-1)}\} \in X^k$ of $\{\mathsf{T}^{(\kappa)}\}_{\kappa=1}^{k-1}$ satisfies the condition

$$\min_{i=0,1,\dots,k-1} \bar{y}^{(i)} < \bar{x}^{(\kappa)} < \max_{i=0,1,\dots,k-1} \bar{y}^{(i)}$$

where $\kappa = 0, 1, \dots, k-1$.

Proof. If we show $\min_{i=0,1,\dots,k-1} \bar{y}^{(i)} < \bar{x}^{(\kappa)}$ ($\kappa = 0, 1, \dots, k-1$), then $\bar{x}^{(\kappa)} < \max_{i=0,1,\dots,k-1} \bar{y}^{(i)}$ can be shown by a similar argument.

We define $\bar{y}^{(p)} := \min_{i=0,1,\dots,k-1} \bar{y}^{(i)}$ and denote by $\mathsf{T}^{(p)}$ the operator that has $\bar{y}^{(p)}$ as a fixed point. Also denote by $\mathfrak{T}(\mathsf{T}^{(p)})$ a composite operator $\mathsf{T}^{(p-1)} \circ \mathsf{T}^{(p-2)} \circ \dots \circ \mathsf{T}^{(0)} \circ \mathsf{T}^{(k-1)} \circ \mathsf{T}^{(k-2)} \circ \dots \circ \mathsf{T}^{(p)}$. Suppose that $\hat{x} \in (-\infty, \bar{y}^{(p)}] \cap X$ is a fixed point of $\mathfrak{T}(\mathsf{T}^{(p)})$. Then, for arbitrary $n \in \mathbb{Z}_+$, $(\mathsf{T}^{(p)})^n \hat{x} \in [\hat{x}, \bar{y}^{(p)}]$ holds. If the fixed point $\bar{y}^{(p+n)}$ (mod M) of $\mathsf{T}^{(p+n)}$ (mod M) satisfy $\bar{y}^{(p+n)} > \bar{y}^{(p)}$, $\mathsf{T}^{(p+n)} \circ (\mathsf{T}^{(p)})^n \hat{x} > \hat{x}$ holds by Lemma 2. Since any operator $\mathsf{T}^{(i)}$ where $i \neq q$ has a fixed point $\bar{y}^{(i)} \geq \bar{y}^{(p)}$, $\tilde{x} := \mathsf{T}^{(p-2)} \circ \dots \circ \mathsf{T}^{(0)} \circ \mathsf{T}^{(k-1)} \circ \mathsf{T}^{(k-2)} \circ \dots \circ \mathsf{T}^{(p)} \hat{x} \geq \mathsf{T}^{(p+n)} \circ (\mathsf{T}^{(p)})^n \hat{x} > \hat{x}$. For $\bar{y}^{(p)}$to be a fixed point of $\mathfrak{T}(\mathsf{T}^{(p)})$, there must exist $\tilde{\mathsf{T}} \in \mathfrak{T}$ such that $\hat{x} = \tilde{\mathsf{T}}\tilde{x}$ holds. It means $\tilde{\mathsf{T}}\hat{x} - \hat{x} = \tilde{\mathsf{T}}\hat{x} - \tilde{\mathsf{T}}\tilde{x} < 0$. Then, from Lemma 2, the fixed point $\tilde{y}$ of $\tilde{\mathsf{T}}$ satisfies $\tilde{y} < \hat{x} \leq \bar{y}^{(p)}$. This contradicts to the definition of $\bar{y}^{(p)}$. Therefore, in the region $(-\infty, \bar{y}^{(p)}] \cap X$, the fixed point of $\mathfrak{T}(\mathsf{T}^{(p)})$ cannot exist. By a similar argument, it is shown that it does not exist in the region $\left[\max_{i=0,1,\dots,k-1} \bar{y}^{(i)}, +\infty\right) \cap X$. □

In our example, there are two contraction mappings. We denote them by $\overline{\mathsf{T}}$ and $\underline{\mathsf{T}}$. $\underline{\mathsf{T}}$ operates on points B and C and $\overline{\mathsf{T}}$ operates on points A and D. In general, $\mathsf{T}^{(t \bmod M)}$ is a function of $\eta^{(t \bmod M)}$ and $v_{(t \bmod M)} - u_{(t \bmod M)}$. Since $\eta^{(t \bmod M)}$ is fixed for the moment, let us write the operator $\mathsf{T}^{(t \bmod M)}$ as $\mathsf{T}^{(t \bmod M)}(v_{(t \bmod M)} - u_{(t \bmod M)})$ to be explicit on the parameter. Consider two cases $\mathsf{T}^{(t \bmod M)}(K_1)$ and $\mathsf{T}^{(t \bmod M)}(K_2)$ where $K_1 > K_2$. Suppose $v_t \neq 0$. From (14), we can show

$$\frac{\mathsf{T}^{(t \bmod M)}(K_1)\, v_t}{\mathsf{T}^{(t \bmod M)}(K_2)\, v_t} = \exp\left\{\left(1 - F(x^*)\right) \frac{K_1 - K_2}{v_t}\right\} > 1.$$

Thus, $\mathsf{T}^{(t \bmod M)}(K_1)\, v_t > \mathsf{T}^{(t \bmod M)}(K_2)\, v_t$ for fixed η. This fact implies $\overline{\mathsf{T}} > \underline{\mathsf{T}}$. It is shown in Figure 10(a).

By Proposition 4, all points on the limit cycle is between $\underline{y}$ and $\overline{y}$. Thus, points A and D are on the left-hand side of the fixed point of $\overline{\mathsf{T}}$. Point A is obtained by operating $\overline{\mathsf{T}}$ twice on v_4 in the graph whereas point D is obtained by operating it only once. Thus, point A is closer to fixed point $\overline{y}$. By the same reason,

FIGURE 10. Dynamics of limit cycles

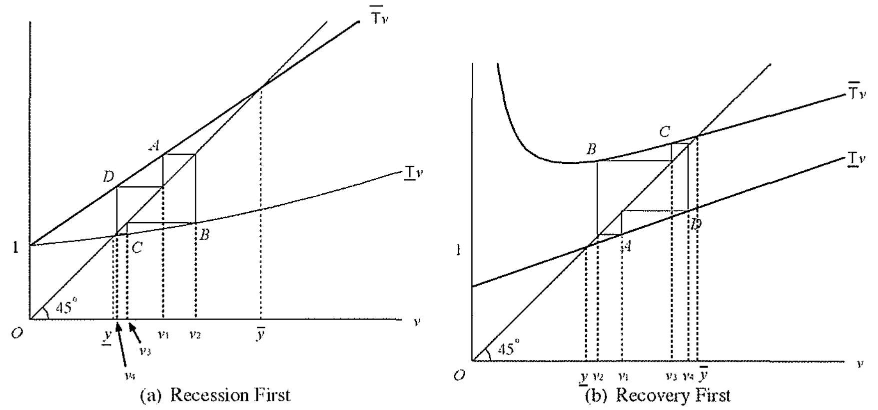
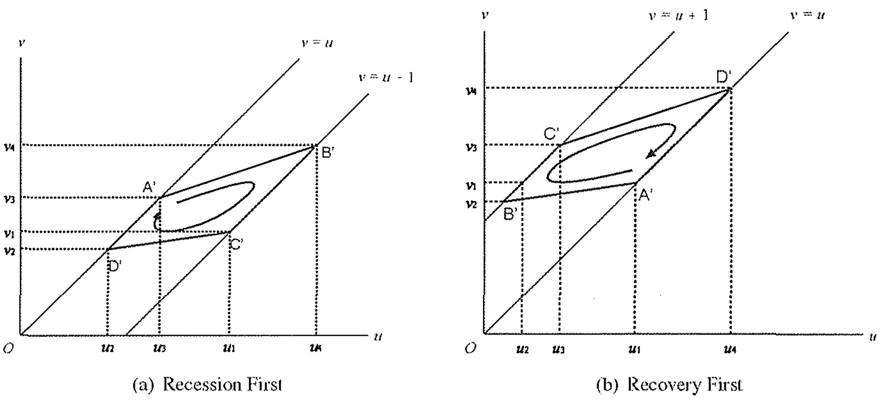


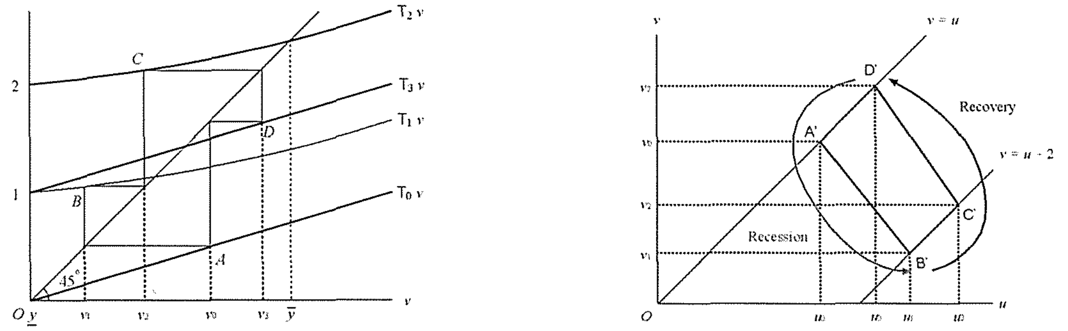
FIGURE 11. Feasible Beveridge cycles



point C is closer to fixed point $\underline{y}$ than point B. Therefore, $v_1 > v_4$ and $v_2 > v_3$. If the economy begins from point C, not A, the diagram changes. It is shown in Figure 10(b). Now, points B and C relates to operator $\overline{\mathrm{T}}$ and points A and D relates to operator $\underline{\mathrm{T}}$. Roles of $\overline{\mathrm{T}}$ and $\underline{\mathrm{T}}$ are swapped. Note that we cannot relabel points since the related isoinflux line of each point is different.

The dynamics of v is translated to the dynamics of u by isoinflux lines. Thus, we have a dynamics of uv plots. It is shown in Figure 11(a). Point A' corresponds to point A in the previous figure and so on. As easily observed, the direction of the cycle is opposite to the empirical cycle. The fact does not change even if we change the direction of the fluctuation of ξ so that ξ fluctuates downwards first instead of upwards. Such a changes moves the uv cycle above the 45 degree line, but does not turn the orientation of the cycle (Figure 11(b)). This is obvious from equation 14. If $v - u$ is greater, the slope of equation 14 is larger. Since η is fixed, it implies that the corresponding mapping T is located above of other equations that have smaller $v - u$. If $v - u$ is higher in period t than period s, it means that the point of the trajectory at period t is on an isoinflux line which is upper-left of the isoinflux line at period s. When the position of an isoinflux line is *relatively* above of other isoinflux lines that the trajectory visits, then v in next period will be larger than the current period. Thus, in the uv plane, as the trajectory goes to upper left (i.e. visits an isoinflux line upper-left) the path tends to shift outward. Note that only *relative* position among related isoinflux

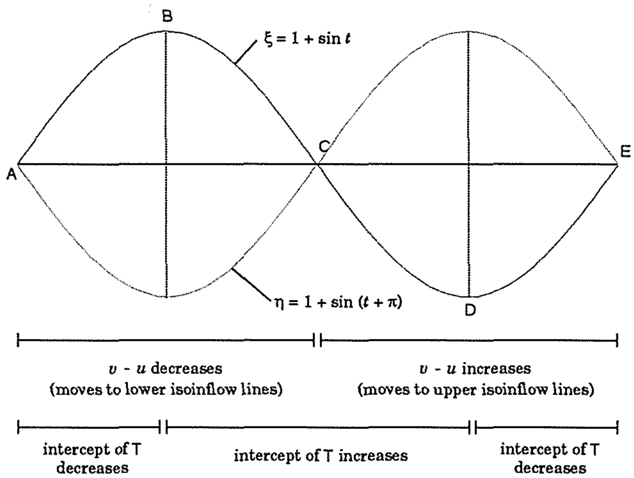
FIGURE 12. More realistic case



(a) Limit cycle in dynamics of v (b) Beveridge cycle

lines is crucial in determining the direction of the shift. The *absolute* position of the isoinflux line, namely the value of $v - u$, is not related. This observation suggests that the fluctuation of vacancies η is necessary to generate an counterclockwise behaviour of the trajectory in the actual data. Therefore, we take another simple example in which η fluctuates.

Consider an example as shown in the figure below. Again, we assume that four matching sessions open within a cycle of inflows. The pattern of job-seeker inflow is unchanged from the previous example. Here, vacancy inflow fluctuates countercyclically to the job-seeker inflow. It is given by $\eta = 1 + \sin(t + \pi)$.



This pattern of ξ and η is similar to the data observed in the actual labour market. In general, ξ is countercyclical and η is procyclical against business cycles. We will see later the symmetric inflow pattern is critical for the emergence of counterclockwise cycle. Figure 12 shows the limit cycle of v and the same movement in the uv plane. The cycle in Figure 12 (b) shows a counterclockwise movement. This property appears not only with the specific functional form used here but appears more generally. We will provide a sufficient condition for the emergence of counterclockwise Beveridge curve. Before doing so, we prove the following lemma.

Lemma 3. *Let $X \subset \mathbb{R}$. Consider a family of contraction mappings $\mathrm{T}(\eta) : X \to X$ which has $\eta \in \mathbb{R}$ as a parameter. Suppose that $\mathrm{T}(\eta)$ is continuous on the parameter and if $\eta < \eta'$ then $\mathrm{T}(\eta) < \mathrm{T}(\eta')$. Suppose η is a continuous function of $t \in \mathbb{R}$ and there exists $T \in \mathbb{R}$ such that for all $n \in \mathbb{Z}$, $\eta(t + nT) = \eta(t)$ Pick up s points $t_i \in [0, T)$ where $i = 0, 1, \ldots, s-1$ and consider a dynamics defined by $x_{k+1} = \mathrm{T}(\eta(t_{k \bmod s}))\, x_k$. Then, it has a unique stable limit cycle $\left\{\bar{x}_s^{(0)}, \bar{x}_s^{(1)}, \ldots, \bar{x}_s^{(s-1)}\right\}$ and if s increases such that $\max|t_{i+1} - t_i| \to 0$ holds, then $\bar{x}_s^{(i)} \to \bar{y}_s^{(i)}$ holds where $\bar{y}_s^{(i)} = \mathrm{T}(\eta(t_i))\, \bar{y}_s^{(i)}$.*

Proof. Suppose that there exists some i such that $\bar{x}_s^{(i)} \not\to \bar{y}_s^{(i)}$. From Lemma 2, there exists some $\varepsilon > 0$ such that $\left|\mathrm{T}\left(\eta\left(t_i\right)\right)\bar{x}_s^{(i)} - \bar{x}_s^{(i)}\right| \geq \varepsilon$. All T's are contraction mappings, thus
(17)

$$\left|\bar{x}_s^{(i)} - \bar{x}_s^{(i-1)}\right| > \frac{\left|\mathrm{T}\left(\eta\left(t_{i-1}\right)\right)\bar{x}_s^{(i)} - \bar{x}_s^{(i)}\right|}{\beta} \geq \frac{\left|\bar{x}_s^{(i+1)} - \bar{x}_s^{(i)}\right| - \left|\mathrm{T}\left(\eta\left(t_i\right)\right)\bar{x}_s^{(i)} - \mathrm{T}\left(\eta\left(t_{i-1}\right)\right)\bar{x}_s^{(i)}\right|}{\beta}.$$

Define $Y := \left[\underline{y}, \bar{y}\right]$. Then, since T is continuous and monotone in terms of the parameter, as s increases so that $\max\left|t_{i+1} - t_i\right| \to 0$ holds, it is possible to achieve that , for any $\eta > 0$ and $\bar{x} \in Y$, $\left|\mathrm{T}\left(\eta\left(t_i\right)\right)\bar{x}_s^{(i)} - \mathrm{T}\left(\eta\left(t_{i-1}\right)\right)\bar{x}_s^{(i)}\right| < \eta$ (Dini). Now, suppose that s is sufficiently large so that, for all i, $\left|\mathrm{T}\left(\eta\left(t_i\right)\right) - \mathrm{T}\left(\eta\left(t_{i-1}\right)\right)\right| < \eta$ holds. Then (17) becomes, for any j,

$$\left|\bar{x}_s^{(j)} - \bar{x}_s^{(j-1)}\right| > \frac{\left|\bar{x}_s^{(j+1)} - \bar{x}_s^{(j)}\right|}{\beta_{j-1}}.$$

Using this recursively, we obtain

$$\left|\bar{x}_s^{(2)} - \bar{x}_s^{(1)}\right| > \frac{\left|\bar{x}_s^{(i+1)} - \bar{x}_s^{(i)}\right|}{\prod_{j=0}^{i-1}\beta_j} \geq \frac{\varepsilon}{\prod_{j=0}^{i-1}\beta_j}.$$

On the other hand, since $\mathrm{T}^{(0)} = \min \mathrm{T}$, $\bar{x}^{(0)} > \underline{y}$ holds, i.e. $\mathrm{T}^{(0)}\bar{x}^{(1)} - \bar{x}^{(1)} < 0$. Then,

$$\begin{aligned}\left|\mathrm{T}\left(\eta\left(t_1\right)\right)\bar{x}^{(1)} - \mathrm{T}\left(\eta\left(t_0\right)\right)\bar{x}^{(1)}\right| &= \left|\bar{x}^{(2)} - \bar{x}^{(1)} + \bar{x}^{(1)} - \mathrm{T}\left(\eta\left(t_0\right)\right)\bar{x}^{(1)}\right| \\ &> \left|\bar{x}^{(2)} - \bar{x}^{(1)}\right| \geq \frac{\varepsilon}{\prod_{j=0}^{i-1}\beta_j}.\end{aligned}$$

This contradicts to the fact that, for any $\eta > 0$ and i, $\left|\mathrm{T}\left(\eta\left(t_i\right)\right) - \mathrm{T}\left(\eta\left(t_{i-1}\right)\right)\right| < \eta$. Therefore, for all i, $\bar{x}_s^{(i)} \to \bar{y}_s^{(i)}$must hold. □

Theorem 2. *Suppose that η and ξ are time continuous and $d\eta/dt \gtreqless 0 \Leftrightarrow d\xi/dt \lesseqgtr 0$. Then, if matching sessions are held sufficiently frequently within a cycle of η and ξ, then the point that the trajectory on the uv plane crosses a particular isoinflux line from the right comes upper right of the point that the path crosses it from the left in the uv plane.*

Proof. Since η is time continuous, for any $\varepsilon > 0$ and t, if the interval of the matching session is made sufficiently short, then it is possible to make $\left|\eta^{(t \bmod M+1)} - \eta^{(t \bmod M)}\right| < \varepsilon$. That is,

$$\begin{aligned}&\eta - \varepsilon + \exp\left\{-\left(1 - \frac{v_{(t \bmod M)} - u_{(t \bmod M)}}{v_t}\right)\left(1 - F\left(x^*\right)\right)\right\}v_t \\ &< \ v_{t+1} < \eta + \varepsilon + \exp\left\{-\left(1 - \frac{v_{(t \bmod M)} - u_{(t \bmod M)}}{v_t}\right)\left(1 - F\left(x^*\right)\right)\right\}v_t.\end{aligned}$$

From $v_{(t \bmod M)} - u_{(t \bmod M)} = v_0 - u_0 + \sum_{i=1}^{t \bmod M}\left(\eta^{(i)} - \xi^{(i)}\right)$ and $d\eta/dt \gtreqless 0 \Leftrightarrow d\xi/dt \lesseqgtr 0$, the differentiation by $\eta^{(t \bmod M)}$ of the left-most or the right-most hand side of the above equation gives

$$1 + \left(1 - F\left(x^*\right)\right)K_t\exp\left\{-\left(1 - \frac{v_{(t \bmod M)} - u_{(t \bmod M)}}{v_t}\right)\left(1 - F\left(x^*\right)\right)\right\} > 0$$

where K_t is a positive constant. Therefore, taking ε sufficiently small, T can be ordered by $\eta^{(t \bmod M)}$. Namely, if $\eta_1 > \eta_2$, then $\mathrm{T}\left(\eta_1\right) > \mathrm{T}\left(\eta_2\right)$.

Now, suppose that the uv path passes over the isoinflux lines $v - u = k_1$ and $v - u = k_2$ where $k_1 < k_2$, and it is currently on the line $v - u = k$ where $k_1 < k < k_2$ after passing over $v - u = k_2$ for the first time. Then $\eta^{(t \bmod M)} - \xi^{(t \bmod M)} < 0$. Since η and ξ cyclically fluctuates, the path must return to a point on $v - u = k_2$ from Theorem 1. It means that the path crosses $v - u = k$ with $\eta - \xi > 0$. From $d\eta/dt \gtreqless 0 \Leftrightarrow d\xi/dt \lesseqgtr 0$, the η in the latter case has larger value. From Proposition 3, it implies that, if matching session is held frequently enough, then the fixed point of v in the latter case must be larger than the former case. □

The theorem says that when the trajectory forms a simple cycle, namely when it crosses any isoinflux line at most twice within a cycle, then the direction of such a cycle is counterclockwise under the conditions provided by the theorem. Generally, when the path crosses any isoinflux line from the right, its crossing point should be upper right of any points where the path crosses it from the left, and vice versa. Note that we assumed no changes in matching technology. The cycle is brought by the asymmetry between job-seekers and vacancies embedded in the matching process.

5. Unbalanced Inflows

In this section, we consider a case where inflows are not balanced in long run. We define norm $\|\mathsf{T}\|$ of an operator $\mathsf{T} : X \to X$ by $\|\mathsf{T}\| := \sup_{x\in X} |\mathsf{T}x|$. The following lemma allows the operator not being constant in the contraction mapping theorem.

Lemma 4. *Let $\{\mathsf{T}_1, \mathsf{T}_2, \dots, \mathsf{T}_n, \dots\}$ be a sequence of contraction mappings from a Banach space X to itself with modulus $\beta \in (0, 1)$. For an arbitrary initial value $x_0 \in X$, consider a dynamics of $\{x_i\}_{i=0}^{\infty} \in X^{\infty}$ defined by*

$$\forall i = 1, 2, \dots, \quad x_i = \mathsf{T}_i x_{i-1}.$$

If $\{\mathsf{T}_n\}_{n=1}^{\infty}$ converges to T_∞ under the norm defined above, then the sequence of operators $\{\mathsf{T}_n\}_{n=1}^{\infty}$ has a unique stable fixed point $\bar{x} \in X$ and $\bar{x}$ is a fixed point of T_∞.

Furthermore, if one takes sufficiently large n, then for any initial value $x \in X$ and any $\varepsilon > 0$,

$$\|\mathsf{T}_n \cdot \mathsf{T}_{n-1} \cdots \mathsf{T}_1 x_0 - \bar{x}\| < \beta^n \|x_0 - \bar{x}\| + \varepsilon \tag{18}$$

holds.

Proof. For any $n = 1, 2, \dots,$

$$\begin{aligned} \|x_{n+1} - x_n\| &= \|\mathsf{T}_{n+1}x_n - \mathsf{T}_n x_{n-1}\| \\ &= \|(\mathsf{T}_{n+1}x_n - \mathsf{T}_n x_n) + (\mathsf{T}_n x_n - \mathsf{T}_n x_{n-1})\| \\ &\leq \|\mathsf{T}_{n+1}x_n - \mathsf{T}_n x_n\| + \|\mathsf{T}_n x_n - \mathsf{T}_n x_{n-1}\| \\ &\leq \|\mathsf{T}_{n+1}x_n - \mathsf{T}_n x_n\| + \beta \|x_n - x_{n-1}\| \end{aligned}$$

holds. The third line is from the triangle inequality and the fourth line if from the fact that T_n is a contraction mapping. Using this recursively, we obtain

$$\begin{aligned} \|x_{n+1} - x_n\| \leq\ & \|\mathsf{T}_{n+1}x_n - \mathsf{T}_n x_n\| + \beta \|\mathsf{T}_{n+1}x_n - \mathsf{T}_n x_n\| + \beta^2 \|\mathsf{T}_n x_{n-1} - \mathsf{T}_{n-1}x_{n-1}\| \\ & + \cdots + \beta^{n-2} \|\mathsf{T}_2 x_1 - \mathsf{T}_1 x_1\| + \beta^{n-1} \|x_1 - x_0\|. \end{aligned}$$

Since $\{\mathsf{T}_n\}$ converges under the norm defined above, it is a Cauchy sequence and there exists maximum in the difference. We denote it by $K := \max_{i=1,2,\dots} \|\mathsf{T}_{i+1} - \mathsf{T}_i\|$. On the other hand, since $\{\mathsf{T}_n\}$ is a Cauchy sequence, for any $\eta > 0$, there exists N, for any $\nu > N$,

$$\forall x \in X, \ \|\mathsf{T}_\nu x - \mathsf{T}_{\nu-1}x\| < \eta$$

holds. Now, if one takes n large enough corresponding to the value of η, then one can find $N \leq n$ which satisfies the above condition, and obtain

$$\begin{aligned} \|x_{n+1} - x_n\| &< \eta + \beta\eta + \cdots + \beta^{n-N}\eta + \beta^{n-N+1}K + \cdots + \beta^{n-2}K + \beta^{n-1}\|x_1 - x_0\| \\ &= \frac{1-\beta^{n-N+1}}{1-\beta}\eta + \beta^{n-N+1}\frac{1-\beta^{N-2}}{1-\beta}K + \beta^{n-1}\|x_1 - x_0\|. \end{aligned}$$

If we take $n = N$, then

$$\|x_{n+1} - x_n\| < \eta + \beta\frac{1-\beta^{n-2}}{1-\beta}K + \beta^{n-1}\|x_1 - x_0\|.$$

If we take η sufficiently small so that $\eta + \beta\frac{1-\beta^{n-2}}{1-\beta}K < \varepsilon$ holds and take n sufficiently large correspondingly, then the condition (18) is satisfied. Then, $\{x_i\}_{i=0}^{\infty}$ is a Cauchy sequence and have a limit point $\bar{x} \in X$.

Next, we show that $\bar{x}$ is a fixed point of T_∞. For any n,

$$\begin{aligned}\|\mathsf{T}_\infty\bar{x}-\bar{x}\| &\leq \|\mathsf{T}_\infty\bar{x}-\mathsf{T}_n\bar{x}\|+\|\mathsf{T}_n\bar{x}-x_n\|+\|x_n-\bar{x}\| \\ &\leq \|\mathsf{T}_\infty\bar{x}-\mathsf{T}_n\bar{x}\|+\beta\|\bar{x}-x_{n-1}\|+\|x_n-\bar{x}\|\end{aligned}$$

The first line is from the triangle inequality and the second if from the fact that T_n is a contraction mapping. Since T_n converges to T_∞, the first term of the last line converges to zero and, from the proof in the previous paragraph, x_n converges to $\bar{x}$, which means the second and third terms converges to zero. Since n is arbitrary, it implies $\|\mathsf{T}_\infty\bar{x}-\bar{x}\|=0$, i.e. $\mathsf{T}_\infty\bar{x}=\bar{x}$. Thus, $\bar{x}$ is a fixed point of T_∞.

By a similar argument, one can show that the fixed point of T_∞ which is also a limit point of $\{\mathsf{T}_n\}$ is unique. If there exists another fixed point $\hat{x}\neq\bar{x}$ which is a limit point of $\{\mathsf{T}_n\}$, then for any n,

$$\begin{aligned}\|\hat{x}-\bar{x}\| &= \|\mathsf{T}_\infty\hat{x}-\mathsf{T}_\infty\bar{x}\| \\ &\leq \|\mathsf{T}_\infty\hat{x}-\mathsf{T}_n\hat{x}\|+\|\mathsf{T}_n\hat{x}-\mathsf{T}_n\bar{x}\|+\|\mathsf{T}_n\bar{x}-\mathsf{T}_\infty\bar{x}\| \\ &\leq \|\mathsf{T}_\infty\hat{x}-\mathsf{T}_n\hat{x}\|+\beta\|\hat{x}-\bar{x}\|+\|\mathsf{T}_n\bar{x}-\mathsf{T}_\infty\bar{x}\|\end{aligned}$$

holds. The first and third term in the last line converge to zero, thus taking sufficiently large n,

$$\|\hat{x}-\bar{x}\|\leq\beta\|\hat{x}-\bar{x}\|$$

holds. Since $\beta<1$, the above inequality holds only when $\|\hat{x}-\bar{x}\|=0$, namely $\hat{x}=\bar{x}$. □

In the above lemma, each T_n is a contraction mapping, but it is not guaranteed that T_∞ is also a contraction mapping. Therefore, all the fixed points of T_∞ need not be a limit point of $\{\mathsf{T}_n\}$, but its inverse is true.

The following lemma is a straightforward extension of the generalised contraction mapping theorem proven above.

Lemma 5. *Let $\{\mathsf{T}_0,\mathsf{T}_1,\dots,\mathsf{T}_n,\dots\}$ be a sequence of contraction mappings from a Banach space X to itself with modulus $\beta_i\in(0,1)$ where $i=0,1,\dots$. Suppose that there exists some k such that, for any $\kappa=0,1,\dots,k-1$, $\lim_{n\to+\infty}\|\mathsf{T}_{\kappa+kn}\|=:\left\|\mathsf{T}_\infty^{(\kappa)}\right\|$ exists and holds. Define a dynamics by*

$$\forall t=0,1,\dots,\quad x_{t+1}=\mathsf{T}_t x_t$$

for any initial value $x_0\in X$. Then the sequence of operators $\{\mathsf{T}_t\}_{t=1}^{\infty}$ has a unique stable limit cycle $\{\bar{x}^{(0)},\bar{x}^{(1)},\dots,\bar{x}^{(k-1)}\}\in X^k$ and $\bar{x}^{(\kappa)}$ for $\kappa=0,1,\dots,k-1$ is a fixed point of $\mathsf{T}_\infty^{(\kappa-1)}\circ\mathsf{T}_\infty^{(\kappa-2)}\circ\cdots\circ\mathsf{T}_\infty^{(0)}\circ\mathsf{T}_\infty^{(M)}\circ\mathsf{T}_\infty^{(k-1)}\circ\cdots\circ\mathsf{T}_\infty^{(\kappa)}$. Furthermore, for any initial value $x\in X$,

$$\|\mathsf{T}^n x_0-\bar{x}\|<\left(\prod_{i=1}^{\infty}\beta_i\right)\|x_0-\bar{x}\|$$

holds.

Proof. From Lemma 1, the composite function of contraction mappings is a contraction mapping, thus the composite mapping of the subsequent k periods $\overline{\mathsf{T}}_i:=\mathsf{T}_i\circ\mathsf{T}_{i+1}\circ\cdots\circ\mathsf{T}_{i+M-1}$ for $\forall i=0,1,\dots$ is a contraction mapping. Therefore, for any $i=0,1,\dots,k-1$, consider a sequence of operators $\{\overline{\mathsf{T}}_i,\overline{\mathsf{T}}_{i+k},\overline{\mathsf{T}}_{i+2k},\dots\}$ and Lemma 4 implies the proposition of this lemma. □

Using these lemmas, we can explore the behaviour of the trajectory on the uv plane when the inflows are not balanced.

Theorem 3. *Suppose that inflow of vacancies to the labour market has a cycle of period S. We denote a series of vacancy inflow within a cycle by $\{\eta^{(0)},\eta^{(1)},\dots,\eta^{(S-1)}\}\in\mathbb{R}_{++}^{S}$. Also, suppose that inflow of job-seekers has a cycle of period T and a series of job-seeker inflow within a cycle is denoted by $\{\xi^{(0)},\xi^{(1)},\dots,\xi^{(T-1)}\}\in\mathbb{R}_{++}^{T}$. Then, if*

$$\frac{\sum_{i=0}^{S-1}\eta^{(i)}}{S}\leq\frac{\sum_{i=0}^{T-1}\xi^{(i)}}{T} \tag{19}$$

holds, then v converges to a cycle of period S and $u\to+\infty$. On the other hand, if

$$\frac{\sum_{i=0}^{S-1}\eta^{(i)}}{S}\geq\frac{\sum_{i=0}^{T-1}\xi^{(i)}}{T} \tag{20}$$

holds, then u converges to a cycle of period T and $v \to +\infty$.

Proof. The theorem is proved by a similar argument to Theorem 1. From equation (19), for some $K \geq 0$,

$$\frac{\sum_{i=0}^{S-1} \eta^{(i)}}{S} + K = \frac{\sum_{i=0}^{T-1} \xi^{(i)}}{T} \tag{21}$$

Then, $(M/T)\sum_{i=0}^{T-1} \xi_i - \sum_{i=0}^{M-1} m_{t+i} = X_t + MK$ holds where $X_t := (M/S)\sum_{i=0}^{S-1} \eta_i - \sum_{i=0}^{M-1} m_{t+i}$, and equations (11) and (12) are denoted by

$$\begin{aligned} v_{t+M} &= v_t + X_t \\ u_{t+M} &= u_t + X_t + MK \end{aligned}$$

Putting $x_{n-1} := \sum_{i=0}^{n-1} X_{j+Mi}$, it becomes, for any $j = 0, 1, \ldots, M-1$ and $n \in \mathbb{N}$,

$$v_{j+Mn} = v_j + x_{n-1} \tag{22}$$

$$u_{j+Mn} = u_j + x_{n-1} + nMK \tag{23}$$

where $v_1, \ldots, v_{M-1}$ and $u_1, \ldots, u_{M-1}$ are obtained by recursive application of equations (9) and (10). Then,

$$u_t = u_{t \bmod M} - v_{t \bmod M} + v_t + (t\,\mathrm{div}M)MK.$$

where $t\,\mathrm{div}M := \lfloor t/M \rfloor$. Using this,

$$\begin{aligned} v_{t+1} &= \eta^{(t \bmod S)} + \exp\left\{-\left(1 - \frac{v_{t \bmod M} - u_{t \bmod M} - (t\,\mathrm{div}M)K}{v_t}\right)\left(1 - F(x^*)\right)\right\} v_t \\ &=: \mathsf{T}_t v_t \end{aligned}$$

Since $v_{t \bmod M} - u_{t \bmod M}$ is bounded as $t \to +\infty$, the coefficient of the second term of T_t becomes $\lim_{t\to\infty} \exp\left[-\left\{1 - \left[v_{t \bmod M} - u_{t \bmod M} - (t\,\mathrm{div}M)K\right]/v_t\right\}\left(1 - F(x^*)\right)\right] = 0$. Then, if we take a bounded and sufficiently large domain that includes all sequence and a limit point, then $\|\mathsf{T}_t - \mathsf{T}_{t+S}\| \to 0$. Then, from Lemma 5, the sequence of operators $\{\mathsf{T}_t\}_{t=0}^{\infty}$ has a unique limit cycle $\{\bar{v}^{(0)}, \bar{v}^{(1)}, \ldots, \bar{v}^{(S-1)}\} \in \mathbb{R}^S$. It means that v converges to a cycle of period S. On the other hand, since v converges, equation (22) implies $x_{n-1} \to 0$. However, since $nMK \to \infty$, (23) implies that u diverges to infinity.

The second half of the theorem is proved by applying a similar argument on the transition equation of u. Equation (20) means that now we have $K \leq 0$ in equation (21), which implies

$$v_t = v_{t \bmod M} - u_{t \bmod M} + u_t - (t\,\mathrm{div}M)MK$$

where $K \leq 0$. From (2),

$$\begin{aligned} u_{t+1} &= \xi^{(t \bmod T)} + \left[1 - \frac{v_t}{u_t}\left(1 - e^{-[1-F(x^*)]u_t/v_t}\right)\right] u_t \\ &= \xi^{(t \bmod T)} + \left[1 + A(u_t, t)\left(1 - \exp\left\{[1 - F(x^*)]\, A(u_t, t)^{-1}\right\}\right)\right] u_t \end{aligned}$$

where $A(u_t, t) := \left[u_{t \bmod M} - v_{t \bmod M} + (t\,\mathrm{div}\,M)MK\right]/u_t - 1 \leq 0$.

Since $\lim_{t\to+\infty} A(\cdot, t)\left(1 - \exp\left\{\left(1 - F(x^*)\right) A(u_t, t)^{-1}\right\}\right) = 0$, then for a bounded and sufficiently large domain that includes all sequence and a limit point, $\|\mathsf{T}_t - \mathsf{T}_{t+M}\| \to 0$. Then, from Lemma 5, the sequence of operators $\{\mathsf{T}_t\}_{t=0}^{\infty}$ has a unique limit cycle $\{\bar{x}^{(0)}, \bar{x}^{(1)}, \ldots, \bar{x}^{(M-1)}\} \in X^M$ and a uv path which begins from any initial value (u_0, v_0) converges to it.

Since u converges, (21) implies $x_{n-1} + nMK \to 0$, which means $x_{n-1} \to +\infty$ since $nMK \to -\infty$. Thus, (22) implies that v diverges to infinity. □

The theorem shows that when there is imbalance in inflows, the trajectory tends to shift outside. The sign of the imbalance decides whether the shift occurs *upward* or *rightward*, but never shifts the path *inside* (i.e. downward or leftward). As seen in the previous section, abstracting cyclical effect, shift never occurs when inflows are balanced. For a path once shifted out to come back, inflows must balance and one must wait until the matching process brings it back onto the Beveridge curve.

6. General Inflows

The following theorem refers to a general case where periodicity of inflows are not assumed. In the previous theorems, we employed the notion of isoinflux line to reduce the problem into the one which includes only one variable. In a general problem, there is no guarantee that a future point regresses to a given isoinflux line which past points visited. The notion of isoinflux line is not very useful. However, we still use a similar notion, but we cannot reduce the number of variables to one.

Theorem 4. *Consider sequences $\{\eta_t\}_{t=0}^{\infty} \in \mathbb{R}_{++}^{\infty}$ and $\{\xi_t\}_{t=0}^{\infty} \in \mathbb{R}_{++}^{\infty}$. Both are bounded and have strictly positive lower bound. Then, if $\sum_{t=0}^{\infty} (\eta_t - \xi_t)$ is bounded, then $\{(u_t, v_t)\}_{t=0}^{\infty}$ is globally stable in the sense of Liapnov.*

Proof. The transition equations of u and v are

$$\begin{aligned} v_{t+1} &= \eta_t + \exp\left\{-\frac{u_t}{v_t}\left[1 - F(x^*)\right]\right\} v_t \\ u_{t+1} &= \xi_t + \left[1 - \left(1 - \exp\left\{-\frac{u_t}{v_t}\left[1 - F(x^*)\right]\right\}\right)\frac{v_t}{u_t}\right] u_t. \end{aligned} \tag{24}$$

Defined the right hand side of the simultaneous equations as $\mathsf{T}_t\binom{u_t}{v_t} : \mathbb{R}_+^2 \to \mathbb{R}_+^2$. First, we show that u_t/v_t is bounded away from zero. The intercept of the isoinflux line at time t is given by

$$u_t - v_t = u_0 - v_0 + \sum_{i=0}^{t} \xi_i - \sum_{i=0}^{t} \eta_i =: B_t. \tag{25}$$

Since $\sum_{t=0}^{\infty} (\eta_t - \xi_t)$ is bounded, there exist $\underline{B}$ and $\overline{B}$ such that $B_t \in [\underline{B}, \overline{B}]$. If $B_t \geq 0$, then $\frac{u}{v} \geq 1 > 0$. If $B_t < 0$, then $\underline{B} < 0$. We denote the lower bound of η by $\underline{\eta} > 0$ and the lower bound of ξ by $\underline{\eta} > 0$. Then, we have

$$\inf \frac{u}{v} = 1 + \inf \frac{B_t}{v} \geq 1 + \frac{\underline{B}}{\inf v} \geq 1 + \frac{\underline{B}}{\max\{\underline{\xi} - \underline{B}, \underline{\eta}\}} > 0.$$

Therefore, u_t/v_t is always bounded away from zero. We denote the lower bound of u_t/v_t by $\underline{\mathfrak{B}} > 0$.

By a symmetric argument, v_t/u_t is also bounded away from zero. It implies that u_t/v_t is bounded above. We denote the upper bound of u_t/v_t by $\overline{\mathfrak{B}} < \infty$.

We define $\overline{\mathsf{T}}$ by

$$\overline{\mathsf{T}}\begin{pmatrix} v \\ u \end{pmatrix} := \begin{pmatrix} \overline{\eta} \\ \overline{\xi} \end{pmatrix} + \begin{pmatrix} \exp\{-\underline{\mathfrak{B}}(1 - F(x^*))\} & 0 \\ 0 & 1 - \dfrac{1 - \exp\{-\underline{\mathfrak{B}}[1 - F(x^*)]\}}{\overline{\mathfrak{B}}} \end{pmatrix}\begin{pmatrix} v \\ u \end{pmatrix}.$$

Then, T_t satisfies

$$\mathsf{T}_t\begin{pmatrix} v \\ u \end{pmatrix} \leq \overline{\mathsf{T}}\begin{pmatrix} v \\ u \end{pmatrix}.$$

The inequality shows that inequality holds for each element of the vector. Since $\overline{\mathsf{T}}$ is a monotonically increasing contraction mapping from $\mathbb{R}^2$ to itself, it has a unique fixed point. We denote a fixed point of a contraction mapping T by $\bar{v}(\mathsf{T})$. Then, $\bar{v}(\mathsf{T}_t) \leq \bar{v}(\overline{\mathsf{T}})$.

Denote a projection onto v axis by $\mathfrak{V}[\cdot]$ and a projection onto u axis by $\mathfrak{U}[\cdot]$. If $v_t \geq \mathfrak{V}[\bar{v}(\overline{\mathsf{T}})]$, then for any $\tau \geq t$, $v_\tau < \mathfrak{V}[\overline{\mathsf{T}}v_t]$ holds. If $v_t \leq \mathfrak{V}[\bar{v}(\overline{\mathsf{T}})]$, then for any $\tau \geq t$, $v_\tau < \mathfrak{V}[\bar{v}(\overline{\mathsf{T}})]$. Similarly, If $u_t \geq \mathfrak{U}[\bar{u}(\overline{\mathsf{T}})]$, then for any $\tau \geq t$, $u_\tau < \mathfrak{U}[\overline{\mathsf{T}}u_t]$ holds. If $u_t \leq \mathfrak{U}[\bar{u}(\overline{\mathsf{T}})]$, then for any $\tau \geq t$, $u_\tau < \mathfrak{U}[\bar{u}(\overline{\mathsf{T}})]$. Then, (u, v) has an upper bound $(\max\{\mathfrak{V}[\overline{\mathsf{T}}v_0], \mathfrak{V}[\bar{v}(\overline{\mathsf{T}})]\}, \max\{\mathfrak{U}[\overline{\mathsf{T}}v_0], \mathfrak{U}[\bar{v}(\overline{\mathsf{T}})]\})$.

On the other hand, $v_0 \geq 0$ and $\mathsf{T}_t v_t$ is a mapping from $\mathbb{R}_{++}$ to $\mathbb{R}_{++}$. Thus, v_t has 0 as a lower bound. The same holds for u. Therefore, $\{(u_t, v_t)\}_{t=0}^{\infty}$ is bounded. □

7. Examples

Our matching market model is open-ended. We do not consider other mechanisms working outside. Even wage determination process is external. Therefore, the model is applicable to either state of equilibrium or disequilibrium. The results obtained so far, especially in Section 4 and 5, provide a guideline how the trajectory look like. However, it does not give an idea about the actual shape of the path. In this section, I'd like to provide some examples for typical inflow patterns.

Figure 13 illustrates some examples when the condition of Theorem 1, $\sum_{i=0}^{S-1}\eta_i/S = \sum_{i=0}^{T-1}\xi_i/T$, holds. Figure 13 (a) shows the trajectory in the case that the inflow of job vacancies is constant at $\eta = 1$ and the inflow of job-seekers fluctuates to satisfy $\xi = 1 + \sin t$. All figures assume that the labour market opens with a time frequency of 0.2. In (b), inflows have the same frequency and amplitude: $\eta = 1 + \cos t$ and $\xi = 1 + \sin t$. (c) is the case in which inflows of vacancies and job-seekers are symmetric, i.e. the condition in Theorem 2 $d\eta/dt \gtreqless 0 \Leftrightarrow d\xi/dt \lesseqgtr 0$ is satisfied. (d) is the case when there is a difference in frequency between the inflows. Such difference in frequency makes the trajectory to circulate gradually. (e) is the case when the frequency of inflows are increased compared to (d). (f) is the case when the inflow is not a simple harmonic motion but show a complex oscillation. The vacancy inflow is fixed at $\eta = 2$ and the job-seekers inflow follows $\xi = 2 + \sin(2t) + \sin(3t)$.

Figure 14 shows the cases when the inflows are not balanced. Figure 14 (a) shows the case when the condition $\sum_{i=0}^{S-1}\eta_i/S \geq \sum_{i=0}^{T-1}\xi_i/T$ in Theorem 3 holds. As it predicts, v diverges and u converges to a periodic cycle. Similarly, Figure 14 (b) shows the case that the condition $\sum_{i=0}^{S-1}\eta_i/S \leq \sum_{i=0}^{T-1}\xi_i/T$ holds. u diverges and v converges to a periodic cycle.

8. Beveridge Curve

Finally, we will provide more formal illustration of Section 3. It confirms the common understanding that Beveridge curve is a collection of fixed points. As a special case of Theorem 1, we have the following result.

Corollary 4. *Suppose that $\eta_t = \xi_t = k$ holds for any period t where $k > 0$ is a constant. Consider two sequences $\{(u_i, v_i)\}_{i=0}^{\infty}$ and $\{(u_i', v_i')\}_{i=0}^{\infty}$ such that $(u_0, v_0) \neq (u_0', v_0')$ and $v_0 - u_0 = v_0' - u_0'$. Then, two sequences converges to the same fixed point on the uv plane.*

Proof. Put $S = T = 1$ in Theorem 1. □

The corollary says that any two trajectories beginning from different points on the same isoinflux line converges to the same point on the isoinflux line if inflows are constant. Thus, we can well-define the Beveridge curve as follows.

Definition 1. The *Beveridge curve* $B(k)$ is a set of fixed points in the uv plane to which the trajectory from any initial point converges when $\eta_t = \xi_t = k$ for all t.

Proposition 5. *The Beveridge curve is continuous and strictly convex to the origin.*

Proof. The Beveridge curve is a set of fixed points of a mapping $\mathsf{T}v = k + [1 - b(v, K)]\, v$ on $u = v - K$ for any $K \in \mathbb{R}$, where $b(\cdot\,,\cdot)$ is the matching ratio of vacancies and $b_v > 0$ and $b_K < 0$. At the fixed point $(\bar{u}, \bar{v})$,

$$\bar{v} = k + [1 - b(\bar{v}, \bar{v} - \bar{u})]\, \bar{v} \tag{26}$$

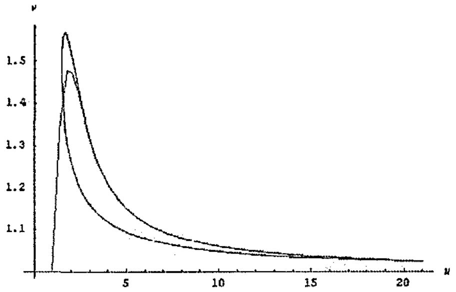
holds. From the implicit function theorem, $\bar{u} = \bar{u}(\bar{v})$ is a continuous function and satisfies

$$\begin{aligned} \frac{d\bar{u}}{d\bar{v}} &= \frac{b + (b_v + b_K)\,\bar{v}}{b_K \bar{v}} < 0 \\ \frac{d^2\bar{u}}{d\bar{v}^2} &= -\frac{b}{b_K \bar{v}^2} > 0. \end{aligned}$$

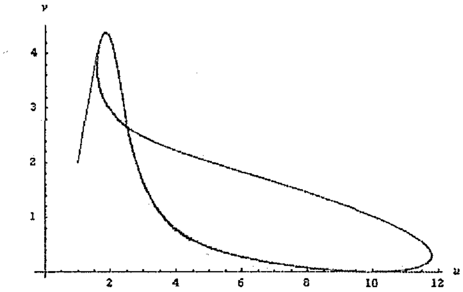
The Beveridge curve is a decreasing and convex function. Therefore, the Beveridge curve is strictly convex to the origin. □

The following proposition says k can be interpreted as a shift parameter of the Beveridge curve $B(k)$.

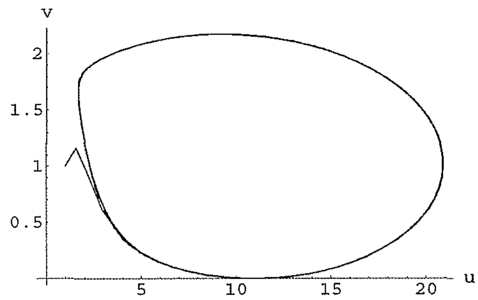
FIGURE 13. Examples of trajectories when the inflows are balanced



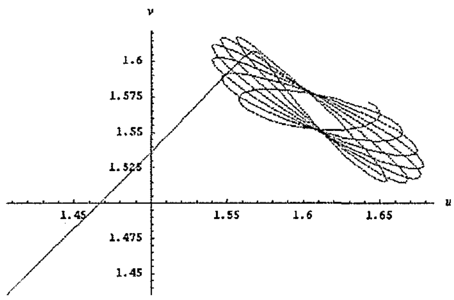
(a) $\xi = 1 + \sin t,\ \eta = 1$



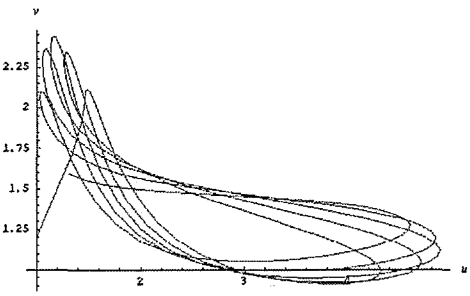
(b) $\xi = 1 + \sin t,\ \eta = 1 + \cos t$



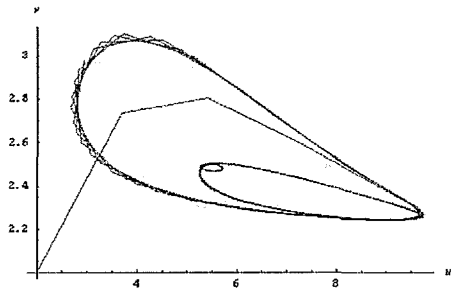
(c) $\xi = 1 + \sin t,\ \eta = 1 + \sin(t - \pi)$



(d) $\xi = 1 + \sin t/100,\ \eta = 1 + \cos(1.1t)/70$

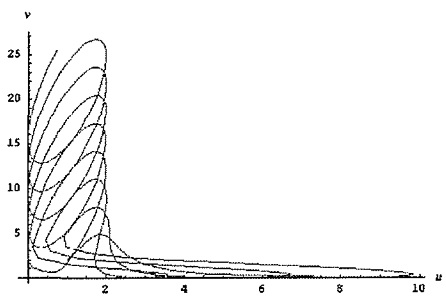


(e) $\xi = 1 + \sin t/5,\ \eta = 1 + \cos(1.1t)/5$

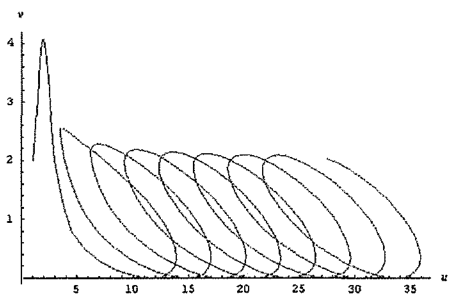


(f) $\xi = 2 + \sin(2t) + \sin(3t),\ \eta = 2$

FIGURE 14. Examples of trajectories when the inflows are not balanced



(a) $\xi = 1 + \sin t,\ \eta = 1.1 + \cos t$



(b) $\xi = 1.1 + \sin t,\ \eta = 1 + \cos t$

Proposition 6. *When $k' \neq k''$, $B(k') \cap B(k'') = \emptyset$. Furthermore, the higher k the Beveridge curve $B(k)$ has, the more upper-left it is located in the uv plane.*

Proof. Applying the implicit function theorem on 26, we have

$$\frac{d\bar{v}}{dk} = -\frac{1}{b + v(b_1 + b_2)} > 0$$

for any fixed k. Therefore, on the same isoinflux line, the Beveridge curves with different k do not share a point and the Beveridge curve with higher k has higher $\bar{v}$ on the isoinflux line. □

Beveridge curves $B(k)$ that have different value of k does not cross. Note that the Beveridge curve shifts out if we have more inflows.

For the Beveridge curve not to degenerate, $\max \sum \eta_t - \sum \xi_t \neq \min \sum \eta_t - \sum \xi_t$ is necessary. If $|\max\{\sum \eta_t - \sum \xi_t\} - \min\{\sum \eta_t - \sum \xi_t\}| \to 0$, then the path converges to an isoinflux line, which means the path converges to the intersection point of the isoinflux line and the Beveridge curve. It means that as the amplitude of the inflow cycle converges to zero, the frequency that the labour market opens must increase to infinite.

9. CONCLUSION

In this paper, we studied a matching process which arises when there is a cost to collect information about agents in the labour market. We found that its implication is consistent with data in many ways including the behaviour on the uv plane. The main reason that the behaviour on the uv plane becomes not so trivial is that the it plots two *stock* variables. It is considered that wage rate under the existence of labour market friction becomes a function of the uv ratio. Since a shock does not reflect immediately to the uv ratio, wage rate responds slowly. It deteriorates the adjustment power of wage rate between inbalance of labour supply and demand.

The actual matching process may contain more frictions than the simple information asymmetry proposed in Section 2. The matching ratio of job-seekers in 2000 was 6.25% and the matching ratio of vacancies was 10.13% in Japan.[9] These numbers do not change drastically irrespective of the phase of business cycles. They look much lower than our matching model predicts. It may simply reflect the fact that the interval between matching sessions is too long. Or, it may be because of other factors which belong to the behavioural pattern of market participants. The average matching ratio of a marriage broker is also more or less 10%. Brokers can theoretically attain complete matching by screening and they do adopt meeting after screening as a major mediation method. Even so, the matching ratio is as low as 10%. The last case suggests that people think important *the way* to be matched, at least in certain matching processes.

APPENDIX. RATIONALE OF THE ADVERTISEMENT PROCESS

Note that communication necessary here is only mutual revelation of agent's own properties. It is only a bundle of "*advertisement*" of the self properties in opposite directions. The properties may be a result of bilateral negotiation, but we can safely abstract it as though it is found on encounter if the advertisement includes sufficient information.[10] The advertisement does not necessarily have to be made independently in pairs. Only requirement is that advertisement becomes bilateral between agents who are searching before matching is made. Various measures of advertisement with various scope are available, with different costs. A personnel section chief may invite his friend engineer to a new job opened in his company while talking in a pub. Advertisement cost is almost negligible in this case (maybe a pint of beer), but the scope of advertisement is minimal. Unless the personnel section chief chances to know an appropriate person in advance, he would rather choose media of broader scope, probably paying additional cost, because such personal contact with unknown people sums up to huge amount of cost. However, by the economy of scale, there *are* efficient media whose scope is sufficiently wide and whose cost is sufficiently low such as newspaper ads and job-placement journals. To abstract the difference of prior knowledge of matching candidates, we assume that no agents have personal connection. In addition, we assume that there are only

[9] The source of the data is the record of the Public Employment Security Office excluding new school graduates.

[10] When the receiver of advertisement observes its content, he can correctly expect the outcome of negotiation as he knows the characteristics of the advertiser and also himself.

TABLE 1. Advertisement strategy

		Firm	
		Advertise	No Advertise
Worker	Advertise	$(\Omega_w^s - c_w,\ \ \Omega_f^s - c_f)$	$(\Omega_w^a - c_w,\ \ \Omega_f^r)$
	No Advertise	$(\Omega_w^r,\ \ \Omega_f^a - c_f)$	$(B,\ \ 0)$

two types of advertisement media available, the one whose scope covers all searching agents with low cost and the other whose scope is minimal with no cost, simply contacting one of candidates already known.

Now, workers and firms each have two choices, to advertise or not to advertise, which is shown in Table 1. Only representative worker and firm are written in the matrix. Corresponding to the state who advertises, the different matching process arises. If both workers and firms do not advertise, then no one can find candidates. No production occurs, no profits in the hand of firms, workers only receiving unemployment benefit $B \geq 0$. We assume that no cost is needed to keep a job vacancy. If both workers and firms advertise, we obtain a stable matching as an outcome. If only one side advertises, advertisers set deadline and wait for application from readers of the advertisement. If only some of each group advertise, then the same procedure occurs. The expected monetary utility obtained when entered in the matching market which arises for a particular combination of Advertise and No Advertise is denoted by Ω's in the table. Expected utility of a worker is denoted by a subscript w and that of a firm by f. When both advertise, the arising matching is a stable matching and its expected utility is denoted by superscript s. When only one side advertises, the matching process becomes such that readers of advertisement which advertisers make respond to it. The expected utility of an advertiser is denoted by superscript a and that of a reader by superscript r. The cost of advertisement is $c_w \geq 0$ for a worker and $c_f \geq 0$ for a firm.

Consider next the case that both workers and firms advertise. Firms know all workers and their profiles, and the same holds to workers. Therefore, the search process is undertaken under perfect information, which implies that the outcome is one of stable matchings.

In general, when there are multiple stable matchings, there is at least an agent who has incentive to misrepresent his/her preference to obtain a better matching. However, Roth (1984) showed that a stable matching obtained by misrepresented preferences is also stable under true preferences. Therefore, the possibility of misrepresentation would not interfere the presumption that the expected outcome is stable. We denote by Ω_w^s the expected discounted value of future worker's utility measured by money when matched through the stable matching process. Similarly, that of a firm is denoted by Ω_f^s. Advertisement cost for a worker is c_w and that for a firm is c_f. Then, the payoff is $\Omega_w^s - c_w$ for the worker and $\Omega_f^s - c_f$ for the firm. The results by Dagsvik (2000) and Yokota (2001) suggest that a stable matching process asymptotically matches all agents in the smaller group. In addition, the utility of each agent after matching process is highest among *feasible* mates. Agent A is *feasible* for B if all agents ranked higher than B by A rejects A. Therefore, Ω_w^s and Ω_f^s are higher than other Ω's in the matrix.

In the off-diagonal elements of the matrix, only one group advertises. The advertisers do not have any information about the advertisement readers, whereas the readers *do* obtain information about the advertisers through the advertisement. If one of the readers wants to make a pair with one of the advertisers, he must "visit" the advertiser.[11] If he does not, nothing happens. The "visit" is a type of advertisement with minimal scope and we assume that the readers can do it with minimal cost (like that personnel chief in the pub). This is sufficient for readers since they already know that the advertiser is seeking for a partner. After the advertisement, the matching process carries on as follows. Advertisement readers decide to which advertisers they apply. They apply to an advertiser who they think best. After application, selection procedure begins which binds the applicant and prevents him from joining selection procedures of other advertisers. Advertisers select the most preferable candidate from the applicants and form a pair with him. The rest is rejected and goes to find a new advertisement. In this matching process, an advertisement reader will apply to only one firm, since he cannot join multiple selection procedures that subsequently follow. The assumption that the selection procedure does not allow an applicant to join other selection procedures

[11]The reader need not physically visit the advertiser. However, he needs to show his will to be matched with the advertiser *and also* his profile which is necessary for selection. In many cases, the required profiles include nonverbal factors which cannot be written down on a résumé. The reader would have to be interviewed at least before final selection.

may be too strong. Even though the number is limited, an applicant can, and will, join different selection procedures in reality, taking into account the risk of rejection. In this case, an applicant will be approved from multiple advertisers, some ranking him top, others keeping him as a substitute candidate. Then, the final stage of the selection becomes a variant of stable matching problem where the list of preference order of each agent is truncated. The length of the preference list is restricted by the physical condition that how many selection procedures an agent can join in simultaneously. To do so, there must exist multiple selection procedures proceeding whose expiration date is almost the same, above all. This condition is easily satisfied for a matching market of newly graduating workforce, but may not hold for other matching processes. Given that there exist sufficient number of selection opportunities, the upper limit of multiple attendance is determined by various conditions such as the occupying time in a day and the frequency of each selection stage, transportation cost, and the advertiser's policy whether he gives priority to confronting his competitors or he gives precedence to candidates' welfare by giving insurance against risk. The upper limit is physically predetermined by these conditions. It does not increase when the market size grows. In general, when only one group advertises and attendance to other selection procedures is not allowed, the matching does not reach to complete matching. Utility of an applicant conditional upon match is the same as the case of stable matching process since both obtain the highest feasible mate. Utility of an advertiser conditional upon match is lower than or equal to the stable matching process. In the unidirectional advertisement process, readers who are feasible in the stable matching process may not visit the advertiser. A reader who are not feasible in the stable matching process will not visit the advertiser again in the unidirectional advertisement. Anyway, the utility of an agent conditional upon match is lower than or equal to the case of stable matching process. Because of lower possibility to match, the expected utility that can be obtained through the unidirectional advertisement process is generally lower than the stable matching process. The possibility to attend multiple selection procedures enhances its expected utility, but still does not change the property of incomplete matching. Therefore, the superiority of the stable matching process is immutable.

We write the expected utility measured by money of an advertiser by Ω_i^a $(i = w,\ f)$. When the advertiser is a worker, it is Ω_w^a and when it is a firm, we write Ω_f^a. Similarly, we denote by Ω_i^n $(i = w,\ f)$ the expected utility of an advertisement reader measured by money. Then, the advantage of a stable matching process is expressed by the conditions $\Omega_i^s > \Omega_i^a$ and $\Omega_i^s > \Omega_i^n$. An advertiser must pay advertisement cost, c_w if he is a worker, c_f if he is a firm. An advertisement reader does not have to pay any cost. In the unidirectional advertisement process, how accurately advertisement media can transmit necessary information plays a critical role. Necessary information for a worker to choose a job would be: kind of work he is going to engage in, speciality the job requires, wage rate, fringe benefit, working hours, so on. All of these can be written down in the advertisement. Contrarily, many properties a firm wants to know about a particular worker belong to nonverbal factors. Especially, when the job is a moderately general work, the measure to evaluate a worker's ability belongs to *tacit knowing* (Polanyi (1958)).[12] Therefore, if workers are advertisers and firms are readers, there is a huge risk on the side of firms that the top-ranked worker evaluated through the advertised information was not actually so good. Taking into account this error, we expect $\Omega_f^n \ll \Omega_f^s$. There is no such big error for workers, so $\Omega_w^n < \Omega_w^s$ but their difference is not so big as the case of firms. Rather, since additional information of workers via advertisement is not much use for firms, we expect $\Omega_w^n \fallingdotseq \Omega_w^s$. The asymmetry of required information between workers and firms also brings asymmetry in cost structure. A firm only need publicise objective data of the job if it wants to advertise. In addition, many properties about a firm are already known to people through everyday production activities. It will save the cost of job advertisement for a firm. However, a worker must promulgate various supporting evidences which hint his property. If workers advertise, they are forced to make exertions to distinguish

[12] Once, I have engaged in recruiting new graduates to a job, which requires flexibility and adaptation to new situations, several times. To collect information about students which would be used later in the selection process, we called interested students to an informal meeting and prompted them to discuss various problems which loosely relate to the job, while giving information about the job to the students. About ten colleagues were observing the meeting independently, sometimes motivating students. After the session, we evaluated and ranked students from the view point of aptitude for the job. With a few exceptions of students ranked at bottom, the ranking amazingly coincided among colleagues without knowing *why* we ranked students in that way. The evaluation could be shared only by the colleagues who attended. If we were asked to show the output of the session, then we could only show the ranking of students as an objective result. We might have been able to provide additional comments about each student, but they would have remained only supporting evidences.

themselves from others. The cost of advertisement for a worker tends to be high. Therefore, we expect to have the relations $\Omega_f^s - c_f > \Omega_f^n$ and $\Omega_w^s - c_w < \Omega_w^n$. We assume that the advertisement cost of a firm is not so high that it exceeds the benefit which is obtained from advertisement: $c_f < \Omega_f^a$. Thus, $\Omega_f^a - c_f > 0$. Then, the unique outcome of Table 1 is that firms advertise and workers do not advertise.

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