Dishonest Sellers and Costly Information*

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1. Introduction

Recently, the economics of information has drawn much attention. The purpose of this paper is to study the competitive general equilibrium in the case where information is distributed asymetrically among agents. When fututre periods or uncertainty is considered, the proposition about Pareto optimality of competitive equilibrium is valid only if the market is complete in the sense that there exist a large number of contingent markets. But this presumption may hardly be satisfied.

For example, temporary equilibrium models take the case when future markets are lacked and focus on the efficiency of stationary states [Grandmont (1982), Grandmont and Younes (1972), and Kodaira (1980)]. However, people generally do not need to agree on probabilities of future events to obtain Pareto optimality or at least *ex ante* optimality. Starr (1973) provides the conditions under which people do have homogeneous expectations sufficient to obtain *ex post* optimality. In his case, people know perfectly the characteristics of commodities that they are buying or selling.

However, the assumption seems too much demanding that the buyers know as well what they are buying as the sellers do. The main source of imperfect information is that both good products and bad ones are sold on the same market and that it is very difficult and almost impossible for buyers

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to distinguish the good from the bad. Another source is the fact that people may not know the exact assessment of goods without consuming. This is particularly true when the goods are new products.

Here, we consider an economy in which there are intrinsic differences in commodities and sellers who notice the differences try to sell commodities to consumers who do not perceive the differences without meaningful brands or signals. In this sense, sellers are said "dishonest." We may call this situation an Akerlof's (1970) economy.

In next section, we define a short run equilibrium concept in such an economy and proves its existence under certain conditions (especially, "optimistic" expectation about qualities plays an important role). Then, a long run equilibrium is discussed in which the revision process of expectations leads to a stationary state (section 3). Finally, costly information is introduced and the efficiency is studied.

2. Model and short run equilibrium

Consider a pure exchange economy with two classes of commodities. The first commodity class is a class of commodities in the sense that they are traded in separate markets (one to one relationship between commodities and markets). The second is a group of commodities that are traded in one market (the "amalgam" market). Without loss of generality, we can assume that there exist three commodities, one of which (good x) is the first category good and the other (goods y and z) the second. Suppose, for simplicity, that every consumer prefers good y to z (for example, riped lemons and overriped or rotten lemons). Then, consumers face uncertainty concerning the proportin of good y that they can expect from the purchase of "amalgam."

We assume that there exist many but finite number of agents in the economy so that the perfect competition prevails. Each agent, indexed by $i=1, \ldots, n$, is characterized by his initial endowment, preference relation defined over consumption vectors, and expectation about the proportion. Let

$$w^i=(w^i_x,\ w^i_y,\ w^i_z)\epsilon R^3_+$$

be the vector of endowments of agent i and x^i be the quantity of good x

that he consumes. Goods y and z are traded as the amalgam in the second market and consumers are not sure how much good y (hence z) is contained in what they are buying. Let y^i(z^i, respectively) be the quantity of good y (z, respectively) that consumer i consumes out of his initial holdings and m^i be the quantity of amalgam that he buys from the market. Writing θ the proportion of good y contained in the amalgam, his consumption vector is

$$(x^i,\ y^i+\theta m^i,\ z^i+(1-\theta)m^i).$$

Assume that he has a subjective probability distribution on this proportion θ, which depends on prices $p=(p_x,\ p_m)\epsilon\Delta$, a unit simplex of R^2; in other words, he has an expectation function ψ^i which associates with each $p\epsilon\Delta$ a probability measure on the unit interval $I=[0,\ 1]$. Then, the expectation function ψ^i may be written as a mapping from Δ into the space $\mathcal{M}(I)$ of probability measure on I. The space Δ is endowed with the usual topology, $\mathcal{M}(I)$ with the weak topology. The image of p under the mapping ψ^i is denoted by $\psi^i(\cdot\ ,\ p)$. Here we assume the followings:

Assumption 1. The mapping ψ^i is continuous from Δ into $\mathcal{M}(I)$, $i=1,\dots,n$.
Assumption 2. Von Neumann-Morgenstern utility function u^i from R^3_+ into R is bounded, concave and continuous. Furthermore, u^i is strictly monotone.

Then, he has an expected utility function (Belnoulli index) v^i defined by

$$(1)\quad v^i(x^i,\ y^i,\ z^i,\ m^i)=\int_I u^i(x^i,\ y^i+\theta m^i,\ z^i+(1-\theta)m^i)\ \mathrm{d}\,\psi^i(\theta,\ p).$$

Lemma 1. The function v^i from R^4_+ into R is bounded, concave, continuous and strictly monotone.
(Proof) See Grandmont (1972, 1982).

An action $a^i=(x^i,\ y^i,\ z^i,\ m^i)$ is called technologically feasible if $a^i\epsilon A^i=\{(x,\ y,\ z,\ m)\,|\,x,\ y,\ z,\ m\geq 0,\ y\leq w^i_y,\ z\leq w^i_y\}$. An action is said to satisfy the budget constraint if $a^i\epsilon B^i(p)=\{(x,\ y,\ z,\ m)\,|\,p_x x+p_m(y+z+m)\leq p_x w^i_x+p_m(w^i_y+w^i_z)\}$. Then, an action a^i is called feasible if $a^i\epsilon\Gamma^i(p)=A^i\cap B^i(p)$. Let us call $\Gamma^i(p)$ the budget correspondence.
Lemma 2. The budget correspondence $\Gamma^i(p)$ from $\Delta\times R^3_+$ into R^4 is contin-

uous and non-empty.

(Proof) Consider a sequence of $p^{\nu} \equiv (p_x^{\nu}, p_m^{\nu}) \in \Delta$, $\nu = 1, 2, 3, \cdots$ converging to $p^0 \equiv (p_x^0, p_m^0) \in \text{int } \Delta$ and the corresponding sequence of $a^{\nu} \equiv (x^{\nu}, y^{\nu}, z^{\nu}, m^{\nu}) \in \Gamma^i(p^{\nu})$ with $\{(p^{\nu}, a^{\nu})\}$ converging to (p^0, a^0). Letting $\nu \to \infty$,

$$p^0 a^0 \leq p^0 a' \text{ for any } a' \in \{a \in \Gamma^i | a \geq a^0\}.$$

Therefore,

$$(p^0, a^0) \in \text{the graph of } \Gamma^i(p).$$

Hence, Γ^i has a closed graph. Since the set Γ^i is compact by the assumption the correspondence Γ^i is upper semicontinuous.

For the lower semicontinuity, it is senough to show that $a \in \Gamma^i$ and $\{p^{\nu}\} \to p^0$ imply the existence of $\{a^{\nu}\}$ converging to a^0 such that $a^{\nu} \in \Gamma^i(p^{\nu})$. Now, there exists $a^* \in \Gamma^i$ such that $p\ a^* < p\ \bar{a}$. The convergence of $\{p^{\nu}\}$ to $\bar{p}$ implies that for large ν, $p^{\nu}\ a^* < p^{\nu}\ \bar{a}$. Therfore, $a^* \in \Gamma^i(p^{\nu})$ for large ν. Let a^{**} be an arbitrary point of $\Gamma^i(p^{\nu})$. Define

$$a^{\nu} = t^{\nu}\ a^{**} + (1 - t^{\nu}) a^*$$

where $t^{\nu} \in [0, 1]$ is a maximal real number such that $a^{\nu} \in \Gamma^i(p^{\nu})$. It remains to show that the sequence $\{a^{\nu}\}$ converges to a^{**}. From the definition of a^{ν}, this is true if and only if $\{t^{\nu}\} \to 1$. Suppose the contrary. Since $t^{\nu} \in [0, 1]$, there exists a subsequence $\{t^{\nu}\}$ converging to $t^* < 1$. But from the definition of $x^{\nu'}$

$$p^{\nu'}\ a^{\nu'} = p^{\nu'}\ a^* \text{ for any } \nu'.$$

Suppose the sequence of $a^{\nu'}$ converges to $a^{0'}$ as the price sequence $\{p^{\nu'}\} \to p^0$. Therefore,

$$p^0\ a^{0'} = p^0 \{t^*\ a^{**} + (1 - t^*) a^*\} = p^0\ \bar{a}.$$

Here, $p^0\ a^0 < p^0\ \bar{a}$, hence $p^0\ a^* > p^0\ \bar{a}$. That is

$$a^* \notin \Gamma^i(p^0),$$

which leads to a contradiction.

Suppose

$$p = (p_x, p_m) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

which clearly lies in the interior of the price space Δ. The non emptyness is obvious. (Q. E. D.)

Therefore, the consumer's problem is to choose an action a^i from his feasible set $\Gamma^i(p)=A^i \cap B^i(p)$ so as to maximise his expected utility $v^i(a^i)$ defined by (1). Define the decision correspondence of consumer i as

(2) $D^i: \Delta \rightarrow R^4_+$

given by

$$D^i(p)=\{a^{i*}=(x^{i*}, y^{i*}, z^{i*}, m^{i*})\in\Gamma^i(p)\,|\,v^i(a^{i*})\geq v^i(a) \quad \text{for all } a\in\Gamma^i(p)\}.$$

Lemma 3. Decision correpodence $D^i(p)$ defined by (2) is upper semicontinuous (hence, compact valued).

(Proof) By the definition (2)

$$D^i(p)\equiv\Gamma^i(p)\cap C^i(p)$$

where $C^i(p)\equiv\{z\in\Gamma^i(p)\,|\,v^i(a^i)\geq v^i(a)$ for any $a\in\Gamma^i(p)\}$. Since $\Gamma^i(p)$ is shown to be lower semicontinuous in Lemma 2, it remains to prove that the correspondence C^i is continuous on the price p, which is convex and compact.

The upper semicontinuity of C^i follows from Lemma 1. For the lower semicontinuity, it suffices to show that the existence of $z\in C^i$ and the convergence of $\{p^\nu\}$ to p imply that there exists a sequence $\{z^\nu\}$ converging to z such that $z^\nu\in C^i(p^\nu)$. Now, there exists $z\in\Gamma^i$ such that $v^i(z)\geq v^i(\bar{z})$ and $\{p^\nu\}\rightarrow p$ imply that $v^i(z^\nu)\geq v^i(\bar{z})$ for large ν. Therefore, $z\in C^i(p^\nu)$ for large ν. Let z' be an arbitrary point of C^i. Define

$$z^\nu=t^\nu\ z'+(1-t^\nu)z$$

where $t^\nu\in[0, 1]$ is a maximal real number such that $z^\nu\in C^i$.

Claim that $z^\nu\rightarrow z'$. From the definition of z^ν, this is true if and only if $\{t^\nu\}\rightarrow 1$. Suppose not. Then, there exists a subsequence $\{t^{\nu'}\}$ of $\{t^\nu\}$, converging to $t'<1$. Suppose

$$z^{\nu'}\in C^i(p^{\nu'})\rightarrow\bar{z} \text{ as } p^{\nu'}\rightarrow p.$$

But by the continuity of expectation and the convexity of preference relation, $\bar{z}\in C^i(p)$, which leads to the desired contradiction.

Finally, the compactness follows from the upper semicontinuity as the correspondence is non empty and bounded (Lemma 2). (Q. E. D.)

Lemma 4. The decision correspondence $D^i(p)$ does not have an equilibrium with a price vector at the boundary of the price space.

(Proof) Suppose not, then there exists a sequence of $p^k \in \Delta$ tending some $p' \in$ bdry Δ and a corresponding sequence of $\{a^{i*k}\} \in D^i(p^k)$. And one can also find $p \in \Delta$ such that the value of $p\ a^{ik}$ is bounded above. But it is easily shown that, in such a case, the sequence $\{a^{ih}\}$ is itself bounded. The asymptotic cone of the above sequence reduces to the origin. Without loss of generality, assume that the sequence in question converges to a^{i0}. By the continuity of D^{i}, this implies

$$a^{i0} \in D^i(p'),$$

leading to a contradiction to the assumption and Lemma 2, since $p' \in$ bdry Δ. (Q. E. D.)

Now, we are ready to discuss the existence of the short run equilibrium. First, let us assume

Assumption 3. $w^i_x > 0$

$$w^i_y + w^i_z > 0.$$

Assumption 4. There exist $\epsilon > 0$ and $\eta > 0$ such that

$$\int_0^{\epsilon} d\psi^i(\theta,\ p) < 1 - \eta$$

for all $p \epsilon \Delta$ and all $m^i \epsilon R_+$.

Assumption 3 is a kind of cheaper point assumption. But we should pay attention to the fact that either w_y or w_z (but not both) can be zero. Assumption 4 requires agent i to believe that the proportion θ of the desired good (good y) in the amalgam is always positive with positive probability. In other words, agent i is optimistic.

A short run competitive equilibrium is an array of a price vector $p^* \epsilon \Delta$ and a set of actions $(a^{1*}, \ldots, a^{n*})$ such that

(3) $a^{i*} \epsilon D^i(p^*),\ i = 1, \ldots, n$

(4) $\sum_{i=1}^{n} x^{i*} \leq \sum_{i=1}^{n} w^i_x$

$$\sum_{i=1}^{n} (y^{i*} + z^{i*} + m^{i*}) \leq \sum_{i=1}^{n} (w^i_y + w^i_z)$$

(5) $p^*_x \left(\sum_{i=1}^{n} (x^{i*} - w^i_x) \right) = 0$

$$p^*_m\left(\sum_{i=1}^{n}\{(y^{i*}+z^{i*}+m^{i*})-(w^i_y+w^i_z)\}\right)=0.$$

Condition (3) implies that each agent maximises his expected utility within his own budget constraint (a rational behavior). Condition (4) is called material balance, meaning that aggregate demand cannot exceed aggregate supply. And condition (5) is the Kuhn-Tucker condition.

Theorem 1. There exists a short run competitive equilibrium with a strictly positive price vector for a pure exchange economy that satisfies assumptions 1 to 4.

(Proof) For each $p\epsilon\Delta$, let

$$\zeta(p)=\begin{pmatrix}\sum_{i=1}^{n}(x^i(p)-w^i_x)\\ \sum_{i=1}^{n}\{(y^i(p)+z^i(p)+m^i(p))-(w^i_y+w^i_z)\}\end{pmatrix}$$

be the aggregate excess demand correspondence. All the assumptions made guarantees that this is convex, compact and upper semicontinuous and moreover that this satisfies Walras' law[1]. Therefore, we can apply Grandmont's Market Equilibrium Lemma [Grandmont (1982, Lemma 1, page 888)] to obtain the result. (Q. E. D.)

3. Revision of expectation and long run equilibrium

Although the proportion θ is unknown parameter to them before consumption, buyers become aware of the exact value of θ after consumption. Namely, consumers can gather information about θ at a short run equilibrium and will revise their expectations accordingly. A long run equilibrium will be a state of economy such that the revision process leads to the stationary expectation and then to the same short run equilibrium in all future periods. Though the formulation of revision process might be very important, here we assume a simple version[2]; at the end of each period, consumers know the short run equilibrium price vector p_t and the market proportion $\bar{\theta}_t$ of

1) I. e., $pz=0$ for all $z\epsilon\zeta(p)$ and $p\epsilon\Delta$.

2) Fuchs (1977) is the first rigid treatment of evolution of expectation functions in the framework of temporary general equilibrium.

good y in the amalgam. They revise expectation functions according to the process

(6) $\psi^i_{t+1}(\cdot,\cdot)=H^i[\psi^i_t(\cdot,\cdot),\ p_t,\ \bar{\theta}_t]$.

Since the expectation function $\psi^i(\cdot,\cdot)$ is a mapping from Δ into $\mathfrak{M}(I)$, let $\mathfrak{S}$ be the space of continuous functions from Δ into $\mathfrak{M}(I)$. We endow $\mathfrak{S}$ with the compact open topology[3].

Consider a sequence of short run equilibria. During each period, agents are assumed to have the same endowments and the same von Neumann-Morgenstern utility functions[4].

A long run competitive equilibrium is an array of a price vector $p^*\epsilon\Delta$, a market proportion $\bar{\theta}^*$ of good y in amalgam, a set of actions $(a^{1*},\dots,a^{n*})$, and a set of expectation functions $(\psi^{1*}(\cdot,p),\dots,\psi^{n*}(\cdot,p))$ such that

(7) $\psi^{i*}(\cdot,p^*)=H^i[\psi^{i*}(\cdot,p^*),\ p^*,\ \bar{\theta}^*]$ for $i=1,\dots,n$

(8) $v^i(a^{i*},p^*,\psi^{i*})=\text{Max } v^i(a,p^*,\psi^{i*})$ for $a\epsilon\Gamma^i(p^*)$

(9) $\sum_{i=1}^{n}x^{i*}\leq\sum_{i=1}^{n}w^i_x$

$$\sum_{i=1}^{n}(y^{i*}+z^{i*}+m^{i*})\leq\sum_{i=1}^{n}(w^i_y+w^i_z)$$

(10) $p^*_x\left(\sum_{i=1}^{n}(x^{i*}-w^i_x)\right)=0$

$$p^*_m\left(\sum_{i=1}^{n}\{(y^{i*}+z^{i*}+m^{i*})-(w^i_y+w^i_z)\}\right)=0.$$

Assuption 5. The revision process H^i is continuous from $\mathfrak{S}\times\Delta\times I$ into $\mathfrak{S}$, $i=1,\dots,n$.

Note that the Belnoulli index here is difined as

(1′) $v^i(a,p,\psi^i)=\int_I u^i(x,\ y+\theta m,\ z+(1-\theta)m)\ d\psi^i(\theta,p)$.

Lemma 6. The mapping v^i defined by (1′) is continuous from $A^i\times\Delta\times\mathfrak{S}$ into R, $i=1,\dots,n$.

3) The compact open topology is the weakest topology such that if $\{p^\nu\}_\nu$ converges to p^* and $\{\psi^i_\nu(\cdot,p)\}_\nu$ to $\psi^{i*}(\cdot,p)$ in the topology, then $\{\psi^i_\nu(\cdot,p^\nu)\}_\nu$ converges to $\psi^{i*}(\cdot,p^*)$ in the weak topology. See Kelly (1955).

4) This assumption is made for simplicity.

(Proof) Consider converging sequences $\{a_\nu\}_\nu \to a$, $\{p^\nu\} \to p$, $\{\psi^i_\nu\} \to \psi^i$. From the joint continuity, we know that $\{\psi^i_\nu(\theta, p^\nu)\}_\nu \to \psi^i(\theta, p)$ in the weak topology [see Parthasarathy (1967)]. Since $\{u^i(x_\nu, y_\nu + \theta\ m_\nu, z_\nu + (1-\theta)\ m_\nu)\}$ converges continuously to $u^i(x, y+\theta\ m, z+(1-\theta)m)$ and is uniformly bounded, we obtain the result applying Grandmont's (1972) lemmata. (Q. E. D.)

Using the Arrow and Debreu (1954) compactification, we can show the following.

Lemma 7. Under assumptions 1 to 5, the trancated action correspondence $\bar{a}^i(p, \psi^i)$, $i=1, \dots, n$, is upper semicontinuous, nonempty, and convex valued from $\Delta \times \mathfrak{S}$ into a fixed compact set.

Let

$$A = \prod_{i=1}^{n} [0,\ w^i_y] \times \prod_{i=1}^{n} [0,\ w^i_z].$$

The market proportion of good y is a function of actions taken by traders. Let λ_θ be the correspondence from A into I defined as follows:

$$(11) \quad \lambda_\theta(y^1, \dots, y^n, z^1, \dots, z^n) = \begin{cases} [0,\ 1], & \text{if both } y \text{ and } z \text{ markets are cleared.} \\ \dfrac{\sum_{i=1}^{n} (w^i_y - y^i)}{\sum_{i=1}^{n} (w^i_y - y^i) + \sum_{i=1}^{n} (w^i_z - z^i)}, & \text{otherwise.} \end{cases}$$

The correspondence λ_θ is obviously nonepmty, compact, convex and upper semicontinuous.

We identify a probability measure $\mu^i(\theta)$ on I with a constant expectation function $\psi^i(\theta, p)$ such that $\psi^i(\theta, p) = \mu^i(\theta)$ for all $p \in \Delta$. To each measure $\mu^i(\theta)$, we associate a new measure λ^i in the following way. Regarding μ^i as a constant expectation function, we associate through H^i a new expectation function. We evaluate this expectation function at p, and we consider the constant expectation function that has this value. Assumption 5 implies that λ^i is continuous from $\mathfrak{S}_R$ into itself, where $\mathfrak{S}_R$ is the subset of constant functions in $\mathfrak{S}$.

Here we introduce an auctioneer whose task is to maximise $p\ z$ in Δ, where $z \in \zeta(p)$. Let λ_p be the correspondence that associates to each $(x^i, y^i,$

z^i, m^i), $i=1, \ldots, n$, the set of maxmising prices. It is known to be an upper semicontnuous nonempty, convex, compact-valued correspondence.

Let us construct the correspondence ϕ from $A:=\Delta\times\sum_{i=1}^{n}\mathfrak{S}_R^i\times I\times C$ into itself, where C is a compact ball define by Arrow and Debreu (1954):

λ_θ: $(y^1, \ldots, y^n, z^1, \ldots, z^n)\to\bar{\theta}$

λ_{a^i}: $(p, \mu^i)\to a^i$　　　$i=1, \ldots, n$

λ^i: $(\mu^i, \bar{\theta}, p)\to\mu^i$　　　$i=1, \ldots, n$

λ_q: $(a^1, \ldots, a^n)\to p$　　　$i=1, \ldots, n$

Theorem 2. Under assumptions 1 to 5, there exists a long run competitive equilibrium.

(Proof) By Ascoli's theorem [Kelley (1955, page 233)], $\mathfrak{S}_R$ is compact. Then A is a compact and convex set. Since ϕ is an upper semicontinuous, nonempty, convex, compact valued correspondence, the Schauder-Tychonoff theorem implies that it has a fixed point. It is immediate to show the fixed point turns out a long run competitive equilibrium. (Q. E. D.)

Remark: This long run equilibrium will stay at the equilibrium price and consumers' expectations will be fulfilled if the auctioneer announces the equlibrium price vector at the beginning of each period. But the stability properties of this kind may change over time and we cannot expect a long run equilibrium with "invariant" stability properties without further restrictive assumptions, where "invariant" means that the entire expectation function of every agent is stationary [see Fuchs (1976) for further discussion].

For a short run equilibrium to be meaningful, we need to show that the set of equilibrium prices and equilibrium actions behave continuously for continuous changes of expectation functions. Let $\mathfrak{S}$ be the short run equilibrium correspondence from $\prod_{i=1}^{n}\mathfrak{S}^i$ into $\Delta\times C$.

Theorem 3. $\mathfrak{S}$ is an upper semicontinuous compact valued, and nonempty correspondence.

(Proof) The nonemptyness of $\mathfrak{S}$ follows from Arrow and Debreu (1954) and lemma 7. Consider a sequence $\{(\psi^1_\nu, \ldots, \psi^n_\nu)\}_\nu$ of expectation functions converging to $(\psi^{1*}, \ldots, \psi^{n*})$ and a sequence $\{(p^\nu, a^1_\nu, \ldots, a^n_\nu)\}_\nu$ of corresponding short run equilibria converging to $(p^*, a^{1*}, \ldots, a^{n*})$. Since $(a^1_\nu, \ldots, a^n_\nu)$ satisfies the feasibility condition for each ν, so does $(a^{1*}, \ldots, a^{n*})$. Since the trancated action correspondence $\bar{d}^i(p, \psi^i)$ is continuous for $i=1, \ldots, n$, $a^i_\nu \epsilon \bar{d}^i(p^\nu, \psi^i_\nu)$ implies $a^{i*} \epsilon d^i(p^*, \psi^{i*})$ for $i=1, \ldots, n$. Since the concavity of v^i implies that this fixed point $(p^*, a^{1*}, \ldots, a^{n*})$ is a short run equilibrium, $\mathfrak{S}$ is a closed graph correspondence. Here $\Delta \times C$ is compact, then the result follows. (Q. E. D.)

4. Costly information and efficiency

Buyers may want to determine the quality of purchase before they buy them. To allow for this possibility, we introduce a special kind of costly information in the economic system. Let us difine a unit of information as the possibility of identifying in the market one unit of good y out of the amalgam purchased. And suppose that each bit of information is sold by experts and that a consumer can hire them to shop for him. In other words, the expert is an identification technology with fixed coefficient. Let c the cost to identify one unit of good y.

Writing e^i the quantity of good y bought through experts, the maximising problem of consumer i is;

(12) Max $v^i(x^i, y^i, z^i, m^i, e^i)$

$$= \int_I u_i(x^i, y^i + e^i + \theta m^i, z^i + (1-\theta)m^i) d\psi^i(\theta, p)$$

subject to $p_x x^i + (p_m + c)e^i + p_m m^i \leq p_x w^i_x + p_m(w^i_y - y^i) + (w^i_z - z^i)$

$$x^i, y^i, z^i, m^i, e^i \geq 0$$

$$y^i \leq w^i_y$$

$$z^i \leq w^i_z$$

A short run competitive equilibrium with expert is an array of a price vector $p^* = (p^*_x, p^*_m) \epsilon \Delta$ and a set $(a^{1*}, \ldots, a^{n*})$ of actions that are solutions to the consumers' maximisation problem (12) such that

(13) $\sum_{i=1}^{n} (x^{i*} + c\ e^{i*}) \leq \sum_{i=1}^{n} w^i_x$

$$\sum_{i=1}^{n}(y^{i*}+e^{i*})\leq\sum_{i=1}^{n}w_y^i$$

$$\sum_{i=1}^{n}z^{i*}\leq\sum_{i=1}^{n}w_z^i$$

$$(14)\quad \sum_{i=1}^{n}(y^{i*}+z^{i*}+m^{i*}+e^{i*})\leq\sum_{i=1}^{n}(w_y^i+w_z^i)$$

$$(15)\quad p_x^*\left(\sum_{i=1}^{n}(x^{i*}+c\ e^{i*}-w_x^i)\right)=0$$

$$p_m^*\left(\sum_{i=1}^{n}(y^{i*}+z^{i*}+m^{i*}+e^{i*}-w_y^i-w_z^i)\right)=0.$$

The above formulation of experts seems to be satisfactory only if there is perfect competition in the production of information and if it costs no more than c to find the right commodity once the consumer can identify the good. Although the search cost is in fact random, we will use a certainty equivalent approach for simplicity.

Assumption 6. The information cost c is a positive continuous function of the excess supply $s_y=\sum_{i=1}^{n}(w_y^i-y^i)$ of good y defined on $(0, \sum_{i=1}^{n}w_y^i)$. Moreover, $c(s_y)$ goes to infinity as s_y approaches to zero.

Theorem 4. There exists a short run competitive equilibrium with experts which has a strictly positive price vector under assumptions 1 to 4 and 6.

(Proof) Let $\bar{R}_+$ be the compactified positive real line. Then $c(s_y)$ is considered a function from the compact convex set $[0, \sum_{i=1}^{n}w_y^i]$ into $\bar{R}_+$. Consider the correspondence ζ from $\Delta\times\bar{R}_+\times\bar{R}_+$ into R^3 defined as follows;

$$(16)\quad \zeta(p,\ c)=\begin{pmatrix}\sum_{i=1}^{n}(x^i(p,\ c)-w_x^i)\\ \sum_{i=1}^{n}(y^i(p,\ c)+z^i(p,\ c)+m^i(p,\ c)+e^i(p,\ c)-w_y^i-w_z^i)\end{pmatrix}$$

Then the procedure similar to that of theorem 1 can be applied. (Q. E. D.)

Remark: This result allows equilibria with an infinite search cost, which is another form of market breakdown [Akerlof (1970)].

To deal with efficiency we have to compare alternative economic systems:

a competitive economy with the amalgam market, the same economy with experts, and others. An economic system is defined as a set of institutions, a choice of economic behaviors, and a concept of equilibrium. It may be summarized by the set of its equilibria. Then, an economic system A is said to be efficient if there exists no other feasible economic system B such that an equilibrium of A is Pareto inferior to an equilibrium of B.

In the classical theory of value the largest feasible set is defined by technological constraints. But here it is unrealistic to consider reaching technological efficiency because of the dishonesty of sellers who do not reveal the quality of what they sell. If extraeconomic penalties were available, one might attempt to force honest behavior by fear of high penalties, and we would then have to take into account the psychological and processing costs of such measures[5]. For the moment, let us assume that we are constrained by the dishonesty of sellers.

Because of the remaining uncertainty, we cannot define allocation *ex ante*; we can only difine imputations in terms of expected utility (Belnoulli index). It is appropriate to compare the *ex ante* imputations for the long run equilibria in the different institutional frameworks, since expected imputations will correspond in the long run to the "average" imputations. Unfortunately, our conclusion concerning efficiency is premature. If we donot specify expectation functions in the definition of economic systems but simply say that the expectations of consumers must be fulfilled at the equilibrium (the rational expectation hypothesis), then it can happen that the equilibrium with experts is Pareto inferior to the equilibrium without experts. The reason for this result is that the availability of experts leads to the direct purchase of good y so that expectations about the remaining amalgam must be scaled down to be fulfilled. Meanwhile resources have been used to hire the experts. The decentralized behavior of consumers induces the purchase of expert service and leads to this Pareto inferior state. There may exist excessive purchase of information by competitive agents

5) This problem is similar to that of "free rider" in the allocation of pulic good. For the extraeconomic penalties, the same difficulties apply as the Groves mechanism.

such as those suggested by Hirshleifer (1971). Experts might decrease uncertainty with positive effects: thus the trade-off between risk aversion and information costs would be more complicated.

The aim is to study the consequences of making costly information available to consumers in a framework of perfect competition, namely, on the assumption that sellers and buyers are unidentified. The concept of information is not always meaningful without an associated concept of search cost. However, it appears likely that making information available to buyers will encourage sellers to differentiate themselves. If such a differentiation is possible, then the competitive equilibrium might not be the right tool for analyzing this kind of problems.

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