Asset Bubbles in a Simple Model of Endogenous Growth*

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Possibility of competitive equilibria with bubbles is examined in the endogenous growth model of the Diamond (1965) economy. The results show that bubbly equilibria exist and bubbles reduce the speed of economic growth.

1. Introduction

For some time, it has been discussed whether the market price of an asset can exceed its fundamentals (i.e., the present value of dividends). In an overlapping-generations model, Tirole (1985) has shown that there is a competitive equilibrium in which an asset is priced over its fundamentals. The condition he derived for bubbly equilibria is that an economy grows at a higher rate than the interest rate in almost every period. Notice that his analysis is based on the Diamond (1965) economy of exogenous growth so that there is no productivity (i.e., per capita income) growth in the long run. At

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the same time, the total output and capital stock grow at an exogenous rate.

This note examines the possibility of bubbly equilibria in an endogenous growth model of the Diamond (1965) economy. Also the effects of bubbles on the growth rate are studied. Our results show the possibility of competitive equilibria with bubbles in the Diamond economy. However, for almost every case, the size of bubbles decreases asymptotically along a competitive path. Furthermore, the existence on bubbles reduces the endogenously determined growth rate.

The other studies in the same direction as ours include Michel (1992) and Yanagawa-Grossman (1992). Although there are several differences in details, there is a common result among Michel (1992), Yanagawa-Grossman (1992), and ours. That is, the presence of bubbles reduces the speed of economic growth.

2. Model

This section provides a brief description of our model and assumptions.

Consumers
In each period \( t (t = 0, 1, 2, \ldots) \), a representative consumer is born and lives for two periods of \( t \) and \( t+1 \). While she is young, she earns the wage income, \( w_t \), from her price-inelastic supply of labor. When she is old, she is retired from the labor market. Although some of the income, \( w_t \), is spent for the current consumption, \( c_t \), the remaining amount is saved for her future consumption, \( c_t' \). Given the budget constraint
(1) \( c_t^\sigma = (1 + r_{t+1})(w_t - c_t^\sigma) \),

the consumer chooses a consumption profile \((c_t^\sigma, c_t^\gamma)\) to maximize her utility \(u = u(c_t^\sigma, c_t^\gamma)\). By assuming homotheticity for \(u(\cdot, \cdot, \cdot)\), we obtain the homogeneity of degree one in income for the saving function

(2) \( \forall \lambda > 0, \ s(\lambda w_t, r_{t+1}) = \lambda s(w_t, r_{t+1}) \)

where \(s(w_t, r_{t+1}) = w_t - c_t^\sigma(w_t, r_{t+1})\).

**Firms**

To produce the output, firms demand both labor and capital stock as inputs. While labor service, \(L(=1)\), is supplied by the young generation, the old lends the capital stock, \(K_t\), to the firms. The production technology is described by a constant returns to scale production function

(3) \( Y_t^\sigma = F(K_t, A(K_t)L), A'(\cdot) > 0 \).

Notice that \(A(K)\) represents the productivity improvements due to capital accumulation, which is a common feature in the endogenous growth models such as Romer (1986).

Although an increase in \(K\) improves the labor productivity at the aggregate level, each firm takes accounts of its private marginal products. Thus, the profit maximization of an individual firm leads to the first-order conditions:

(4) \( r_{t+1} = f'(k_t) \),
where \( k_t = K_t / A(K_t) \) and \( f(.) = (K_t / A(K_t))L, \) 1).

Besides the above assumptions, the following condition (6) is assumed to simplify our analysis.

(6) \( A(K_t) = K_t / a. \)

Substitution of (6) into (4) and (5) yields

(7) \( r_{t+1} = f'(a) = r \) for all \( t, \)

(8) \( w_t = f(a) - af'(a) = w \) for all \( t. \)

3. Bubbles and Growth

Consider an intrinsically worthless paper asset which is held by the old generation at the period 0. The asset, which does not yield any dividends, can be traded to the young generation only if the young believe they can resell the asset to the next generation at the price of comparable assets. We define \( B_t \) as the aggregate of the intrinsically worthless asset the young generation would buy at the period \( t. \) Because of this asset, the young hold their wealth in the form of capital stock and the paper asset:

(9) \( s(A(K_t)Lw_t, r_{t+1}) = K_{t+1} + B_t. \)

Using the homogeneity (2) as well as (6) - (8), we may rewrite (9) as
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$$K_{t+1} = A(K_t) L [s(w_t, r_{t+1})] - b_t,$$  where $b_t = B_t / A(K_t)L$,

$$K_{t+1} / K_t = [s(w, r) - b_t] / a,$$

(10) \( g_t = (K_{t+1} - K_t) / K_t = \frac{s(w, r) - b_t}{a} - 1 \)

which gives us the endogenously determined growth rate in the model.

Now we turn to the conditions in which bubbles are sustained. First of all, the paper asset has to earn the same rate of returns as other assets: $B_{t+1} = (1 + r_{t+1}) B_t$. Secondly, the equilibrium condition (9) or (10) should be satisfied. Immediately we get

![Figure 1](image-url)
\[ (1) \quad b_{t+1} = b_t \frac{1 + r}{1 + g_t}. \]

Finally, a difference equation on \( \{b_t\} \) is derived from (10) and (11).

\[ (12) \quad b_{t+1} = \frac{a(1 + r)b_t}{s(w, r) - b_t}. \]

A typical path \( \{b_0, b_1, b_2, \ldots\} \) generated by (12) is illustrated on the Figure 1.

From (12), we obtain

**Proposition 1:**

A competitive equilibrium with a positive sequence \( \{b_t\} \) can exist only if \( 1 + r < s(w, r)/a. \)

**Proof:** It is clear from the context and omitted here. The same applies for the rest of our propositions.

**Proposition 2:**

If \( b_0 > b^*=s(w, r)-a(1+r) \), then any path \( \{b_t\} \) with the initial value \( b_0 \) is infeasible.

**Proposition 3:**

If \( 0 < b_0 < b^* \), then a path \( \{b_t\} \) is (i) monotonically decreasing, and (ii) converging to 0.
Proposition 4:

There are two steady states. One is the equilibrium with no bubbles. The other is the one with $b_t = b^*$ for all $t$.

Turning to the effects of bubbles on the growth rate, we have the following proposition from (10).

Proposition 5:

The growth rate along a bubbly path is increasing over time. However, it is lower than the rate at the equilibrium with no bubbles:

$$g_t = \frac{s(w, r) - b_t}{a} - 1 < \frac{s(w, r)}{a} - 1.$$

Proposition 5 simply reflects the fact that the bubble is a transfer of wealth from the old generation to the young, and is not a productive asset in our model.
REFERENCES


